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ALGEBRA OF THE CURRENTS AND DISPERSION RELATIONS

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ALGEBRA OF THE CURRENTS AND DISPERSION RELATIONS

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Recently great success has been achieved in studying the form-factor properties by the methods based on the algebra of charges and dispersion relations [1], on the one hand, and the algebra of the current density [2], on the other. In this paper we shall show the equivalence of both methods, in particular with respect to the so-called "summation rule". In fact, we shall consider a more general case of the algebra generated by the current density Fourier components which in the limit of zero momentum reduces to the usual algebra of charges.

Following [1] we shall postulate for current-like operators A , B , and C equal time commutators of the type

$$[A_\lambda(x),\ B(y)]|_{x_0=y_0} = \delta(\vec{x}-\vec{y})C_\lambda(x)\ . \qquad (1)$$

Then, by using invariance under translations and expansion in a complete set of the state vectors we are led to the following equations for the matrix elements:

$$\begin{aligned}\langle q|C_\lambda(0)|p\rangle &= \sum_n (2\pi)^3\delta(\vec{p}+\vec{k}-\vec{q})\langle q|A_\lambda(0)|n\rangle\langle n|B(0)|p\rangle - \\ &-\sum_n (2\pi)^3\delta(\vec{p}_n-\vec{k}-\vec{p})\langle q|B(0)|n\rangle\langle n|A_\lambda(0)|p\rangle\ .\end{aligned} \qquad (2)$$

Now let us show, that the dispersion approach gives the same result (2). For this purpose we define the operators

$$Q(\vec{k}) = \int A_0(\vec{x},0)e^{-i\vec{k}\vec{x}}\,d^3x$$

$$Q^r(k) = \int \theta(x_0)\partial_\lambda(A_\lambda(x)e^{ikx})\,d^4x$$

$$Q^a(k) = \int \theta(-x_0)\partial_\lambda(A_\lambda(x)e^{ikx})\,d^4x$$

One can see that

$$Q^r(k) = -Q(\vec{k}) \qquad \text{if} \quad \mathrm{Im}\,k_0 > 0$$

$$Q^{a}(k) = Q(\vec{k}) \qquad \text{if } \operatorname{Im} k_0 < 0 \ . \tag{3}$$

Using Eqs. (1) and (3) we get

$$\langle q | C_0(0) | p \rangle = \pm \int \theta(\mp x \ ) \partial_\lambda (\langle q | [A_\lambda(x), B(0)] | p \rangle e^{ikx} \ ) d^4x \tag{4}$$

$$\text{if } \pm \operatorname{Im} k_0 < 0 \ .$$

Note, that the right-hand side of this equation does not depend on k and in this sense it is a trivial analytical function. But we can obtain nontrivial analytical properties, if define two auxiliary functions

$$T^{r,a}(\ell, k) = \mp \int \theta(\pm x_0) e^{i\ell x} \partial_\lambda (\langle q | [A_\lambda(x), B(0)] | p \rangle e^{ikx}) d^4x \ . \tag{5}$$

In the limit $\ell \to 0$ this equality goes over into Eq.(4). It is convenient to rewrite the expression (5) for $T^{r,a}$ in the form

$$T^{r,a}(\ell, k) = T_1^{r,a}(\ell + k) + ik_\lambda T_{2\lambda}^{r,a}(\ell + k) \ ,$$

where

$$T_1^{r,a}(\ell + k) = \mp \int \theta(\pm x_0) e^{i(\ell+k)x} \partial_\lambda \langle q | [A_\lambda(x), B(0)] | p \rangle d^4x$$

$$T_{2\lambda}^{r,a}(\ell + k) = \mp \int \theta(\pm x_0) e^{i(\ell+k)x} \langle q | [A_\lambda(x), B(0)] | p \rangle d^4x \tag{6}$$

The local commutativity guarantees the relativistic invariance of the functions (6) and allows to perform their analytical continuation into the complex plane of argument $\nu = \frac{q}{m}(\ell + k)$, where m is the mass of the particle, whose momentum is q . This leads to a dispersion relation for the functions

$$\operatorname{Re} T(\nu) = \frac{P}{\pi} \int_{-\infty}^{\infty} \frac{\operatorname{Im} T(\nu')}{\nu' - \nu} d\nu' \ . \tag{7}$$

The translation invariance and completeness of the state-vector set lead to the following expression

$$\operatorname{Im} T_{2\lambda} = \frac{1}{2i} \Sigma (2\pi)^4 \delta(\ell + k + q - p_n) \langle q | A_\lambda(0) | n \rangle \langle n | B(0) p \rangle - \\ - \frac{1}{2i} \sum_n (2\pi)^4 \delta(\ell + k - p + p_n) \langle q | B(0) | n \rangle \langle n | A_\lambda(0) | p \rangle \tag{8}$$

and

and

$$\mathrm{Im}\, T_1 = \frac{1}{2} \sum_n (2\pi)^4 \delta(\ell + k + q - p_n) \langle q | (q - p_n)_\lambda A_\lambda(0) | n \rangle \langle n | B(0) | p \rangle -$$

$$- \frac{1}{2} \sum_n (2\pi)^4 \delta(\ell + k - p + p_n) \langle q | B(0) | n \rangle \langle n | (p_n - q)_\lambda A_\lambda(0) | p \rangle \quad .$$

Then the substitution of (8) into (7) gives in the limit $\ell \to 0$ the result (2).

So, the algebra of charge together with the local commutativity yields the same results as the algebra of current density.

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