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THE ORBITAL FRACTIONAL PARENTAGE COEFFICIENTS FOR TWO AND THREE NUCLEONS IN THE 2 SLD SHELL IN THE SU(3) CLASSIFICATION SCHEME

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It is well-known [1], that it is possible to introduce a basis in the space of orbital wave functions for k nucleons in the 2 s|d shell of the harmonic oscillator, so that under transformations belonging to $U(6) \supset SU(3) \supset SO(3)$ the wave functions of this basis

$$\phi_{1\ldots k}([f]\beta\,(\lambda)\alpha LM(r)) \qquad (1)$$

transform according to the irreducible representations[x)] $[f]$, (λ) and L of these groups. The transformation properties of these functions with respect to nucleon permutations are determined by the Yamanouchi symbol r. Here α and β are the so-called additional quantum numbers.

The wave functions (1) may be expressed in terms of those for k-1 nucleons by means of the orbital fractional parentage coefficients

$$\phi_{1\ldots k}([f]\beta(\lambda)\alpha LM|r) =$$

$$= \sum_{\bar\beta(\bar\lambda)\bar\alpha\bar L\bar\ell} ([\bar f]\bar\beta(\bar\lambda)\bar\alpha\bar L;\bar\ell\,|\}[f]\beta(\lambda)\alpha L)[\phi_{1\ldots k-1}([\bar f]\bar\beta(\bar\lambda)\bar\alpha\bar L(\bar r))\phi_k(\bar\ell)]^L_M \,. \qquad (2)$$

These coefficients are the products of two factors, each of which depends only on a smaller number of variables and has a simple group-theoretical meaning :

$$([\bar f]\bar\beta(\bar\lambda)\bar\alpha\bar L;\bar\ell\,|\}[f]\beta(\lambda)\alpha L) =$$

$$= ([\bar f]\bar\beta(\bar\lambda);[1](20)|[f]\beta(\lambda))((\bar\lambda)\bar\alpha\bar L;(20)\bar\ell\,|(\lambda)\alpha L). \qquad (3)$$

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x) We denote and irreducible representation of $SU(3)$ $(\lambda) = (\lambda_1\lambda_2)$ instead of Elliot's notation $(\lambda\mu)$. As far as $SO(3)$ is concerned, we follow Fano and Racah [2], except of some unimportant changes in notation.

