ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E- 2246

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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BARYON ANNIHILATION IN BROKEN $\tilde{U}(12)$ SYMMETRY

ЯФ, 1966, т3, в3, с 541-544.

1965

3509/1 up.

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Submitted to Jad.Phys.

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The groups $\tilde{U}(12)$ and $SL(6)$ suggested in [1-7] as relativistic generalizations of the $SU(6)$ symmetry, have been extensively applied to study baryon currents [5,8,9], scattering amplitudes etc.

It has been pointed out (cf. [7] and also [5,8,9,13]) that matrix elements contain not only so-called regular amplitudes, constructed from the wave-functions of the initial and final particles, but also irregular ones, explicitly containing the particle momenta, considered as tensors, transforming according to the adjoint representation of the given group. If these "irregular" couplings are excluded, the S-matrix violates the unitarity condition [10-12]. Several hundred independent irregular amplitudes exist for meson baryon scattering in the $\tilde{U}(12)$ symmetry, and even more in $SL(6)$. However, for the two-meson annihilation of a baryon-antibaryon pair at rest, most of these amplitudes vanish.

Two-meson annihilation at rest, leading to two pseudoscalar particles, has been considered in a number of papers [14-19]. Only regular invariants were considered in [14-17] and it was shown that they all vanish for annihilation at rest, so that two-meson annihilation should be forbidden. However, relations between the cross-sections for the production of various types of pseudoscalar mesons were calculated on the basis of additional assumptions, like pole approximations, etc. The contribution of irregular amplitudes to the process was estimated in [18]. The $p\bar{p}$ -annihilation at rest into two pseudoscalar or vector mesons was considered in [19] in the framework of the non-relativistic $SU(6)$ symmetry. The authors obtain a number of relations between the cross sections, most of which disagree badly with the experimental data. They argue that one of the reasons of this disagreement is the application of a non-relativistic theory and stress the necessity of a relativistic treatment.

In this paper we calculate in detail the octet baryon-antibaryon annihilation at rest into two arbitrary mesons, taking all amplitudes, consistent with the intrinsically broken $\tilde{\tilde{U}}(12)$ symmetry into account. We show that the predictions of this theory are also at variance with the existing experimental results, although the agreement is somewhat better than in [19].

Let us consider the invariant amplitudes, surviving for annihilation at rest. We take the following arguments into account:

1) It follows from the Bargmann-Wigner equations and purely kinematical arguments, that all amplitudes in which there is at least one summation between the baryon function indices e.g.

$$\bar{\Psi}^{ABE} P^{C}_{1A} P^{F}_{2D} Q^{K}_{B} \Psi_{EFK} \Phi^{L}_{C} P^{M}_{1L} P^{N}_{2M} \Phi^{D}_{N}$$

are equal to zero. Here P_1, P_2 are the 143-dimensional baryon momenta Q_1, Q_2 the meson ones, $Q = Q_2 - Q_1$ and Ψ^{ABC}, Φ^{D}_{E} the baryon and meson wave functions, transforming according to the 364 and 143-dimensional representations of $\tilde{\tilde{U}}(12)$ respectively.

2) At rest we have $P_1 = P_2 = \frac{Q_1 + Q_2}{2}$ so that many invariants, in general independent, reduce to simpler ones.

3) The mesons grouped in one $\tilde{\tilde{U}}(12)$ multiplet, must obey a generalized Pauli principle, i.e. the total amplitude must be symmetrical with respect to the interchange of the two meson functions (including the unitary parts). It follows that all amplitudes, antisymmetrical with respect to the mentioned interchange, must be multiplied by antisymmetrical functions of the kinematical invariants s, t and u. Let us consider these coefficients as functions of s and $\nu = t - u$. We have $\nu \to -\nu$ under the meson interchange and thus, for the antisymmetrical coefficients

$$f(s,\nu) = -f(s,-\nu)$$

$$f(s,0) = 0 .$$

However, at rest $\nu = t - u = 2(p_1, q_2 - q_1) = 0$ and all antisymmetrical amplitudes vanish.

The most general amplitude of the considered process can be written as

$$M = f_1(s,\nu)\bar{\Psi}^{ABC} Q^{E}_{A} Q^{F}_{B} \Psi_{EFD} (\bar{\Phi}^{D}_{K} \Phi^{K}_{C} + \bar{\Phi}^{K}_{C} \Phi^{D}_{K}) +$$

$$+ f_2(s,\nu)\bar{\Psi}^{ABC} Q^{E}_{A} Q^{F}_{B} \Psi_{EFD} (\bar{\Phi}^{L}_{K} Q^{K}_{L} \Phi^{D}_{C} - \bar{\Phi}^{D}_{C} Q^{L}_{K} \Phi^{K}_{L}) . \qquad (1)$$

The second amplitude contributes only to processes in which ω or ϕ mesons are created.

