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With the help of the SU(6) group[1] one can explain many features of the nonrelativistic elementary particle phenomena. However many authors emphasized the principal necessity of finding a higher relativistic group containing the SU(6) group as a subgroup. Such a group was proposed independently by many authors[2,3,4,5] – the inhomogeneous linear unimodular group in 6 dimensions ISL(6). There are 36 (or in some papers 72) translation operators among the generators of this group. These translation operators transform as a $D(\bar{6},6)$ representation of the group[2,3], they can be represented as components of a 6 x6 hermitian matrix[2,4]

$$P_{ab} = \sigma_{\mu,\alpha\beta} \lambda^{i}_{A,k} P_{\mu,A} , \qquad (1)$$

where the indices take on the following values

$$a,b = 1, ..., 6 \quad ; \quad \alpha,\beta = 1,2 \ ; \quad i,k = 1,2,3 \ ;$$

$$\mu = 0,1,2,3 \ ; \quad A = 0, 1, ..., 8 \ ;$$

σ_1 , σ_2 and σ_3 are the Pauli matrices, $\sigma_0 = I$, the λ_A are the well known matrices of the SU(3) group, $\lambda_0 = \sqrt{\frac{2}{3}} I$. The operator $P_{\mu,A}$ transforms as a four-vector in Lorentz space for any fixed A, and as an octet-vector and a singlet in SU(3) space for any fixed μ . The commutation relations among the $P_{\mu,A}$ and T_A operators are the following (the operators T_A are the generators of the SU(3) group)[3]:

$$\begin{aligned} [T_A , P_{\rho,B}] &= 2i F^{A}_{BC} P_{\rho,C} , \\ [T_A , T_B] &= 2i F^{A}_{BC} T_C , \end{aligned} \qquad (2)$$

$$[P_{\rho,A} , P_{\mu,B}] = 0 .$$

One can see, that the operators $P_{\mu,0}$, $P_{\mu,3}$ and $P_{\mu,8}$ can be diagonalized simultaneously with Y and T_3. It was proposed[2] that for physical particle states all the other components of the momentum operator ($P_{\mu,1}$, $P_{\mu,4}$, ... , $P_{\mu,7}$) have vanishing eigenvalues. An essential assumption of paper[2] is that $P_{\mu,0}$ and similarly $P_{\mu,3}$ and $P_{\mu,8}$ have the same eigenvalues within one multiplet. Defining the mass operator, as a quadratic combination of the 36 momenta one can obtain a mass splitting with very attractive mass formulas[2]. Nevertheless, we should like to show a principal difficulty of this method.

Assume, that we describe the meson fields by some representation of the ISL (6) group. Some of the components of the meson field operator describe the pseudoscalar mesons. These components transform as an octet representation of SU(3) (later on we shall discuss the case when SU(3) is violated so that the octet transformation character is lost; if the breakdown of the symmetry is spontaneous the octet transformation character is preserved). We denote the pseudoscalar meson field operators by ϕ_A .

The field operators satisfy the commutation relations:

$$[T_B , \phi_C] = 2iF^B_{CD}\phi_D \tag{3}$$

(this relations follow from the transformation properties of ϕ_C). We denote the eigenvalues of the operators $P_{\mu,A}$ by $\hat{P}_{\mu,A}$. They are defined for A = 0,3,8 (in what follows we do not assume even the equality of the eigenvalues of operators $P_{\mu,A}$ on different states $\phi_C|0>$). The condition, that the operators $P_{\mu,A}$ have vanishing eigenvalues on physical states for $A \neq 0,3$ or 8 may be expressed by the following equation:

$$[P_{\mu,A} , \phi_C]|E> = \hat{P}_{\mu,A}(C)\phi_C|E> , \tag{4}$$

where

$$\hat{P}_{\mu,A} = 0$$

if $A \neq 0,3,8$, $|E>$ is some physical state.

It is possible to substitute (4) by a weaker condition,

$$<F|[P_{\mu,A} , \phi_C]|E> = \hat{P}_{\mu,A}(C)<F|\phi_C|E> , \tag{5}$$

where $\hat{F}_{\mu,A} = 0$, if $A \neq 0,3,8$, $|E>$ and $<F|$ are physical states.

The operators

$$P_{\rho,A} , T_B , \phi_C$$

satisfy the Jacobi identity:

$$[P_{\rho,A}[T_B , \phi_C]] + [\phi_C,[P_{\rho,A} , T_B]] + [T_B,[\phi_C , P_{\rho,A}]] . \tag{6}$$

Taking eq. (6) between states $<E|$ and $|0>$ and using equations (2), (3), (4) we obtain

$$F^B_{CE}\hat{P}_{\rho,A}(E) - F^A_{BD}\hat{P}_{\rho,D}(E)\delta_{CE} - F^B_{CE}\hat{P}_{\rho,A}(C) = 0 . \tag{7}$$

The same equation can be obtained assuming only the validity of condition (5) instead of (4) and assuming the supplementary condition

$$T_B|0> = 0 . \tag{8}$$

Somewhat later we shall discuss the problems connected with the validity of (8). The consequences of eq. (7) are the following:

1. The second term of equation (7) equals zero for A = 0,3 or 8. So we obtain

$$\hat{P}_{\rho,A}(E) = P_{\rho,A}(C) . \tag{9}$$

Eq. (9) confirms the fact that the operator $P_{\rho,A}$ have the same eigenvalues for different members of the multiplet.

2. Putting A = 1, 2,4,5,6,7 into eqs. (7) we can obtain easily

$$\hat{P}_{\rho,3} = \hat{P}_{\rho,8} = 0 . \tag{10}$$

If eqs. (4) or eq. (5) and (8) do not hold then we cannot obtain eq.(7) and (10), but it leads to a spontaneous breakdown of the symmetry[6] independently of the existence of the 36 momentum operators, what means in our case that the mean value of the $P_{\mu,0}$ operators will depend on the octet index of the physical particle in question, so we cannot get easily a nice mass formula. On the other hand if eq. (10) holds the masses of the pseudoscalar octet do not split. In a similar way one can prove the same fact for vector mesons too. The above considerations are valid by no means only for the spontaneous breakdown of the ISL(6) symmetry. Of course it is possible to get nontrivial mass formulas with an external breakdown of the symmetry: putting a noninvariant term into the Hamiltonian, which leads to the change of commutation relations (2). Such a violation is very well-known and extensively used in the theory of SU(3) and SU(6) symmetries. However, in this case all the calculations must be made very carefully, taking into account the breakdown of commutation relations among the generators.

A similar investigation may be made in different relativistic group theoretical extensions of the SU(6) group.

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