Table 1

k = 3 orbital fractional parentage coefficients

					$\bar{I}=2$											$\bar{I}=0$									
			$\bar{F}$ $[\bar{f}]$			(21) [11]				(11) [2]							(21) [11]				(11) [2]				
			$(\bar{\lambda})$			(21)				(40)			(02)				(21)				(40)			(02)	
r	[f]	(λ)	L / $\bar{L}$		F	D	P		G	D	S		D	S		F	D	P		G	D	S		D	S
(321)	[111]	(30)	F		$2\sqrt{\frac{1}{3\cdot5}}$	$\frac{2}{3}$	$\sqrt{\frac{1}{3\cdot5}}$									$-\frac{1}{3}\sqrt{2}$									
			P		$-\frac{2}{3}\sqrt{\frac{2\cdot7}{3\cdot5}}$	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{2^3}{3}\sqrt{\frac{1}{3\cdot5}}$											$-\frac{1}{3}\sqrt{\frac{1}{3}}$							
		(03)	F		$2\sqrt{\frac{2}{3\cdot5}}$	$-\frac{1}{3}\sqrt{2}$	$\sqrt{\frac{2}{3\cdot5}}$									$\frac{1}{3}$									
			P		$\frac{1}{3}\sqrt{\frac{7}{3\cdot5}}$	$-\frac{2^2}{3}\sqrt{\frac{1}{3}}$	$-\frac{2}{3}\sqrt{\frac{2}{3\cdot5}}$											$-\frac{2}{3}\sqrt{\frac{2}{3}}$							
(211) (121)	[21]	(41)	H		1				1																
			G		$\sqrt{\frac{1}{2\cdot3}}$	$\sqrt{\frac{5}{2\cdot3}}$			$-\frac{1}{3}\sqrt{\frac{11}{2\cdot7}}$	$-\sqrt{\frac{5}{2\cdot7}}$										$\frac{1}{3}\sqrt{5}$					
			F		$-\sqrt{\frac{1}{2\cdot5}}$	$\sqrt{\frac{1}{2\cdot3}}$	$\sqrt{\frac{2}{5}}$		$-\sqrt{\frac{5}{2\cdot7}}$	$3\sqrt{\frac{1}{2\cdot7}}$						$\sqrt{\frac{1}{3}}$									
			D		$-\frac{2}{3}\sqrt{\frac{1}{5}}$	$-\frac{1}{3}\sqrt{\frac{1}{2}}$	$\sqrt{\frac{7}{3\cdot5}}$		$2\sqrt{\frac{3}{5\cdot7}}$	$\sqrt{\frac{1}{2\cdot3\cdot7}}$	$-\sqrt{\frac{7}{3\cdot5}}$						$\frac{1}{3}\sqrt{\frac{7}{2}}$				$\sqrt{\frac{1}{2\cdot3}}$				
			P		$\frac{2^2}{3}\sqrt{\frac{1}{3\cdot5}}$	$-\frac{1}{3}\sqrt{\frac{7}{3}}$	$\frac{1}{3}\sqrt{\frac{2\cdot7}{3\cdot5}}$			-1								$\frac{1}{3}\sqrt{\frac{2\cdot7}{3}}$							
		(22)	G		$\sqrt{\frac{5}{2\cdot3}}$	$-\sqrt{\frac{1}{2\cdot3}}$			$\sqrt{\frac{11}{2\cdot7}}$	$\sqrt{\frac{1}{2\cdot5\cdot7}}$			1							$\sqrt{\frac{1}{5}}$					
			F		$\sqrt{\frac{1}{2\cdot5}}$	$\sqrt{\frac{1}{2\cdot3}}$	$-\sqrt{\frac{2}{5}}$		$-3\sqrt{\frac{1}{2\cdot7}}$	$-\sqrt{\frac{5}{2\cdot7}}$			1			$\sqrt{\frac{1}{3}}$									
			D′	-1	$-\sqrt{\frac{7}{2\cdot3\cdot5}}$	$\frac{1}{2}\sqrt{\frac{7}{3}}$	$\sqrt{\frac{1}{2\cdot3}}$	$\frac{\sqrt{5}}{3}$	$-\frac{3}{5}\sqrt{\frac{1}{2}}$	$\frac{3}{2}\sqrt{\frac{1}{5}}$	$\frac{1}{5}\sqrt{\frac{1}{2}}$	$-\frac{2}{3}$		$-\sqrt{\frac{1}{2}}$	-1		$-\frac{1}{2}\sqrt{\frac{1}{3}}$		$\frac{\sqrt{5}}{3}$		$\frac{1}{2}\sqrt{\frac{7}{5}}$		$-\frac{2}{3}$	$\sqrt{\frac{1}{2}}$	
