

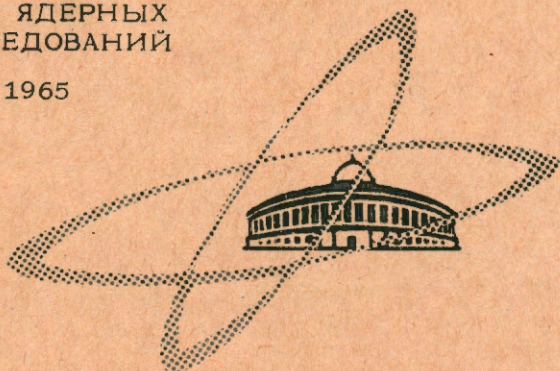
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ЭКЗ. ЧИТ. ЗАЛА

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ИНСТИТУТ
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SELF-CONSISTENT CALCULATION
OF THE K^* -RESONANCE PARAMETERS

ЛАБОРАТОРИЯ ГЕОРЕГИОНСКОЙ ФИЗИКИ

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Направлено в ЖЭТФ

ОИИ
БИБЛИОТКА

1. Introduction

The bootstrap method proposed by Chew and Frautschi about four years ago^{1/} is at present rather widely known. The essence of this method, its advantages and disadvantages are presented in the report by Shirkov^{2/}. The bootstrap method is used to describe almost all presently available meson and baryon resonances.

In the present paper the bootstrap method is used to find the K^* -resonance parameters ($T = \frac{1}{2}$, $J = 1$ the mass M_{K^*} is 888 MeV, the width Γ_{K^*} is 50 MeV). This problem was already considered in a number of papers^{3-5/}. Capps^{3/} treated a three-channel problem (πK , $\pi\eta$, ηK) but he has not succeeded in finding good agreement with experimental data especially in estimating the resonance width. It was also noted when the vector particle exchange occurs further approximations in the $\frac{N}{D}$ method lead to divergences. In author's opinion the bootstrap method may be improved by using the Regge representation for the asymptotic behaviour of the amplitude. In papers by Diu et al^{4/} both the single-channel and two-channel problems were considered. In both cases the authors have not succeeded in obtaining convincing arguments of the existence of bootstrap solutions. The two essential features were noted there:

- the dependence of the results on the cut-off and, as a consequence, the influence of the choice of the asymptotic behaviour on the bootstrap solution,
- in the two-channel problem there arises the problem of stability of the obtained solutions. Fulco et al.^{5/} have considered the effect of neighboring inelastic channels on the resonance width. The authors have obtained interesting results pointing to the fact that the resonance width becomes narrow by taking into account neighboring channels (in particular, the K^* -resonance width decreased from 210 MeV down to 150 MeV in taking into account the second channel). However, their results essentially depend on the cutoff as well. The authors think that the model can be improved by considering more carefully the asymptotic behaviour of the amplitudes and taking into account inelastic processes.

A series of papers by Capps^{6/} was devoted to the finding of bootstrap solution in the framework of the $SU(3)$ symmetry. It was found that such solutions may exist if we assumed beforehand the existence of six types of mesons (η , π , ω , ρ , K , K^*) and ascribe to them all known quantum numbers (isospin, strangeness and so on). However the bootstrap equation themselves do

not lead to the well-known meson octets and are satisfied by smaller sets of particles. A detailed calculation of the K^* -meson characteristics is not given there.

In the present papers the K^* -resonance parameters are determined by the Balaz method according to which the asymptotic behaviour of the amplitudes is described by the Regge poles from the crossing channels, and the effect of inelastic processes is taking into account by introducing a certain function into the two-particle unitarity condition. As far as in the one channel case the problem depends on many parameters (the slope of the Regge trajectory, the coupling constants $\rho_{\pi\pi}$ and $\rho_{K\bar{K}}$, the account of inelastic processes), there is a relative freedom in the choice of the parameters to obtain the bootstrap solution. The relative freedom implies here the restriction of the above parameters in the limits of reasonable values. Thus, in the present paper solutions close to the experimental data are found as well as the dependence of these solutions on the choice of the parameters is investigated. In particular, the obtained solutions turn out to be essentially dependent on the choice of the point of comparison. A similar result has been obtained in some other papers, e.g. ^[8].

