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THE OBTAINING OF APPROXIMATE EQUATIONS FOR THE SCATTERING MATRIX ELEMENTS IN THE RELATIVISTIC THREE-BODY PROBLEM

The present paper is a continuation of papers[1-3] where the relativistic three-body problem has been discussed. Using the equations for the transition operators we obtain a system of equations for the scattering amplitude matrix elements. In particular, the approximations for three-particle scattering amplitudes are considered which appear to be respectively valid for the cases of strong and weak coupling inside two-particle bound states.

We shall consider three relativistic particles with different masses m_i, m_j and m_k. Assume that between the particles there exists only pairing interaction what allows us to use the results of the previous papers. In the present paper we employ also the Jacobi coordinates in the x-representation and the momenta conjugated to the latter.

Earlier[3] it was shown that the S-matrix elements are expressed in terms of the matrix elements of the scattering amplitude as follows:

$$S_{ij} = \delta_{(i)(j)} + T_{ij} \tag{1}$$

T_{ij} can be written in the form

$$T_{ij} = \chi^{+0}_{(i)} M_{ij} \chi^{0}_{(j)} \tag{2}$$

where M_{ij} satisfy the equations

$$M_{ij} = \sum_{\alpha \neq i} K_\alpha + \sum_{\beta \neq j} M_{i\beta}\, g_\beta\, K_\beta \; . \tag{3}$$

Here g_i is the two-particle Green function; K_α are the pair kernels, and χ^0_i is the solution of the two-particle Bethe-Salpeter equation for the i-th two-particle subsystem. In this paper we restrict ourselves to consideration of M_{ij} determined by eq. (3) which were denoted in ref.[3] by M^+_{ij} because the use of the second type operators M^-_{ij} has given us so far nothing new. As was noted earlier the operators can be also determined by the following relations

$$g_i M_{ij} g_j = g_i (K - K_i) g = g(K - K_j) g_j \; , \tag{4}$$

where g is the full three-particle Green function, and

$$g_i = g_0 + g_0 K_i g_i \quad . \tag{5}$$

From (4) it is seen that

$$g_i M_{ij} g_j = g_i M_{ik} g_k \quad . \tag{6}$$

The M_{ij} determined in such a way do not practically depend on the second index what indicates that there exists some connection between the M_{ij} for various j . From (6) it follows that eq. (3) can be rewritten in the form:

$$M_{ij} = (K - K_i) + M_{ij} g_j (K - K_j) \tag{7}$$

Thus, we have obtained a system of uncoupled equations for all M_{ij} .

In what follows we shall restrict ourselves to consideration of the problem of scattering of a particle on the two-particle bound state. We are going to consider two approximations:

1. Weak coupling inside the two-particle bound state. In that case the bound state mass $\mu_i < m_j + m_k$ and μ_i are of the same order that $m_j + m_k$.

2. Strong coupling. In that case $\mu_i \ll m_j + m_k$. Consider the first case. For definiteness we write down, e.g. the equation for M_{11} . Owing to (7) we have

$$M_{11} = K_2 + K_3 + M_{11} g_1 (K_2 + K_3). \tag{8}$$

Due to weak coupling we neglect the interaction inside the bound state putting $g_1 = g_0$. A formal solution of eq. (8) in our approximation is of the form

$$M_{11} = (K_2 + K_3)(1 - g_0 \; (K_2 + K_3))^{-1} \; . \tag{9}$$

Taking into account that

$$K_i = T_i \, (1 + g_0 T_i)^{-1} \; ; \; T_i = S_i^{-1} \hat{T}_i^{\,6} \, , \tag{10}$$

where $\hat{T}_i$ is the two-particle scattering amplitude, we obtain

$$M_{11} = \sum_{\substack{n=1 \\ i \neq k = 2,3}}^{\infty} \underbrace{T_i \, g_0 \, T_k \, g_0 \, T_i \; \dots}_{n} \tag{11}$$

The expansion (11) is an expansion in multiplicity of interactions. So, e.g. the first two terms correspond to the single scattering of an incident particle on each of the particles forming the bound state, the second two terms correspond to the consecutive scattering first on one particle and then on another and so on.

