ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
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ИССЛЕДОВАНИЙ

Дубна

E- 2120

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ITERATION METHOD IN NONRENORMALIZABLE FIELD THEORY

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1965

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Submitted to JETP

1. Introduction

In the previous work [1], which in the following will be referred to as I, we have studied the approximate Edwards equation for the vertex function in the nonrenormalizable field theory of the interaction between scalar particles and vector ones. This equation is represented graphically in Figure. Like in I we consider the case $k_\mu = 0$ in which the calculations are considerably simpler. The invariant function

$$F(p^2) = \frac{1}{2p^2} p_\mu \Gamma_\mu (p,0); \tag{1.1}$$

satisfies the following equation

$$F = A + K^{(0)}F + K'F ; \tag{1.2}$$

where

$$A = Z + \frac{a\lambda^2}{m^2p^2} \int \frac{d^4q}{(2\pi)^4} \frac{2(pq)^2}{(q^2+M^2)^2} F(q^2); \tag{1.3}$$

$$K^{(0)}F = \frac{a\lambda^2}{m^2p^2} \int \frac{d^4q}{(p-q)^4} \left[\frac{(p^2-q^2)^2}{(p-q)^2}(pq) - 2(pq)^2\right] \frac{F(q^2)}{(q^2+M^2)^2} ; \tag{1.4}$$

$$K'F = \frac{4a\lambda^2}{p^2} \int \frac{d^4q}{(2\pi)^4} \frac{p^2q^2(pq)-(pq)^3}{(p-q)^2[(p-q)^2+m^2]} \cdot \frac{F(q^2)}{(q^2+M^2)^2} . \tag{1.5}$$

Here Z is the renormalization constant of the vertex, λ is the coupling constant, $a = 1$ for the theory involving SU(2) symmetry, $a = 3/2$ for the theory with SU(3) symmetry, p and q are the Euclidean four-momenta. The constant A, generally speaking, is fixed by the normalization condition on the mass shell.

Here we have divided the kernel in Eq. (1.2) into two parts: 1) the most singular part (for large p and q) i.e. the integral in the right-hand side of (1.3) and the kernel $K^{(0)}$, 2) the less singular part K'. The most singular part is of positive dimension in the variables p and q (for $p, q \to \infty$) and gives therefore power divergences by iterations of eq. (1.2). The kernel K' is dimensionless in the large variables p and q and corresponds, in this sense, to kernels, which one faces in renormalizable theories.

The main idea of the work I consists in choosing the solution of the following equation with the most singular kernel as a zero approximation

$$F^{(0)} = A + K^{(0)} F^{(0)} . \qquad (1.6)$$

To calculate the exact function F we then transform formally Eq.(1.2) by multiplying both sides of Eq. (1.2) by the resolvent $R = (1-K^{(0)})^{-1}$ x). Using Eq. (1.6) we obtain following equation:

$$F = F^{(0)} + (1-K^{(0)})^{-1} K'F = F^{(0)} + RK'F ; \qquad (1.7)$$

This equation being solved by iterations gives the correction functions $F^{(n)}$ which were considered in I. We have proved there, that Eq. (1.6) for $A \neq 0$ has the unique solution $F^{(0)}(p^2)$ decreasing for $p^2 \to \infty$ and having the logarithmic branch point in the coupling constant of the kind $\lambda^2 \log \lambda^2$. In I we have also shown the existence of any iteration $F^{(n)}$ of Eq. (1.7), but the convergence of the series of iterations

$$F = \sum_{n=0}^{\infty} F^{(n)} ; \qquad (1.8)$$

has not been considered. Furthermore, the expansion of $F^{(0)}$ for small λ was found in the case $M = 0$ only. Now we study the properties of Eq. (1.7) and of the iteration solution (1.8) in more detail. This consideration enables us, in particular, to calculate the expansion of the exact function F for small λ.

