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The theory based on the group SU (6) makes it possible to explain a large number of experimental data. It has been shown in a number of papers,[1-10] that the groups SL (6) and $\tilde{\tilde{U}}(12)$ are possible relativistic generalizations of the SU(6) symmetry. It has also been shown that the wave equations even for free particles break the SL (6) and $\tilde{\tilde{U}}(12)$ symmetries, so that it is meaniglless to construct an S-matrix, invariant with respect to these groups. However, if the usual four-momenta are considered as components of the tensors[1] $(P)^{(a\alpha)}_{(\dot{b}\dot{\beta})}$, $(P)^{(\dot{a}\dot{\alpha})}_{(b\beta)}$ transforming like the corresponding SL (6) spinors, the corresponding wave equations are SL (6) invariant. Starting from this point Nguyen van Hieu[7] suggested a method for investigating the SL (6) symmetry, making it possible to take into account the so called intrinsic breaking[4] due to the wave equations. This method was called the spurion formalism in[7] and consists of the following. Constructing the matrix elements of scattering processes and currents, or investigating wave equations, we first consider the particle four-momenta p_μ to be components of the tensors $(P)^{(a\alpha)}_{(\dot{b}\dot{\beta})}$ and $(\hat{P})^{(\dot{a}\dot{\alpha})}_{(b\beta)}$. The matrix elements of the scattering processes or currents contain not only the wave functions of the initial and final particles, but also these 36-dimensional momenta. In this case we demand that the matrix elements and wave equations should be invariant with respect to SL (6) and then we perform the following substitution in the obtained invariant expressions:

$$P^{\dot{B}}_{A} \to \hat{p}^{\dot{b}}_{a}\,\delta^{\beta}_{\alpha} \qquad\qquad P^{B}_{\dot{A}} \to \hat{p}^{b}_{\dot{a}}\,\delta^{\beta}_{\alpha}$$

i.e. we set all additional components equal to zero. Nguyen van Hieu and Smorodinsky[11] have shown that this formalism applied to investigating the structure of currents in SL(6) symmetry gives a series of new predictions.

In this paper we investigate the general structure of the baryon vector and axial current matrix elements in the framework of the $\tilde{\tilde{U}}(12)$ symmetry group. Since the wave equations will be invariant with respect to the $\tilde{\tilde{U}}(12)$

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[1] Here α, β are unitary indices, a, b spin indices. For details confront.[7]

group only if we introduce 143-component momenta, we shall use them and construct invariant matrix elements, in which we finally put all additional components equal to zero.

Let us consider the matrix element of the current $J^{(a\alpha)}_{(b\beta)}$ where $a = 1,2,3,4,$ $\alpha = 1,2,3$; transforming according to the regular representation of $\tilde{\tilde{U}}(12)$, between baryon states, belonging to the multiplet 364 in $\tilde{\tilde{U}}(12)$ (the multiplet 56 in SU (6)). We denote the momenta of the initial and final baryons by p and q respectively, and first consider them as components of the 143 component tensors $(P)^A_B$ and $(Q)^A_B$ We denote

$$(K)^A_B = (P)^A_B - (Q)^A_B, \qquad (L)^A_B = (P)^A_B + (Q)^A_B$$

For these momenta we have the following relation

$$(P)^A_B (Q)^B_C + (Q)^A_B (P)^B_C = 2\delta^A_C (pq)$$

Since the current J^A_B is hermitian

$$\overline{(J^A_B)} = J^B_A$$

its matrix element satisfies the relation

$$\overline{\langle Q \mid J^A_B \mid P \rangle} = \langle P \mid J^B_A \mid Q \rangle \qquad (1)$$

Invariance considerations and the generalized Bargmann-Wigner equations for the given baryon multiplet imply that the most general matrix element of the current J^A_B, satisfying (1), can be written as x)

$$\langle Q \mid J^A_B \mid P \rangle = f_1(\kappa)\, \bar\psi(Q)_{BCD}\, \psi(P)^{ACD} +$$

$$+ f_2(\kappa) [\, \bar\psi(Q)_{DEF} \left(\frac{K}{m}\right)^D_B \psi(P)^{AEF} - \bar\psi(Q)_{BEF} \left(\frac{K}{m}\right)^A_D \psi(P)^{DEF}] + \qquad (2)$$

$$+ f_3(\kappa)\, \bar\psi(Q)_{DEF} \left(\frac{K}{m}\right)^D_B \left(\frac{K}{m}\right)^A_G \psi(P)^{GEF} + f_4(\kappa) \left(\frac{L}{m}\right)^A_B \bar\psi(Q)_{CDE}\, \psi(P)^{CDE}$$

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x) Actually a fifth term $f_5(\kappa)(K^A_C L^C_B - L^A_C K^C_B)\, \bar\psi(Q)_{CDE}\psi(P)^{CDE}$ should be added, but it can not contribute to vector or axial currents.