As usual, we reduce the amplitude (1) with respect to the subgroup $SU(3)\times \mathcal{L}$. We use the relations[1]

$$\Psi_{ABC} = (\epsilon_{abd} N^d_c - \epsilon_{cad} N^d_\sigma)\Psi_{[\alpha\beta]\gamma} + (\epsilon_{bcd} N^d_a - \epsilon_{cad} N^d_b)\Psi_{[\beta\gamma]\alpha} \tag{2}$$

$$\Phi^B_A = P^b_a[(1-\frac{i\hat{q}}{\mu})\gamma_5]_{\alpha\beta}\,\phi + V^b_a[(1-\frac{i\hat{q}}{\mu})\gamma_\mu]_{\alpha\beta}\,\xi_\mu$$

$$\Psi_{[\alpha\beta]\gamma} = [(1-\frac{i\hat{p}}{m})\gamma_5 C]_{\alpha\beta}\,\Psi_\gamma \tag{3}$$

where N^b_a, P^b_a, V^b_a are the unitary parts and ψ_γ, ϕ, ξ_μ the baryon, pseudoscalar and vector meson wave functions respectively, and reduce (1) to the form

$$\begin{aligned} M = {} & a_1(\bar\Psi i\hat{q}\Psi)(\bar\phi\phi) + a_2(\bar\Psi i\hat{q}\Psi)(\xi^+\xi) + \frac{1}{\kappa^2}a_3(\bar\Psi i\hat{q}\Psi)\cdot \\ & \cdot(q\xi^+)(q\xi) + a_4(\bar\Psi i\hat{\xi}\Psi)(\xi^+ q) + a_5(\bar\Psi i\hat{\xi}^+\Psi)(\xi q) + a_6(\bar\Psi\gamma_5\Psi)\cdot \\ & \cdot\epsilon_{\mu\nu\rho}\,\xi^+_\mu\,\xi_\nu\,q_\rho\,p + a_7(\bar\Psi\gamma_5\Psi)\bar\phi(q\xi) + a_8(\bar\Psi\gamma_5\Psi)\phi(q\xi^+) + \\ & + a_9(\bar\Psi\gamma_\sigma\Psi)\bar\phi\epsilon_{\mu\nu\rho\sigma}\,\xi_\mu q_\nu p_\rho + a_{10}(\bar\Psi\gamma_\sigma\Psi)\phi\epsilon_{\mu\nu\rho\sigma}\,\xi^+_\mu q_\nu p_\rho \end{aligned} \tag{4}$$

where

$$a_1 = 3g_1(\bar N N_F \bar P P_F)$$

$$a_2 = 3g_1(\bar N N_F \bar V V_F)$$

$$a_3 = -6g_1\kappa(\bar N N_{\frac{2}{3}F-D-\kappa F}\cdot\bar V V_F) + 2g_2[(\bar V)(\bar N N_{F+\kappa(D-\frac{2}{3}F)}\cdot V) - (V)(\bar N N_{F+\kappa(D-\frac{2}{3}F)}\bar V)]$$

$$a_4 = 3g_1(\bar N N_{D-\frac{2}{3}F}\bar V V_F) - g_2[3(\bar N N_{D-\frac{2}{3}F}V) - (\bar N N)(V)](\bar V)$$

$$a_5 = 3g_1(\bar N N_{D-\frac{2}{3}F}\bar V V_F) + g_2[3(\bar N N_{D-\frac{2}{3}F}\bar V) - (\bar N N)(\bar V)](V)$$

$$a_6 = -2g_1[(\bar N N_{\frac{3}{2}D-F}\bar V V_D) - (\bar N N)(\bar V V)]$$

$$a_7 = 3g_1[\bar N N_{D-\frac{2}{3}F}\bar P V_F] + g_2[3(\bar N N_{D-\frac{2}{3}F}\bar P) - (\bar N N)\bar P](V)$$

$$a_8 = 3g_1[\bar N N_{D-\frac{2}{3}F}\bar V P_F] - g_2[3(\bar N N_{D-\frac{2}{3}F}P) - (\bar N N)(P)](\bar V)$$

$$a_9 = -2g_1[(\bar N N_{\frac{3}{2}D-F}\bar P V_D) - (\bar N N)(\bar P V)]$$

$$a_{10} = -2g_1[(\bar N N_{\frac{3}{2}D-F}\bar V P) - (\bar N N)(P\bar V)]$$

Here we have

$$g_1 = 32 f_1 \frac{m^2 - \mu^2}{\mu}\left(1 + \frac{m}{\mu}\right)$$

$$g_2 = 128 f_2 \frac{m}{\mu}(m^2 - \mu^2)$$

$$\kappa = \frac{m}{\mu}$$

and e.g.