2. Resolvent of the kernel $K^{(0)}$

We have shown in I that, after integrating over angular variables in the four-dimensional spherical coordinate system, the kernel $K^{(0)}$ of Eq. (1.6) takes the following form

x) Previously[1] Eq. (1.6) was shown to have no solution for $A = 0$. The existence of the resolvent follows from this fact.

$$K^{(0)}(x,y) = \frac{g^2}{12} \frac{y^2}{(y+M^2)^2} \left[\left(\frac{y^2}{x^2} - 2\frac{y}{x} \right) \theta(x-y) + \left(\frac{x^2}{y^2} - 2\frac{x}{y} \right) \theta(y-x) \right] ; \qquad (2.1)$$

where

$$g^2 = \frac{3a\lambda^2}{8m^2\pi^2} , \quad x = p^2 , \quad y = q^2 . \qquad (2.1a)$$

Owing to the occurence of θ - functions, we may reduce the integral equation (1.6) to the differential one[1]

$$\frac{1}{x} \frac{d^2}{dx^2} \left[\frac{1}{x} \frac{d^2}{dx^2} (x^2 F^{(0)}) \right] + \frac{g^2 F^{(0)}}{x(x+M^2)^2} = \frac{1}{x} \frac{d^2}{dx^2} \left[\frac{1}{x} \frac{d^2}{dx^2} (x^2 A) \right] ; \qquad (2.2)$$

with the boundary conditions

$$F^{(0)}(x) \underset{x \to \infty}{\to} 0 ; \qquad (2.3a)$$

$$F^{(0)}(x) \text{ is bounded for } x \to 0 \qquad (2.3b)$$

Note, that if we replace the constant A in Eq. (1.6) by an arbitrary function $f(x)$, then the corresponding function $\bar{F}^{(0)}(x)$ satisfies Eq. (2.2) in which A is replaced by $f(x)$. We use this fact for finding the resolvent R. Indeed, the resolvent is defined by the relation $\bar{F}^{(0)} = Rf$. Hence, to define R we have to find the solution of Eq. (2.2) for the arbitrary function f.

The equation for $\bar{F}^{(0)}$ may be written in the following form

$$\frac{1}{x} \frac{d^2}{dx^2} \left\{ \frac{1}{x} \frac{d^2}{dx^2} \left[x^2 (\bar{F}^{(0)} - f) \right] \right\} + \frac{g^2 (\bar{F}^{(0)} - f)}{x(x+M^2)^2} = - \frac{g^2 f}{x(x+M^2)^2} . \qquad (2.4)$$

Then we express the solution $\bar{F}^{(0)}$ in terms of f with the aid of the Green function $G(x,y)$ of the boundary value problem (2.2)-(2.3), (for $A = 0$)

$$\bar{F}^{(0)}(x) = f(x) - g^2 \int_0^{\infty} dy \frac{G(x,y)}{y(y+M^2)^2} f(y). \qquad (2.5)$$

The Green function may be easily constructed if one knows the linear independent solutions F_i of the homogeneous equation obtained from Eq. (2.2) by setting $A = 0$ (see, for instance, the book[2]):

$$G(x,y) = \theta(x-y) \sum_{i=1,2} F_i(x) \frac{W_i(y)}{W(y)} - \theta(y-x) \sum_{i=3,4} F_i(x) \frac{W_i(y)}{W(y)} \qquad (2.6)$$

Here $W(y)$ is the Wronskian of the solutions, $F_i(y)$ and $W_i(y)$ are signed minors of the Wronskian, which correspond to $\frac{d^3}{dy^3}F_i(y)$. The linear independent solutions F_i have the following asymptotic behaviour at infinity

$$F_{1,2}(x) \underset{x\to\infty}{\approx} (g^2x)^{-3/8}\exp[-4e^{\pm i\frac{\pi}{4}}(g^2x)^{1/4}];$$
$$F_{3,4}(x) \underset{x\to\infty}{\approx} (g^2x)^{-3/8}\exp[4e^{\pm i\frac{\pi}{4}}(g^2x)^{1/4}]. \qquad (2.7)$$

The solutions $F_{3,4}(x)$ satisfy as well the condition (2.3b). The possibility of such a choice of the linear independent solutions follows from the results of I.