			D″		$-\frac{3}{5}\sqrt{\frac{1}{2}}$	$\frac{1}{2}\sqrt{\frac{1}{5}}$	$-\frac{1}{3}\sqrt{\frac{3\cdot7}{2}}$		$3\sqrt{\frac{3}{2\cdot5\cdot7}}$	$\frac{5}{2}\sqrt{\frac{1}{3\cdot7}}$	$\sqrt{\frac{7}{2\cdot3\cdot5}}$		$2\sqrt{\frac{2}{3\cdot5}}$	$\sqrt{\frac{7}{2\cdot3\cdot5}}$			$\frac{1}{2}\sqrt{\frac{7}{3}}$				$-\frac{1}{2}\sqrt{\frac{1}{3}}$			$\sqrt{\frac{7}{2\cdot3\cdot5}}$	
			S			-1				$-\sqrt{\frac{7}{3\cdot5}}$			$\sqrt{\frac{1}{2\cdot3}}$									$2\sqrt{\frac{2}{3\cdot5}}$			$-\sqrt{\frac{5}{2\cdot3}}$
		(11)	D	-1	$\frac{2^2}{3\cdot5}\sqrt{7}$	$\frac{1}{3}\sqrt{\frac{2\cdot7}{5}}$	$\frac{1}{5}\sqrt{\frac{1}{3}}$						$\sqrt{\frac{7}{3\cdot5}}$	$2\sqrt{\frac{1}{3\cdot5}}$	-1		$\frac{2}{3}\sqrt{\frac{2}{5}}$							$-2\sqrt{\frac{1}{3\cdot5}}$	
			P		$-\frac{2}{3}\sqrt{\frac{2\cdot7}{3\cdot5}}$	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$-\frac{1}{3}\sqrt{\frac{7}{3\cdot5}}$						1					$\frac{2}{3}\sqrt{\frac{1}{3}}$							
(111)	[3]	(60)	I						1																
			G						$-\frac{2}{3}\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2\cdot11}{5\cdot7}}$										$\frac{1}{3}\sqrt{\frac{11}{5}}$					
			D						$\frac{2^2}{5}\sqrt{\frac{1}{7}}$	$-2\sqrt{\frac{2}{5\cdot7}}$	$\frac{1}{5}\sqrt{7}$										$\sqrt{\frac{2}{5}}$				
			S							$2\sqrt{\frac{2}{3\cdot3}}$												$\sqrt{\frac{7}{3\cdot5}}$			
		(22)	G						$\sqrt{\frac{11}{2\cdot7}}$	$\sqrt{\frac{1}{2\cdot5\cdot7}}$			1							$\sqrt{\frac{1}{5}}$					
			F						$-3\sqrt{\frac{1}{2\cdot7}}$	$-\sqrt{\frac{5}{2\cdot7}}$			1												
			D′					$\frac{2}{3}$	$-\frac{3}{5}\sqrt{\frac{1}{2}}$	$\frac{3}{2}\sqrt{\frac{1}{5}}$	$\frac{1}{5}\sqrt{\frac{1}{2}}$	$\frac{\sqrt{5}}{3}$		$-\sqrt{\frac{1}{2}}$					$\frac{2}{3}$		$\frac{1}{2}\sqrt{\frac{7}{5}}$		$\frac{\sqrt{5}}{3}$	$\sqrt{\frac{1}{2}}$	
			D″						$3\sqrt{\frac{3}{2\cdot5\cdot7}}$	$\frac{5}{2}\sqrt{\frac{1}{3\cdot7}}$	$\sqrt{\frac{7}{2\cdot3\cdot5}}$		$2\sqrt{\frac{2}{3\cdot5}}$	$\sqrt{\frac{7}{2\cdot3\cdot5}}$							$-\frac{1}{2}\sqrt{\frac{1}{3}}$			$\sqrt{\frac{7}{2\cdot3\cdot5}}$	
			S							$-\sqrt{\frac{7}{3\cdot5}}$			$\sqrt{\frac{1}{2\cdot3}}$									$2\sqrt{\frac{2}{3\cdot5}}$			$-\sqrt{\frac{5}{2\cdot3}}$
		(00)	S										$\sqrt{\frac{5}{2\cdot3}}$												$\sqrt{\frac{1}{2\cdot3}}$