II. Kinematics. Formulation of the Problem

The scattering amplitude $\pi(q_1) + K(p_1) \rightarrow \pi(q_2) + K(p_2)$ is considered as a function of the variables s, u, t . In the S -channel these variables are of the form

$$\begin{aligned} s &= (p_1 + q_1)^2 = M^2 + \mu^2 + 2k_s^2 + 2\sqrt{(k_s^2 + \mu^2)(k_s^2 + M^2)} \\ u &= (p_1 - q_2)^2 = 2(M^2 + \mu^2) - s - t \\ t &= (q_1 - q_2)^2 = -2k_s^2(1 - z_s), \end{aligned} \quad (1)$$

where M and μ are the masses of the K -meson and the π -meson respectively, k_s and z_s are the momentum and the cosine of the scattering angle in the c.m.s. In the crossing u -channel ($\pi' + K \rightarrow \pi' + K$) s and u interchange. In the t -channel (the annihilation channel $\pi + \pi \rightarrow K + \bar{K}$) the variables s, u, t in the c.m.s. of the third channel are of the form:

$$\begin{aligned} s &= -p^2 - q^2 + 2pqz_t \\ u &= 2(M^2 + \mu^2) - s - t \\ t &= 4(q^2 + \mu^2) = 4(p^2 + M^2) \end{aligned} \quad (2)$$

where z_t is the cosine of the scattering angle in this channel. The connection between the πK and $\pi' K$ scattering amplitudes and the $\pi\pi \rightarrow K\bar{K}$ amplitudes is given by the rule:

$$A^1(s, t) = \sum_I \alpha_{II'} \bar{A}^{I'}(u, t) = \sum_I \lambda_{II'} \bar{A}^{I'}(t, s), \quad (3)$$

where

$$\alpha_{II'} = \frac{1}{3} \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}; \quad \lambda_{II'} = \begin{pmatrix} 1 & \frac{1}{\sqrt{6}} \\ -\frac{1}{2} & \frac{1}{\sqrt{6}} \end{pmatrix} \quad (4)$$

From the Mandelstam representation it follows that the partial wave amplitudes in the S -channel have in the S -plane the following cuts, see Fig. 1.

In the Balaz method these three cuts are approximated by the two cuts along the real axis:

1. the right cut (physical) - $[M + \mu, \infty]$
2. the left cut (unphysical) - $[-\infty, M^2 - \mu^2]$

The problem is to find the amplitude $A_{J=1}^{T=H}(s)$ by means of which the K^* -resonance parameters are determined. Forces on the left cut in the low energy range are given by the diagrams of Fig. 2 and in the high-energy range are defined by the Regge pole from the S -channel. The position of the ρ -meson and the coupling constants $g_{\rho\pi\pi}$ and $g_{\rho K\bar{K}}$ are the given parameters. The position and the width of the K^* -meson are determined from the bootstrap method equations.

III. Derivation of the Bootstrap Method Equations

The amplitude $A_1^H(s)$ is written in the form

$$H_1^H(s) = \frac{1}{s - (M + \mu)^2} A_1^H(s) = \frac{N(s)}{D(s)} \quad (5)$$

where $N(s)$ has the left cut only and $D(s)$ has the right cut only. We write the unitarity condition for the function $H_1^H(s)$:

$$\text{Im} [H_1^H(s)^{-1}] = -\frac{k_s}{\sqrt{s}} [s - (M + \mu)^2] \cdot R(s) \quad (6)$$

where $R(s)$ is the ratio of the total cross section for the $\pi-K$ scattering to the elastic scattering one. The function $R(s)$ implies the contribution of inelastic processes.