It can be easily verified, that the right-hand side of Eq. (2.6) is invariant under any linear nonsingular transformation of the following kind

$$F'_i(x) = \sum_{j=1,2} c_{ij}F_j(x), \quad i=1,2;$$
$$F'_k(x) = \sum_{\ell=3,4} d_{k\ell}F_\ell(x), \quad k=3,4. \qquad (2.8)$$

We shall use the last comment in the following while considering the behaviour of the Green function in different regions of the variables x and y.

The equation (2.5) gives the following representation of the resolvent

$$R(x,y) = \delta(x-y) - g^2\frac{G(x,y)}{y(y+M^2)^2}. \qquad (2.9)$$

In the following we shall use also the other representation of R, which is obtained by immediate solution of Eq. (2.2)

$$Rf = \int_0^\infty dy\, G(x,y)\frac{1}{y}\frac{d^2}{dy^2}\left[\frac{1}{y}\frac{d^2}{dy^2}(y^2 f(y))\right]. \qquad (2.10)$$

In both cases we have to study the function $G(x,y)$.

With the aid of Eqs. (2.6), (2.7) we can easily find the asymptotic behaviour of the Green function for large values of x and y:

$$G(x,y) \underset{\substack{x\to\infty\\ y\to\infty}}{\approx} \frac{x^{-3/8}y^{21/8}}{4g^{3/2}}\{\theta(x-y)\exp[4e^{\frac{i\pi}{4}}(g^2y)^{1/4}-(g^2x)^{1/4})+i\frac{\pi}{4}] +$$
$$+\theta(y-x)\exp[4e^{\frac{i\pi}{4}}(g^2x)^{1/4}-(g^2y)^{1/4})+i\frac{\pi}{4}] + \text{c.c.}\} \qquad (2.11)$$

To find the behaviour of the Green function for small x and y, we make use of the transformation (2.8) and choose the linear independent solutions $F'_i(x)$ and $F'_k(x)$ to have the following behaviour for small x

$$F'_1(x) \underset{x\to0}{\approx} \frac{1}{(g^2x)^2}, \quad F'_2(x) \underset{x\to0}{\approx} \frac{1}{g^2x};$$
$$F'_3(x) \approx g^2x, \quad F'_4(x) \approx (g^2x)^2. \qquad (2.12)$$

Omitting simple calculations we write down the expression for the Green function in the domain of small x and y

$$G(x,y) \underset{\substack{x\to0\\ y\to0}}{\approx} \frac{1}{12}[\theta(x-y)(2\frac{y^4}{x}-\frac{y^5}{x^2})+\theta(y-x)(2xy^2-x^2y)]. \qquad (2.13)$$

By the same calculations one can find the behaviour of $G(x,y)$ for $x\to0$, $y\to\infty$ or for $x\to\infty$, $y\to0$. The resolvent being expressed in terms of the Green function, the obtained asymptotic formulae define the asymptotic properties of the resolvent R.

3. Proof of the convergence of the iteration procedure

It is proved in this section that the kernel $\bar{K} = RK'$ of Eq. (1.7) is square integrable. Using the known properties of the zero approximation $F^{(0)}$ we ground then the applicability of the Fredholm method to this equation and further show the convergence of the iterative series (1.8) for sufficiently small values of λ.

We study first the kernel $K'(x,y)$. Performing the integration over angular variables in Eq. (1.5) we obtain the following expression

$$K'(x,y) = \frac{g^2}{12}\frac{y^2}{(y+M^2)^2}[h(\xi)-h(\xi_0)], \qquad (3.1)$$

where

$$h(\xi) = 3-12\xi^2+8\xi^4-8\xi(\xi^2-1)^{3/2};$$

$$\xi = \frac{x+y+m^2}{2xy}; \quad \xi_0 = \frac{x+y}{2xy}.$$

In the case $x+y \gg m^2$ we get the convenient asymptotic representation of $K'(x,y)$

$$K'(x,y) \underset{x+y \gg m^2}{\approx} \frac{g^2 m^2}{3} \frac{y}{(y+M^2)^2} \left[\frac{y}{x^2}\theta(x-y) + \frac{x}{y^2}\theta(y-x)\right]. \tag{3.2}$$