where m is the baryon mass, $f_i(\kappa)$ functions of the invariant

$$\kappa = \frac{1}{12}(K)^B_A (K)^A_B = k^2$$

Further we express the spinor ψ^{ABC} with the help of the physical wave functions of the particles, pick out the components transforming like octet vector and axial currents and put all the additional components of the momenta equal to zero. Thus we obtain

$$\langle q | J^V_\mu | p \rangle = \frac{\ell_\mu}{2m}\left(f_1 - \frac{k^2}{m^2} f_3 + \frac{k^2}{m^2} f_2\right)(\bar N N)_F +$$

$$+\left(f_1 - \frac{k^2}{m^2} f_3 + 4f_2\right)\left(\bar N \frac{r_\mu}{4m} N\right)_{D+\frac{2}{3}F} + \frac{3\ell^2}{4m^2}\left[\left(f_1 - \frac{k^2}{m^2} f_3 + 4f_2\right)\bar D_\lambda \gamma_\mu D_\lambda - \frac{2\ell_\mu}{m} f_2 \bar D_\lambda D_\lambda\right] + \qquad (3)$$

$$+ \frac{3k_\lambda k_\sigma}{2m^2}\left[\left(f_1 - \frac{k^2}{m^2} f_3 + 4f_2\right)\bar D_\lambda \gamma_\mu D_\sigma - \frac{2\ell_\mu}{m} f_2 \bar D_\lambda D_\sigma\right] +$$

$$+ \frac{1}{m}\left(f_1 - \frac{k^2}{m^2} f_3 + 4f_2\right)\left(\epsilon_{\mu\nu\kappa\lambda}\ell_\kappa k_\lambda \bar D_\nu N + \text{h.c.}\right)$$

$$\langle q | j^A_\mu | p \rangle = \left(f_1 + \frac{k^2}{m^2} f_3\right)\left[\frac{\ell^2}{4m^2}(\bar N \gamma_\mu \gamma_5 N)_{D+\frac{2}{3}F} + \frac{3\ell^2}{4m^2}\bar D_\lambda \gamma_\mu \gamma_5 D_\lambda + \frac{3k_\lambda k_\nu}{2m^2}\bar D_\lambda \gamma_\mu \gamma_5 D_\nu\right] +$$

$$\frac{2k_\mu}{m}\left(-f_2 + 2f_3\right)\left[\frac{\ell^2}{4m^2}(\bar N \gamma_5 N)_{D+\frac{2}{3}F} + \frac{3\ell^2}{4m^2}\bar D_\lambda \gamma_5 D_\lambda + \frac{3k_\lambda k_\nu}{2m^2}\bar D_\lambda \gamma_5 D_\nu\right] - \qquad (4)$$

$$-\left(f_1 + \frac{k^2}{m^2} f_3\right)\frac{\ell^2}{2m^2}(\bar D_\mu N + \text{h.c.}) + \left[\left(f_1 + \frac{k^2}{m^2} f_3\right)\frac{p_\lambda q_\mu}{m^2} - (f_2 - 2f_3)\frac{2p_\lambda k_\mu}{m^2}\right](\bar D_\lambda N + \text{h.c.}).$$

Here N and D_μ are the physical wave functions for the baryon-octet and decimet and the indices F and D indicate the type of coupling.

Note that f_4 gives no contribution. Thus, in the framework of the $\tilde U(12)$ theory the matrix elements of the octet vector and axial currents for particles in the 56-dimensional multiplet of $SU(6)$ can be expressed with the help of 3 independent functions $f_i(\kappa)$, $i = 1,2,3$.

Let us consider the consequencies of some additional, more particular, assumptions. If we assume that the main contribution to the vector form-factor come from a pole diagram, corresponding to the exchange of one vector meson, we have

$$f_3 = 0 , \qquad f_2 = \frac{m}{2\mu} f_1 \qquad (5)$$

where μ is the meson mass. We thus reproduce the results of /9,10/. In particular, the proton magnetic moment will be $1 + 2\frac{m}{\mu}$. In the composite particle model of Bogolyubov et al./12/ one has:

$$f_3 = 0 , \quad f_2 = \tfrac{1}{2} f_1 . \qquad (6)$$

Let us consider some experimental consequences of the obtained results. It follows from (3) that annihilation of the type

$$\bar{p} + p \to e^+ + e^- \qquad \bar{p} + p \to \mu^+ + \mu^-$$

is forbidden at rest. Further, all electromagnetic form-factors can be expressed with the help of $f_1 - \frac{k^2}{m^2} f_3$ and f_2 and these can be connected with the electric and magnetic form-factors of the proton. Thus, if the proton form-factors are known, cross sections of processes

$$e + p \to e + \Delta^+ , \qquad e + n \to e + \Delta^0$$

and also the radiative decay probabilities of the baryon resonances can be calculated. The experimental investigation of the leptonic weak interaction processes: leptonic decays of baryons and the Ω^--hyperon, production of baryons and baryon resonances in neutrino experiments etc. can also permit to check the obtained relations.

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