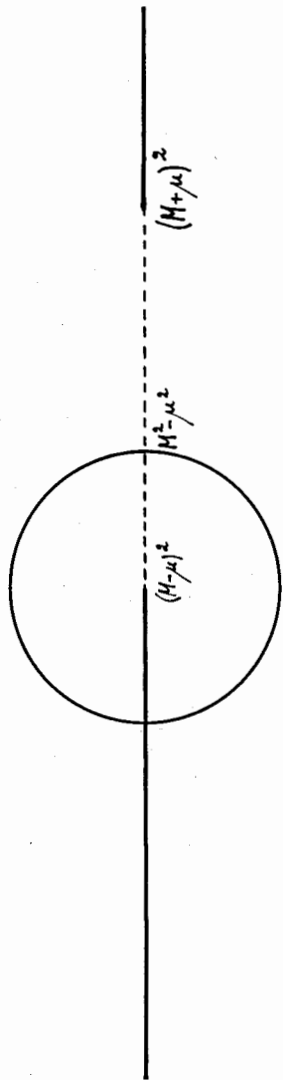


Fig. 1

Position of the cuts of the partial amplitudes of the $\pi K-\pi K$ scattering at the s-plane.

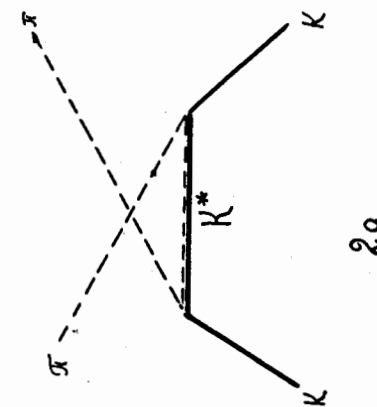
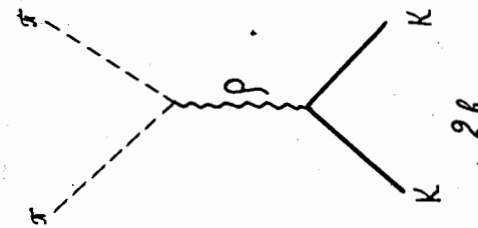


Fig. 2

Diagrams which give the low-energy forces on the left cut.

From eq. (5) we find $\text{Im}N(s)$ (on the left cut) and $\text{Im}D(s)$ (on the right one), after this we write down the equations for $N(s)$ and $D(s)$

$$N(s) = \frac{1}{\pi} \int_{-\infty}^{M^2-\mu^2} ds' \frac{D(s') \text{Im}H_1^{\frac{1}{2}}(s')}{s'-s} \quad (7)$$

$$D(s) = 1 - \frac{s-s_0}{\pi} \int_{(M+\mu)^2}^{\infty} ds' \frac{k'_s}{\sqrt{s'}} \cdot \frac{N(s') \cdot R(s') [s' - (M+\mu)^2]}{(s'-s)(s'-s_0)}$$

The dispersion relation for $D(s)$ is written with one subtraction at the point s_0 . $D(s_0) = 1$. We suppose that the contribution from the left cut can be represented as a sum of the two poles;

$$N(s) = \sum_{i=1,2} \frac{a_i}{s-s_i} \quad (8)$$

where a_i is the residue of the suitable poles. The position of the poles is determined by the way indicated in ref. /7/. They are located at the points $s_1 = -57$ and $s_2 = 10,5$ (if the cut from the t -channel is taken into account) or at the points $s_1 = -57$ and $s_2 = 4,5$ (if the cut from the t -channel is neglected). The second pair of poles corresponds to the case when the influence of the ρ -meson forces reduces practically to zero.