In the case $x + y \ll m^2$ another representation is valid

$$K'(x,y) \underset{x+y \ll m^2}{\approx} \frac{g^2}{12M^4} \left[\left(2\frac{y^3}{x} - \frac{y^4}{x^2}\right)\theta(x-y) + (2xy - x^2)\theta(y-x)\right]. \tag{3.3}$$

It follows from Eq. (2.10) that the kernel $\bar{K}$ may be written in the following form

$$\bar{K}(x,y) = \int_0^\infty dz\, G(x,z) \frac{1}{z} \frac{d^2}{dz^2} \left[\frac{1}{z} \frac{d^2}{dz^2} (z^2 K'(z,y))\right]. \tag{3.4}$$

The asymptotic representations (2.11) and (3.2) give the following asymptotic behaviour of K for $x \to \infty$, $y \to \infty$

$$\bar{K}(x,y) \approx \frac{g^{3/2} m^2}{4} x^{-3/8} y^{-7/8} \{\theta(x-y) \exp[4e^{i\frac{\pi}{4}} ((g^2 y)^{1/4} - (g^2 x)^{1/4}) + i\frac{3\pi}{4}] + \theta(y-x) \exp[4e^{i\frac{\pi}{4}} ((g^2 x)^{1/4} - (g^2 y)^{1/4}) + i\frac{3\pi}{4}] + \text{c.c.}\}. \tag{3.5}$$

For $x \to 0$, $y \to 0$ we find similarly from (2.13) and (3.3) that

$$\bar{K}(x,y) \underset{\substack{x \to 0 \\ y \to 0}}{\approx} \frac{g^2}{12M^4} \left\{\left(2\frac{y^3}{x} - \frac{y^4}{x^2}\right)\theta(x-y) + (2xy - x^2)\theta(y-x)\right\}. \tag{3.6}$$

We do not write down here the asymptotic expressions of $\bar{K}(x,y)$ in the regions $x \to 0$, $y \to \infty$ or $x \to \infty$, $y \to 0$ which can be easily obtained. One can verify that an account of these regions does not change the following results.

Bearing in mind the asymptotic properties of the kernel $\bar{K}$ just found we can show that its norm is finite, i.e. that

$$\|\bar{K}\| = \int_0^\infty dx \int_0^\infty dy\, |\bar{K}(x,y)|^2 < \infty. \tag{3.7}$$

Inasmuch as $F^{(0)}(x)$ is square integrable function, Eq. (3.7) allows us to apply the Fredholm method to Eq. (1.7)[3]. We shall discuss in the following the possibilities which arise from this result. For investigating the convergence of the series (1.8) it is important, however, to know the behaviour of the norm $\|\bar{K}\|$ for small values of λ. Simple computations performed in the Appendix, lead to the following rough estimate.

$$\|\bar{K}\|^2 < C\lambda^4 |\log \lambda| \quad ; \tag{3.8}$$

where C is a dimensionless constant independent of λ. Hence, for sufficiently small values of λ the inequality $\|\bar{K}\| < 1$ is valid, and this is sufficient for the iterative series (1.8) to converge. The convergence is uniform in x in the interval $0 \leq x < \infty$.

As was shown in I, the iteration $F^{(n)}(x)$ decreases for $x \to \infty$ faster than $F^{(n-1)}(x)$. In virtue of the uniform convergence of the iterative series (for sufficiently small λ) its asymptotic behaviour is defined by the asymptotic behaviour of the zero approximation $F^{(0)}(x)$.

The last result is of great importance for proving the transition to the Euclidean metric in Eq. (1.2). This transition was performed by rotating the integration contour in the complex planes of the variables p_0 and q_0 by the angle $\frac{\pi}{2}$ [4]. In I we have proved the possibility of such a transformation for Eq. (1.6). The fact that the asymptotic behaviour of the exact solution $F(x)$ does not differ from that of the zero approximation allows us to prove the possibility of this rotation in the full equation (1.2).

Note that the Fredholm method, the applicability of which was justified above, is very convenient for an investigation of the analytic properties of $F(x)$. Generally, this problem is not different from that considered in[5] and we shall not elaborate it here. In conclusion of this section we would like to note that the Fredholm method may be used for any finite value of λ. This fact makes possible a study of the properties of the vertex function in the strong coupling case.