Of special interest is the case when multiple scatterings may be neglected. This occurs, e.g. in scattering of a nucleon on a deuteron at about 100 MeV energy in the c.m.s. of the deuteron. Then

$$M_{11} = T_2 + T_3 . \qquad (12)$$

From (12) we can obtain the expressions for the amplitudes describing scattering processes on the bound state accompanied or not accompanied by decays as well as scattering processes with production or without production of a bound state. Notice that the considered approximation is a direct analog of the impulse approximation[5].

Now we go over to strong coupling. The approximations we are going to consider would be justified, provided either $\mu \ll m_1 + m_2$ or $m_1, m_2 \to \infty$, μ being finite. We obtain quations making use of the transition $m_1, m_2 \to \infty$. In doing so, the free term of the two particle Bethe-Salpeter equation vanishes ($g_0 \sim \frac{1}{m_1 m_2}$ for spinor particles, $g_0 \sim \frac{1}{m_1^2 m_2^2}$ for scalar ones). Then for the two-particle Green function we have the equation satisfying the homogeneous Bethe-Salpeter equation. Using the expansion of the Green function in the two-particle wave functions and noticing that in this expansion only the bound states contribute we get

$$\hat{g}_1 (P_1 \bar{x}_1 \bar{y}_1) \sim \sum_n \frac{\omega^n_{P_1}(\bar{x}_1)\overset{+}{\omega}{}^n_{P_1}(\bar{y}_1)}{P_{10} - \sqrt{\bar{P}_1^2 + \mu_n^{12}} + i\epsilon} \qquad (13)$$

where $\omega^n_{P_1}(\bar{x}_1)$ obey the equation

$$\omega^n_{P_1}(\bar{x}_1) = \frac{1}{(2\pi)^8} \int g_0(P_1 \ \bar{x}_1 \bar{u}_1) K_1 (P_1 \bar{u}_1 \bar{v}_1) \omega^n_{P_1}(\bar{v}_1) d\bar{u}_1 d\bar{v}_1 . \qquad (14)$$

Starting from the equation for M_{11} (7) we can obtain equations simultaneously for the matrix elements T_{11} and T_{10}. Indeed, from the connection between M_{11} and M_{10} ($g_1 M_{11} g_1 = g_1 M_{10} g_0$)

$$T^{n0}_{10}(P\tilde{p}_1 \tilde{p}'_1 \tilde{\bar{p}}_1) = \int \chi^n_{P\tilde{p}_1} (X_1 \tilde{x}_1 \bar{x}_1) M_{11} (X - Y \ \tilde{x}_1 \bar{x}_1 \tilde{y}_1 \bar{y}_1) \times$$
$$\times \overset{+}{\chi}{}^0_{P\tilde{p}'_1 \bar{p}'_1} (Y \tilde{y}_1 \bar{y}_1) dX dY d\tilde{x}_1 d\tilde{y}_1 d\bar{x}_1 d\bar{y}_1 . \qquad (15)$$

In addition

$$T^{nm}_{11} (P\tilde{p}_1 \tilde{p}'_1) = \int \chi^n_{P\tilde{p}_1} (X \tilde{x}_1 \bar{x}_1) M_{11} (X - Y, \tilde{x}_1 \bar{x}_1 \tilde{y}_1 \bar{y}_1) \times$$
$$\times \overset{+}{\chi}{}^m_{P\tilde{p}'_1} (Y \tilde{y}_1 \bar{y}_1) dX dY d\tilde{x}_1 d\tilde{y}_1 d\bar{x}_1 d\bar{y}_1 , \qquad (16)$$

where

$$\chi^{0}_{P\tilde{p}_1\bar{p}_1}(X\tilde{x}_1\bar{x}_1) = e^{-iPX - i\tilde{p}_1\tilde{x}_1} \quad f_{\frac{M_1}{M}P + \tilde{p}_1, \bar{p}_1}(\bar{x}_1) \tag{17}$$

$$\chi^{n}_{P\tilde{p}_1}(X\tilde{x}_1\tilde{x}_1) = e^{-iPX - i\tilde{p}_1\tilde{x}_1} \quad \omega^{n}_{\frac{M_1}{M}P + \tilde{p}_1}(\bar{x}_1). \tag{18}$$