The parameters a_1 and a_2 are determined from the comparison of the amplitude (5) and its first derivative $\frac{\partial H_1^{\frac{1}{2}}(s)}{\partial s}$ with the calculated function $H_1^{\frac{1}{2}}(s)$ and its first derivative respectively at a certain point of comparison s_{compar} . To reduce the number of parameters we assume $s_{\text{compar}} = s_0$. We calculate the function $H_1^{\frac{1}{2}}(s)$. For fixed s we can write the following dispersion relation for the partial wave

$$A_1^{\frac{1}{2}}(s) = \frac{1}{2\pi k_s^2} \left\{ \int_2^{\infty} A_1^{\frac{1}{2}}(s,t) Q_1 \left(1 + \frac{t'}{2k_s^2}\right) dt' - \int_{(M+\mu)^2}^{\infty} A_u^{\frac{1}{2}}(s,u) \cdot Q_1 \left(-1 - \frac{u' - \frac{(M-\mu)^2}{s}}{2k_s^2}\right) du' \right\} \quad (9)$$

where $A_1^{\frac{1}{2}}(s,t')$ and $A_u^{\frac{1}{2}}(s,u')$ are the imaginary parts of the amplitudes in the t - and u - channels respectively for which the ordinary expansions in the physical channels hold:

$$A_u^{\frac{1}{2}}(s,u') = \sum_{\ell, \ell'} \alpha_{\ell \ell'} A_{\ell u}^{\ell'}(u') P_{\ell}(z_u) \quad (10a)$$

$$A_1^{\frac{1}{2}}(s,t') = \sum_{\ell, \ell'} \lambda_{\ell \ell'} A_{\ell 1}^{\ell'}(t') P_{\ell}(z_t) \quad (10b)$$

One of the restrictions on the choice of the point of comparison $s_{\text{compar}} = s_0$ follows from the consideration of the analyticity domain of $A_u^{\frac{1}{2}}$ and $A_1^{\frac{1}{2}}$. The point of comparison is taken only in that region of the variable s where the expansions (10) are allowed.

In eq. (9) each of the integrals is divided into two parts, i.e. the low-energy part $A_1^{\frac{1}{2}(L)}(s)$ and the high energy one $A_1^{\frac{1}{2}(H)}(s)$. It is supposed that the main contribution to the low-energy part in the u -channel is given by the (2a) diagram which is due to the K^* -meson exchange and therefore in the expansion (10d) there remains only one term with $\ell = i$. The partial wave $A_{1u}^{\ell'}(u')$ is approximated by the Breit-Wigner formula which, in the zero width approximation, is expressed as

$$A_{1u}^{\ell'}(u') = -[u' - (M+\mu)^2] \cdot \pi \cdot \Gamma_1^{\frac{1}{2}} \cdot \delta(u' - u_1) \quad (11)$$

where u_1 is the K^* -resonance position and $\Gamma_1^{\frac{1}{2}}$ is connected with the K^* -resonance width by the equation $\Gamma_{K^*}^{\frac{1}{2}} = \frac{k_r [s_r - (M+\mu)^2]}{s_r} \Gamma_1^{\frac{1}{2}}$. The expression for $A_{\ell t}^{\ell'}(t')$ is found from the diagram (2b) by means of perturbation theory:

$$A_{\ell t}^{\ell'}(t') = 3(pq) \cdot \pi \cdot \Gamma_1^1 \cdot \delta(t' - t_1) \quad (12)$$

where $\Gamma_1^1 = \frac{8}{3} g_{mp} g_{K^* \rho}$, $t_1 = M_{\rho}^2$ is the ρ -meson mass. Using (11) and (12) for the low-energy part $A_1^{\frac{1}{2}(L)}(s)$ of the amplitude (9) we obtain the expression:

$$A_1^{\frac{1}{2}(L)}(s) = \frac{3 \cdot \Gamma_1^1}{4k_s^2} (s + p_r^2 + q_r^2) \cdot Q_1 \left(1 + \frac{t_r}{2k_s^2}\right) + \frac{\Gamma_1^{\frac{1}{2}}}{2k_s^2} \left\{ u_r - (M+\mu)^2 + \frac{2u_r [2(M^2+\mu^2) - u_r - s]}{u_r - (M-\mu)^2} \right\} Q_1 \left(1 + \frac{u_r - \frac{(M-\mu)^2}{s}}{2k_s^2}\right) \quad (13)$$