4. Expansion of the Solution for Small Values of the Coupling Constant

The present section is devoted to the computation of the radiative correction to the vertex function $F(x)$ for small values of λ. In I the expansion of $F^{(0)}(x)$ was found in the approximation $M=0$:

$$F^{(0)}(x)\big|_{M=0} = F_0^{(0)}(x) = A[1 + \frac{g^2 x}{6} \log(m^2 g^2) + \frac{g^2 x}{6}(\log\frac{x}{m^2} + 4\gamma - \frac{10}{3}) + 0(g^2 m^2)] ; \tag{4.1}$$

($\gamma = 0{,}577\ldots$ is the Euler constant, g^2 is connected with λ^2 by (2.1a)). We have noted there without proof that nonanalytic term $\frac{1}{6} g^2 x \log(m^2 g^2)$ does not change by taking into account $M \neq 0$ or the corrective kernel K'. Here we prove this assertion and at the same time compute all terms of the g^2 order in the exact solution $F(x)$.

The method being used allows one, generally, to write down the full expansion of the vertex function $F(x)$ in powers of λ^2 and $\log \lambda^2$. However, Eq. (1.2) evidently does not contain all the diagramms of higher orders (starting from λ^4). Therefore we restrict ourselves to the calculation of the terms being proportional to $\lambda^2 \log \lambda^2$ and λ^2 only.

For this purpose we use the representation of the resolvent in the form (2.9) and write down Eq. (1.7) as follows

$$F(x) = F^{(0)}(x) + \int_0^\infty dy\, K'(x,y) F(y) - g^2 \int_0^\infty dy \int_0^\infty dz \frac{G(x,z) K'(z,y)}{z(z+M^2)^2} F(y). \quad (4.2)$$

One can solve this equation by iterations, assuming that the expansion of the zero approximation $F^{(0)}$ is known. The second order, which we now seek for, is defined by the first two terms of the right-hand side of Eq. (4.2) only, since the contribution of the third term is proportional to λ^4. Let us first calculate the necessary terms of the expansion of $F^{(0)}$ using the expansion (4.1) for $F_0^{(0)}$. It can be easily verified that $F^{(0)}(x)$ satisfies the equation quite analogous to (4.2).

$$F^{(0)}(x) = F_0^{(0)}(x) + \int_0^\infty dy\, K_0'(x,y) F^{(0)}(y) - g^2 \int_0^\infty dy \int_0^\infty dz \frac{G_0(x,z) K_0'(z,y)}{z} F^{(0)}(y); \quad (4.3)$$

where the following notations are introduced

$$K_0'(x,y) = K^{(0)}(x,y) - K^{(0)}(x,y)\big|_{M=0} =$$

$$= -\frac{g^2 M^2}{12} \frac{2y+M^2}{(y+M^2)^2} \left[\left(\frac{y^2}{x^2} - 2\frac{y}{x}\right)\theta(x-y) + \left(\frac{x^2}{y^2} - 2\frac{x}{y}\right)\theta(y-x)\right]; \quad (4.4)$$

$$G_0(x,y) = G(x,y)\big|_{M=0}.$$

Thus, the correction to the terms of the expansion (4.1) are given by the expression

$$A \int_0^\infty dy (K'(x,y) + K_0'(x,y)). \quad (4.5)$$

One can be easily convinced that

$$K' + K_0' = \frac{g^2}{12} \frac{y^2}{(y+M^2)^2} h(\xi) + \frac{g^2}{12} \left[\left(2\frac{y}{x} - \frac{y^2}{x^2}\right)\theta(x-y) + \left(2\frac{x}{y} - \frac{x^2}{y^2}\right)\theta(y-x)\right]. \quad (4.6)$$