The orbital coefficients of fractional parentage for k=3 and k=2 were calculated by the usual methods (see, e.g./3/, and the references quoted there) and are presented in tables 1 and 2. The arrangement of our tables is similar to that of Jahn/4/. The coefficients with $\bar{\Gamma}=2$ are written on the left, the coefficients with $\bar{\Gamma}=0$ on the right-hand side.

Besides the three components of angular momentum, we use five operators $Q(\mu)=\sum_{i=1}^{k} Q_i(\mu)$ where

$$Q_i(\mu) = -[a_i^{+}(1)a_i(1)]^2_{\mu} \qquad (4)$$

as the infinitesimal operators of the algebra of SU(3). The used standard form of the irreducible representations of SU(3) is specified completely by the double bar matrix $(\alpha L\|Q\|\alpha' L')$ in table 3.

Further, the matrix elements of the Wigner central force, between our two-particle states, expressed in terms of the Talmi integrals are presented (table 4).

References

1. J.P.Elliott. Proc. Roy. Soc. A 245, 128 (1958).
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Table 2

k=2 orbital fractional parentage coefficients

Γ [f] (λ)	L	$\bar{\Gamma}=2$: $\bar{F}$ (1) [$\bar{f}$] [1] ($\bar{\lambda}$) (20), $\bar{L}$ = D	$\bar{\Gamma}=2$: $\bar{L}$ = S	$\bar{\Gamma}=0$: (1) [1] (20), $\bar{L}$ = D	$\bar{\Gamma}=0$: $\bar{L}$ = S
(21) [11] (21)	F	1			
	D		$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
	P	-1			
(11) [2] (40)	G	1			
	D	$-\frac{1}{3}\sqrt{2}$	$\frac{1}{3}\sqrt{\frac{7}{2}}$	$\frac{1}{3}\sqrt{\frac{7}{2}}$	
	S	$\frac{2}{3}$			$\frac{1}{3}\sqrt{5}$
(11) [2] (02)	D	$\frac{1}{3}\sqrt{7}$	$\frac{1}{3}$	$\frac{1}{3}$	
	S	$\frac{1}{3}\sqrt{5}$			$-\frac{2}{3}$

Table 3

Standard form of the double bar matrix $(\alpha L \| Q \| \alpha' L')$

(02)	D	S
D	$\sqrt{\frac{5\cdot 7}{3}}$	$-2\sqrt{\frac{5}{3}}$
S	$-2\sqrt{\frac{5}{3}}$	

(03)	F	P
F	$3\sqrt{\frac{2\cdot 7}{5}}$	$-2\sqrt{\frac{3\cdot 7}{5}}$
P	$-2\sqrt{\frac{3\cdot 7}{5}}$	$3^2\sqrt{\frac{1}{5}}$

(11)	D	P
D		$\sqrt{3\cdot 5}$
P	$-\sqrt{3\cdot 5}$	

(20)	D	S
D	$-\sqrt{\frac{5\cdot 7}{3}}$	$2\sqrt{\frac{5}{3}}$
S	$2\sqrt{\frac{5}{3}}$	

(30)	F	P
F	$-3\sqrt{\frac{2\cdot 7}{5}}$	$2\sqrt{\frac{3\cdot 7}{5}}$
P	$2\sqrt{\frac{3\cdot 7}{5}}$	$-3^2\sqrt{\frac{1}{5}}$

(40)	G	D	S
G	$-\sqrt{\frac{2\cdot 3\cdot 5\cdot 11}{7}}$	$2\cdot 3\sqrt{\frac{2\cdot 3}{7}}$	
D	$2\cdot 3\sqrt{\frac{2\cdot 3}{7}}$	$-11\sqrt{\frac{5}{3\cdot 7}}$	$2\sqrt{\frac{2\cdot 7}{3}}$
S		$2\sqrt{\frac{2\cdot 7}{3}}$	

(22)	G	F	D′	D″	S
G		$\sqrt{2\cdot 3\cdot 5}$	$2\sqrt{2\cdot 3}$		
F	$-\sqrt{2\cdot 3\cdot 5}$			$2\sqrt{2\cdot 5}$	
D′	$2\sqrt{2\cdot 3}$			5	2^2
D″		$-2\sqrt{2\cdot 5}$	5		
S			2^2		