Here we notice that when the masses of particles entering the bound state are equal to infinity there is no reason to consider the amplitudes for the production and the decay, since they are all exactly zero. However, we assume that $\mu \ll m_1$, $\mu \ll m_2$ but $m_1, m_2 < \infty$ then T_{01} and T_{10} differ from zero and can be approximately found by means of T_{11} at $m_1 m_2 \to \infty$.

Taking into account eqs. (7), (13) and (16) we get the following equation:

$$T^{nm}_{11}(P\tilde{p}_1\tilde{p}'_1) = \langle K - K_1\rangle^{nm}_{11}(P\tilde{p}_1\tilde{p}'_1) + \sum_{n'}(2\pi)^{-18}\int T^{nm'}_{11}(P\tilde{p}_1\tilde{p}''_1) \times$$
$$\times \frac{S(\frac{m_1}{M}P + \tilde{p}''_1)\langle K - K_1\rangle^{n'm}_{11}(P\tilde{p}''_1\tilde{p}'_1)}{\frac{M_1}{M}P_0 + \tilde{p}''_{10} - \sqrt{[\frac{M_1}{M}\vec{P} + \vec{\tilde{p}}'']^2 + \mu'^2_n} + i\epsilon}\, d\tilde{p}'' \tag{19}$$

where

$$\langle K - K_1\rangle^{nm}_{11}(P\tilde{p}_1\tilde{p}'_1) = (\overset{+}{\chi}{}^{n}_{P\tilde{p}_1}(K - K_1)\chi^{m}_{P\tilde{p}_1})$$
$$\langle K - K_1\rangle^{n0}_{11}(P\tilde{p}_1\tilde{p}'_1\bar{p}_1) = (\overset{+}{\chi}{}^{n}_{P\tilde{p}_1}(K - K_1)\chi^{0}_{P\tilde{p}'_1\bar{p}_1}). \tag{20}$$

The scattering amplitude matrix elements T_{11} from eq. (19) are, obviously, approximate for the case $\mu \ll m_1 + m_2$. Making similar calculations and using eq. (15) we get

$$T^{n0}_{10}(P\tilde{p}_1\tilde{p}'_1\bar{p}'_1) = \langle K - K_1\rangle^{n0}_{11}(P\tilde{p}_1\tilde{p}'_1\bar{p}'_1) + (2\pi)^{-18}\sum_{n'}\int T^{nn'}_{11}(P\tilde{p}_1\tilde{p}''_1) \times$$
$$\times \frac{S(\frac{m_1}{M}P + \tilde{p}''_1)\langle K - K_1\rangle^{n'0}_{11}(P\tilde{p}''_1\tilde{p}'_1\bar{p}'_1)}{\frac{M_1}{M}P_0 + \tilde{p}''_{10} - \sqrt{[\frac{M_1}{M}\vec{P} + \vec{\tilde{p}}'']^2 + \mu'^2_n} + i\epsilon}\, d\tilde{p}''_1 \tag{21}$$

T_{11} entering this equation can be taken out from (19), since corrections to it (because of $m_1 m_2 < \infty$) will be of higher order in the expansion in the inverse powers of the masses of particles forming the bound state.

In conclusion we note that if, instead of the pole expression for g (13) we took into account the scattering states, then we would not able to obtain a separate equation for T_{11}. Thus we have shown that in the case of strong coupling the equations for the three-particle scattering amplitudes are noticeably simplified and reduce to the multichannel two-particle Lippmann-Schwinger equations.

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