Omitting some simple but rather tedious calculations, we present the final form of the expansion of F in the case $M = m$ (which is taken for the sake of simplicity)

$$F(x) = A\Big[1 + \frac{g^2 x}{6}\log(g^2 m^2) - \frac{g^2 x}{6}\left(4\gamma + \frac{10}{3}\right) + \frac{2 g^2 m^2}{3} +$$
$$+ \frac{g^2}{12}(x - 2m^2)\sqrt{\frac{x+4m^2}{x}} \log\left|\frac{(x+2m^2)\sqrt{x(x+4m^2)} + (x+2m^2)^2}{2m^4} - 1\right| + O(g^2)\Big] \quad (4.7)$$

The results presented in this section show that for evaluating the terms being nonanalytic in the coupling constant it is necessary to solve exactly some sufficiently simple differential equations (as was done in I), then the calculation of the following corrections is not more difficult than the usual perturbation theory and reduces to a computation of some convergent integrals.

5. Conclusion

In conclusion we discuss the main results of the present and the previous[1] papers. First we briefly describe the general scheme for the solution of approximate linear equations in nonrenormalizable theories. In doing this we base on the detailed investigation of the equation for the vertex function.

The first step consists in dividing the kernel of an integral equation into the most singular part and the less singular one. The principle of such a division is formulated in the Introduction. Here our approach differs considerably from that of Feinberg and Pais[6]. The essence of their approach consists in choosing such a kernel of the equation for zero approximation, which gives exactly the most divergent terms in each order of perturbation theory. According to the recipe of Feinberg and Pais we should take the following expression instead of (1.4)

$$\frac{a\lambda^2}{m^2 p^2} \int \frac{d^4 q}{(2\pi)^4} \frac{(pq)}{(p-q)^2} F(q^2).$$

For the corresponding zero approximation we then were to obtain quite a simple differential equation of the second order. This equation has a solution, satisfing the boundary conditions. However, the asymptotic behaviour of this solution for large values of x bears no resemblance to the correct one. In particular, this solution does not admit the rotation of the integration contour, which is necessary for the transition to the Euclidean momenta. It is evident therefore that in this case the series of iterations cannot converge. Roughly speaking the main defect of the method of Feinberg and Pais lies in the obvious fact that the terms neglected are as important in the asymptotic region as the ones taken into account. Our rule consists simply in the recognition of equal rights of p and q in the kernel, and this guarantees the correct asymptotic behaviour at infinity.

The next steps, which were described in detail above, allow one to study the solution for any value of the coupling constant. In the case of the weak coupling the solution can be calculated with the arbitrarily high accuracy by using the modified perturbation theory, which enables us to take into account the non-analytic dependence on the coupling constant. By these computations we obtain the expansion in powers of λ^2 and of $\log\lambda^2$ x).

The method described above may be applied to a large class of problems in different nonrenormalizable theories. In particular, one can use this method for investigating the scattering amplitude in nonrenormalizable theories. In this case the problem of how to find the zero approximation also reduces to a differential equation with some boundary conditions. (Similar equations in the theory of nonrelativistic scattering on singular potentials were considered in[8]). All the following steps are also quite similar to the ones discussed above. Note, that the restriction $k_\mu = 0$ we have taken here, is not essential and in considering the case $k_\mu \neq 0$ we meet only the technical difficulties.

We believe the use of the Fredholm method in the equations similar to Eq. (1.7) to be very promising. For $k^2 \neq 0$, the Fredholm denominator depends on k^2 and other parameters of the problem. The zeros of the denominator define the energies of bound states of the system. So the possibility arises to treat the problem of the bound states in nonrenormalizable theories without using auxiliary parameters (cut-off, subtractions, etc).

The authors express their sincere gratitude to N.N.Bogolubov for the fruitful discussions.

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x) T.D.Lee[7] was the first who drew attention to the possibility of finding such terms in nonrenormalizable theories.

APPENDIX

For the estimate (3.8) to obtain, we consider the behaviour of the kernel $\bar{K}$ for $x+y \gg m^2$ in more detail. Choose the constant L of the mass dimension so that $m^2, M^2 \ll L^2 \ll m^2/\lambda^2$. We always may do so for sufficiently small λ. Divide now the domain of integration in (3.7) into two regions: 1. $x < L^2$, $y < L^2$; 2. $x > L^2$ or $y > L^2$.
For small λ the Green function is represented by eq. (2.13) in the region $x, y \lesssim L^2$. Therefore the integral over the first region may be estimated as follows

$$\int_0^{L^2} dx \int_0^{L^2} dy \, |\bar{K}(x,y)|^2 < \lambda^4 C_1 . \tag{A.1}$$

Indeed, λ^4 enters in this integral as a factor, while the integrand and the limits of integration do not depend on λ.