The high-energy contribution of $A_1^{\frac{1}{2}(H)}(s)$ is believed to be defined by the K^* -meson from the S -channel, i.e.

$$A_{\alpha}^{\frac{1}{2}}(s, t) = -\frac{\pi[2\alpha(s)+1]}{2\sin \pi\alpha(s)} \cdot \beta(s)[P_{\alpha}(-z) - P_{\alpha}(z)] \quad (14)$$

A similar expression is written for $A_{\alpha}^{\frac{1}{2}}(s, u)$. The cuts of the functions $P_{\alpha}(-z)$ and $P_{\alpha}(z)$ in the t and u -channels are known. Using (14) we get the following expressions for the imaginary parts of the amplitude $A_{\alpha}^{\frac{1}{2}}$ in the corresponding channels:

$$A_{u\alpha}^{\frac{1}{2}}(s, u) = -\frac{\pi}{2}[2\alpha(s)+1] \cdot \beta(s) \cdot P_{\alpha}\left[1 + \frac{u - (M-\mu)^2}{2k_s^2}\right] \quad (15a)$$

$$A_{t\alpha}^{\frac{1}{2}}(s, t) = -\frac{\pi}{2}[2\alpha(s)+1] \cdot \beta(s) \cdot P_{\alpha}\left[1 + \frac{t}{2k_s^2}\right] \quad (15b)$$

Employing the asymptotic values for $A_{u\alpha}^{\frac{1}{2}}$ and $A_{t\alpha}^{\frac{1}{2}}$ and the functions $Q_1(z_u)$ and $Q_1(z_t)$ substituting them into (9) we get:

$$A_1^{\frac{1}{2}(H)}(s) = -\frac{1}{3} \cdot \frac{[2\alpha(s)+1] \cdot \beta(s) \cdot C_1(\alpha)}{\alpha-1} \left(\frac{t_d}{s_d}\right)^{\alpha-1} \quad (t_d = s_d) \quad (16)$$

where t_d is the lower boundary in both integrals (9), determined from the two considerations: a) first, this boundary must be sufficiently distant in order that we might speak about the asymptotic behaviour of the amplitude and, second, it can not start nearer the singularities of the functions $P_{\alpha}(z_t)$ and $P_{\alpha}(-z_t)$. The point $t_{\alpha} \geq 130$ satisfies these requirements. In our paper $t_{\alpha} = 130$

Suppose^{x/} that

$$C_1(\alpha)[2\alpha(s)+1] \cdot \beta(s) \left(\frac{1}{2k_s^2}\right)^{\alpha-1} = \text{Const} \quad (17)$$

$$\text{Re} \alpha = 1 + \epsilon(s - s_r) \quad (18)$$

where ϵ is the slope of the Regge trajectory. We determine also the residue $\beta(s)$:

$$\beta(s) = [s - (M + \mu)^2] \cdot \Gamma_1^{\frac{1}{2}} \cdot \frac{d\alpha}{ds} \Big|_{s=s_r} \quad (19)$$

Eqs. (16)-(19) yield

$$A_1^{\frac{1}{2}(H)}(s) = \Gamma_1^{\frac{1}{2}} \cdot \frac{s - (M + \mu)^2}{s_r - s} \cdot (t_d)^{\epsilon(s-s_r)} \quad (20)$$

^{x/} In ref./9/ it is shown that the choice of such a condition provides a satisfactory narrowing of the diffraction peak.

Thus, from eqs. (5), (8), (9), (13) and (20) we get the following equations for the determination of the residues a_1 and a_2

$$A_1^{\frac{1}{2}}(s_0) = [s_0 - (M + \mu)^2] \left[\frac{a_1}{s_0 - s_1} + \frac{a_2}{s_0 - s_2} \right];$$