(41)	H	G	F	D	P
H	$-\sqrt{\frac{11\cdot 13}{5}}$	$2\sqrt{\frac{3\cdot 11}{5}}$	$2\sqrt{11}$		
G	$-2\sqrt{\frac{3\cdot 11}{5}}$	$-2\cdot 3\sqrt{\frac{2\cdot 3\cdot 11}{5\cdot 7}}$	$\sqrt{2\cdot 3}$	$2\cdot 3\sqrt{\frac{5}{7}}$	
F	$2\sqrt{11}$	$-\sqrt{2\cdot 3}$	$-2\sqrt{\frac{2\cdot 7}{5}}$	$2\cdot 3$	$2^2\cdot 3\sqrt{\frac{1}{5}}$
D		$2\cdot 3\sqrt{\frac{5}{7}}$	$-2\cdot 3$	$-3\sqrt{\frac{3\cdot 5}{7}}$	$2\sqrt{2\cdot 3}$
P			$2^2\cdot 3\sqrt{\frac{1}{5}}$	$-2\sqrt{2\cdot 3}$	$3^2\sqrt{\frac{1}{5}}$

(60)	I	G	D	S
I	$-\sqrt{\frac{3\cdot 5\cdot 7\cdot 13}{11}}$	$2\sqrt{\frac{3\cdot 5\cdot 13}{11}}$		
G	$2\sqrt{\frac{3\cdot 5\cdot 13}{11}}$	$-3\cdot 5\sqrt{\frac{2\cdot 3\cdot 5}{7\cdot 11}}$	$2^2\sqrt{\frac{3\cdot 11}{7}}$	
D		$2^2\sqrt{\frac{3\cdot 11}{7}}$	$-5\sqrt{\frac{3\cdot 5}{7}}$	$2\cdot 3$
S			$2\cdot 3$	

(21)	F	D	P
F	$-\sqrt{\frac{2\cdot 7}{5}}$	$2\sqrt{\frac{2\cdot 7}{3}}$	$2\sqrt{\frac{2\cdot 7}{5}}$
D	$-2\sqrt{\frac{2\cdot 7}{3}}$	$-\sqrt{\frac{5\cdot 7}{3}}$	$2\sqrt{2}$
P	$2\sqrt{\frac{2\cdot 7}{5}}$	$-2\sqrt{2}$	$7\sqrt{\frac{1}{5}}$

Table 4

Wigner central force matrix elements between two-nucleon states

$$\frac{3}{2^3}I_4 + \frac{1}{2^2}I_2 + \frac{3}{2^3}I_0$$

$$\frac{1}{2}I_3 + \frac{1}{2}I_1$$

$$\frac{3^3}{2^4}I_4 - \frac{3\cdot 7}{2^3}I_3 + \frac{41}{2^3\cdot 3}I_2 - \frac{7}{2^3\cdot 3}I_1 + \frac{5^2}{2^4\cdot 3}I_0$$

$$0$$

$$-\frac{1}{2^2\cdot 3}\sqrt{\frac{7}{2}}I_2 + \frac{1}{2\cdot 3}\sqrt{\frac{7}{2}}I_1 - \frac{1}{2^2\cdot 3}\sqrt{\frac{7}{2}}I_0$$

$$\frac{7}{2^3}I_3 - \frac{3}{2^2}I_2 + \frac{7}{2^3}I_1$$

$$0$$

$$\frac{7}{2\cdot 3}I_2 - \frac{1}{3}I_1 + \frac{1}{2\cdot 3}I_0$$

$$\frac{7}{2^2}I_3 - \frac{5}{2}I_2 + \frac{7}{2^2}I_1$$

$$\frac{3^3\cdot 7}{2^6}I_4 - \frac{3\cdot 5\cdot 7}{2^4}I_3 + \frac{17\cdot 37}{2^5\cdot 3}I_2 - \frac{5\cdot 31}{2^4\cdot 3}I_1 + \frac{13\cdot 19}{2^6\cdot 3}I_0$$

$$\frac{\sqrt{5}}{2\cdot 3}I_2 - \frac{\sqrt{5}}{3}I_1 + \frac{\sqrt{5}}{2\cdot 3}I_0$$

$$\frac{5}{3}I_2 - \frac{2^2}{3}I_1 + \frac{2}{3}I_0$$