In the second region the representation (3.2) for K' is valid. Making use of eq. (3.4) and performing some simple calculations, we get that in this region

$$\bar{K}(x,y) = \frac{g\lambda^2}{8\pi^2} \frac{1}{y^2} \{ -\frac{\partial^2}{\partial y^2} G(x,y) + \frac{4}{y}\frac{\partial}{\partial y} G(x,y) \} . \tag{A.2}$$

Note, that in the second region the Green function $G(x,y)$ depends on g^2 in the following way

$$G(x,y) \approx \frac{1}{g^6} \bar{G}(u,v) ;$$

where

$$\bar{G}(u,v) = G(g^2 x, g^2 y) ;$$

and the dimensionless variables $u = g^2 x$, $v = g^2 y$ are introduced. Then we come to the expression

$$\int\limits_{x>L^2 \text{ or } y>L^2} dx \int dy \, |\bar{K}(x,y)|^2 = \left(\frac{g\lambda^2}{8\pi}\right)^2 \iint\limits_{\substack{u > g^2L^2 \text{ or} \\ v > g^2L^2}} du\,dv \, [\phi(u,v)]^2 ; \tag{A.3}$$

where

$$\phi(u,v) = \frac{1}{v^2} \{ -\frac{\partial^2}{\partial v^2} \bar{G}(u,v) + \frac{4}{v}\frac{\partial}{\partial v} \bar{G}(u,v) \} . \tag{A.4}$$

Bearing in mind that the integral in (A.3) converges we obtain the following estimate of (A.3) (with the aid of eq. (2.13) which is still valid for $x, y \approx L^2$)

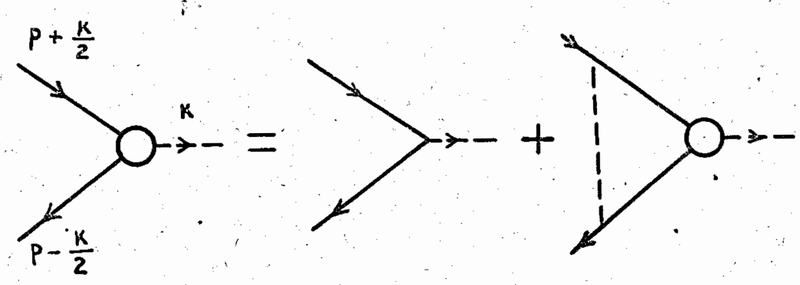
$$\int dx \int dy \, | \bar{K}(x,y) |^2 < \lambda^4 (C_2 |\log L^2 g^2| + C_3) . \qquad (A.5)$$

Now (A.2) and (A.5) give the estimate (3.8).

References

1. B.A.Arbuzov, A.T.Filippov. Preprint JINR P-1910 (1964). Nuovo Cim. in print.
2. E.L.Ince. Ordinary Differential Equations, ONTI, Kharkov, 1939.
3. S.G.Mikhlin. Doklady Academii Nauk USSR 42, 373 (1944).
4. G.C.Wick. Phys.Rev. 96, 1124 (1954).
5. B.A.Arbuzov, A.A.Logunov, A.T.Filippov, O.A.Khrustalev. JETP 46, 1266(1964).
6. G.Feinberg, A.Pais. Phys. Rev. 131, 2724 (1963); 133, B477 (1964).
7. T.D.Lee. Phys. Rev. 128, 899 (1962).
8. B.A.Arbuzov, A.T.Filippov. Phys.Lett., 13, 95 (1964).

Received by Publishing Department
on April 15, 1965.



F i g

The Edwards equation for the vertex function $\Gamma_\mu (p,k)$. The dotted line represents a free vector particle with mass m and the solid one shows a scalar particle with mass M.