$$(\bar{N}N_P \bar{P}P_P) = Sp[\bar{N}N\bar{P}P - \bar{N}NP\bar{P} + N\bar{N}P\bar{P} - N\bar{N}\bar{P}P]$$

Taking the square module of (4), averaging over all factor and specifying the unitary parts to the wave functions of concrete particles, we obtain the results given in tables 1 and 2 for $\bar{p}p$ and $\bar{p}n$ annihilation at rest. The final results are independent of the assumed baryon and meson masses and depend only on the values $|g_1|^2$ and $|g_2|^2$ (except for phase-space considerations). A comparison with experimental data shows that the agreement is very poor, even if all irregular couplings in $\tilde{U}(12)$ are taken into account (a compilation of experimental data on the two-meson annihilation is given in /19/). For pseudoscalar mesons our results agree with the calculations of Hussein and Rotelli /18/.

The authors are indebted to J.A. Smorodinsky for helpful discussions and comments.

Table 1

$p\bar{p}$ annihilation into two mesons

P r o c e s s	Averaged square of matrix element	Cross section (phase-space taken into account)
1	2	3
$p + \bar{p} \to \pi^+ \pi^-$	$\|g_1\|^2$	$\|g_1\|^2$
$K^+ K^-$	$4\|g_1\|^2$	$3{,}4\|g_1\|^2$
$K^0 \bar{K}^0$	$\|g_1\|^2$	$0{,}85\|g_1\|^2$
$\rho^+ \rho^-$	$14\|g_1\|^2$	$9{,}2\|g_1\|^2$
$\rho^0 \rho^0$	$\|g_1\|^2$	$0{,}48\|g_1\|^2$
$K^{+*} K^{-*}$	$23\|g_1\|^2$	$7{,}9\|g_1\|^2$
$K^{0*} \bar{K}^{0*}$	$3{,}6\|g_1\|^2$	$1{,}2\|g_1\|^2$
$\omega\omega$	$\|g_1\|^2 + 2\|g_2\|^2$	$0{,}56(\|g_1\|^2 + 2\|g_2\|^2)$
$\phi_0 \omega$	$5{,}5\|g_1\|^2 + 6{,}5\|g_2\|^2$	$0{,}36\|g_1\|^2 + 4{,}1\|g_2\|^2$
$\rho_0 \omega$	$3{,}2\|g_2\|^2$	$1{,}1\|g_2\|^2$
$\omega\phi$	$5{,}5\|g_2\|^2$	$1{,}6\|g_2\|^2$

1	2	3
$\pi^+\rho^-, \pi^-\rho^+$	$4{,}3\,\lvert g_1\rvert^2$	$3{,}9\,\lvert g_1\rvert^2$
K^+K^{-*}, K^-K^{+*}	$5{,}3\,\lvert g_1\rvert^2$	$4{,}2\,\lvert g_1\rvert^2$
$\bar{K}^0K^{*0}, K^0\bar{K}^{*0}$	$0{,}44\,\lvert g_1\rvert^2$	$0{,}3\,\lvert g_1\rvert^2$
$\eta\rho^0$	$1{,}9\,\lvert g_1\rvert^2$	$1{,}3\,\lvert g_1\rvert^2$
$\chi\rho^0$	$3{,}7\,\lvert g_1\rvert^2$	$1{,}5\,\lvert g_1\rvert^2$
$\pi^0\rho^0$	$2\,\lvert g_1\rvert^2$	$1{,}7\,\lvert g_1\rvert^2$
$\chi\omega$	$1{,}3\,\lvert g_1\rvert^2+0{,}6\,\lvert g_2\rvert^2$	$0{,}25(2\lvert g_1\rvert^2+\lvert g_2\rvert^2)$
$\eta\omega$	$0{,}66\,\lvert g_1\rvert^2+0{,}33\,\lvert g_2\rvert^2$	$0{,}23(2\lvert g_1\rvert^2+\lvert g_2\rvert^2)$
$\pi^0\omega$	$5{,}5\,\lvert g_1\rvert^2+2{,}8\,\lvert g_2\rvert^2$	$2{,}3(2\lvert g_1\rvert^2+\lvert g_2\rvert^2)$
$\chi\phi$	$0{,}33\,\lvert g_2\rvert^2$	forbidden
$\eta\phi$	$0{,}16\,\lvert g_2\rvert^2$	$0{,}09\,\lvert g_2\rvert^2$
$\pi^0\phi$	$1{,}4\,\lvert g_2\rvert^2$	$0{,}96\,\lvert g_2\rvert^2$
$\phi\phi$, $\eta\chi$, $\pi^0\chi$, $\pi^0\eta$	forbidden	

Table 2

$p\bar{n}$ annihilation into two mesons

	Averaged square of matrix element	Cross section (phase-space taken into account)
$\bar{p}n \to \pi^0\pi^-$	$2\,\lvert g_1\rvert^2$	$2\,\lvert g_1\rvert^2$
K^-K^0	$\lvert g_1\rvert^2$	$0{,}85\,\lvert g_1\rvert^2$
$\rho^0\rho^-$	$28\,\lvert g_1\rvert^2$	$16{,}4\,\lvert g_1\rvert^2$
$K^{*-}K^{*0}$	$19\,\lvert g_1\rvert^2$	$6{,}9\,\lvert g_1\rvert^2$
$\phi\rho^-$	$6{,}5\,\lvert g_1\rvert^2$	$2{,}16\,\lvert g_1\rvert^2$
$\rho^-\eta$	$3{,}7\,\lvert g_1\rvert^2$	$2{,}8\,\lvert g_1\rvert^2$
$\rho^-\chi$	$7{,}4\,\lvert g_1\rvert^2$	$2{,}9\,\lvert g_1\rvert^2$
$\omega\pi^-$	$1{,}1\,\lvert g_1\rvert^2+5{,}5\,\lvert g_2\rvert^2$	$4{,}5(2\lvert g_1\rvert^2+\lvert g_2\rvert^2)$
$\phi\pi^-$	$2{,}8\,\lvert g_2\rvert^2$	$1{,}9\,\lvert g_2\rvert^2$
$\pi^0\rho^-$	$5{,}5\,\lvert g_1\rvert^2$	$4{,}6\,\lvert g_1\rvert^2$
$\pi^-\rho^0$	$5{,}5\,\lvert g_1\rvert^2$	$4{,}6\,\lvert g_1\rvert^2$
K^-K^{*0}	$8{,}3\,\lvert g_1\rvert^2$	$6{,}6\,\lvert g_1\rvert^2$
$K^{*-}K^0$	$8{,}3\,\lvert g_1\rvert^2$	$6{,}6\,\lvert g_1\rvert^2$
$\omega\rho^-$	$1{,}1\,\lvert g_1\rvert^2+1{,}3\,\lvert g_2\rvert^2$	$7\,\lvert g_1\rvert^2+8{,}3\,\lvert g_2\rvert^2$
$\pi^-\chi$	forbidden	
$\pi\eta$	— " —	

References

1. A. Salam, R. Delbourgo, J. Strathdee. Proc. Roy. Soc. A284, 146 (1965).
2. R. Delbourgo, M.A. Rashid, A. Salam, J. Strathdee. Proc.Roy. Soc. A285, 312 (1965).
3. B. Sakita, K.C. 'Wali. Phys. Rev. Lett. 14, 404 (1965).
4. M.A.B. Beg, A.Pais. Phys.Rev. 137, B1514 (1965).
5. W. Rühl. Nuovo Cim. 37, 301, 319 (1965). Phys.Lett. 15, 99, 101, 340 (1965).
6. V.G. Kadyshevsky, R.M. Muradyan, A.N. Tavkhelidze, I.T. Todorov. Phys.Lett. 15, 182 (1965).
7. Нгуен Ван Хьеу, Я.Ф. 1 (1965). in print).
8. Нгуен Ван Хьеу Я.А. Смородинский.Я.Ф.1(1965).(in print).
9. П. Винтерннитц, А.А. Макаров, Нгуен Ван Хьеу, Л.Г. Ткачев, М. Углирж, ДАН (1965). (in print).
10. R. Blankenbecler, M.L. Goldberger, K. Johnson, S.B. Treiman, Phys.Rev.Lett. 14, 518 (1965).
11. J.M. Cornwall, P.G. Freund, K.T. Mahanthappa. Phys.Rev.Lett. 14, 515 (1965).
12. M.A.B. Beg, A. Pais. Phys. Rev. Lett. 14, 509 (1965).
13. J.M. Charap, P.T. Mathews. Phys. Lett. 16, 95 (1965).
14. R. Delbourgo, Y.C. Leung, M.A. Rashid, J. Strathdee, Phys.Rev.Lett. 14, 609 (1965).
15. Ngee-Pong Chang, J.M. Shpiz. Phys.Rev. Lett. 14, 617 (1965).
16. Y. Hara. Phys. Rev. Lett. 14, 603 (1965).
17. F. Hussain. Preprint CTP 65/11.
18. F. Hussain, P. Rotelli. Phys. Lett. 16, 183 (1965).
19. F.J. Dyson, Nguyen-huu Xuong. Phys. Rev. Lett. 14; 655 (1965).
20. V. Bargmann, E. Wigner. Proc. Nat. Ac. Sci. Wash. 34, 211 (1948).

Received by Publishing Department
on June 26, 1965.