$$\begin{aligned} \frac{\partial A_1^{\frac{1}{2}}(s)}{\partial s} \Big|_{s=s_0} &= \sum_{i=1,2} a_i \left\{ \frac{1}{s_0 - s_i} - \frac{s_0 - (M + \mu)^2}{(s_0 - s_i)^2} \right\} + \\ &+ \frac{A_1^{\frac{1}{2}}(s_0)}{\pi} \int_{s_d}^{s_d'} ds' \frac{k_s'}{\sqrt{s'}} \cdot \frac{[s' - (M + \mu)^2]}{(s' - s_0)^2 (s' - s_i)} + \frac{A_1^{\frac{1}{2}}(s_0)}{\pi} R(s) \int_{s_d}^{\infty} ds' \frac{k_s'}{\sqrt{s'}} \cdot \frac{[s' - (M + \mu)^2]}{(s' - s_0)^2 (s' - s_i)} \end{aligned} \quad (21)$$

In eq. (21) $R(s)$ is put to be constant. An approximate expression for it will be given below. On the left, instead of $A_1^{\frac{1}{2}}(s_0)$ at the point $s_0 = s_{\text{comp}}$, the sum of the expressions (13) and (20) is inserted, and instead of the derivative $\frac{\partial A_1^{\frac{1}{2}}(s)}{\partial s} \Big|_{s=s_0}$, the derivative of the same sum. Now the function $D(s)$ can be found after substitution of a_1 and a_2 from (21) into eq. (7), then the position and the width of the resonance are determined from the conditions

$$\text{Re} D(s_r) = 0 \quad (22)$$

$$\Gamma_1^{\frac{1}{2}} = - \frac{N(s_r)}{\frac{\partial \text{Re} D(s)}{\partial s} \Big|_{s=s_r}}$$

respectively. The behaviour of $\text{Re} D(s)$ near the resonance is rather well approximated by the expression (see Fig. 3):

$$\text{Re} D(s) = \frac{s - s_r}{s_0 - s_r}$$

from where

$$\Gamma_1^{\frac{1}{2}} = (s_r - s_0) \cdot N(s_r) \quad (23)$$

The quantity $R(s)$ is calculated under the assumption that the contribution of inelastic processes to the $\pi K - \pi K$ scattering is provided by the Pomernchuk pole:

$$A^0(t, s) = \gamma(t) \frac{\pi[2\alpha(s)+1]}{2\sin \pi\alpha(t)} [1 + e^{-i\pi\alpha(t)}] \cdot C_1(\alpha) \left(\frac{s}{\mu^2}\right)^{\alpha(t)} \left(\frac{\mu^2}{2}\right)^{\alpha(t)}$$

Near the resonance the quantity ϵt is rather small and in this approximation the expression for $A^0(t, s)$ takes on the form

$$A^0(t,s) = -3\gamma(0) \left(\frac{\pi\epsilon t}{2} + i \right) \frac{\pi s}{4} e^{\epsilon t \ln \frac{s}{\mu^2}}$$

After simple integration the partial wave $A_{l=1}^{\frac{1}{2}}(s) = \frac{1}{2} \int_{-1}^{+1} dz z^n \cdot z \cdot \frac{A^0(t,s)}{\sqrt{6}}$ is expressed as follows

$$A_{l=1}^{\frac{1}{2}}(s) = - \frac{3\pi \cdot s}{16\sqrt{6}} \cdot \frac{\gamma(0)}{k_s^2 \epsilon \ln \frac{s}{\mu^2}} \{ a_R + i a_I \}$$

where

$$a_R = - \frac{\pi}{2 \ln \frac{s}{\mu^2}} + \frac{\pi\epsilon}{2k_s^2 (\epsilon \ln \frac{s}{\mu^2})^2}; \quad a_I = 1 - \frac{1}{2k_s^2 \cdot \epsilon \cdot \ln \frac{s}{\mu^2}}$$

Making use of the relations:

$$\text{Im } A_l(s) = \frac{\sqrt{s}}{k_s} |A_l(s)|^2 \cdot R(s)$$

and

$$\gamma(0) = - \frac{\sqrt{6}}{12\pi^2} \cdot \sigma_{\text{tot}}$$

where σ_{tot} is the total cross section for the $\pi\pi \rightarrow \text{KK}$ annihilation, it is easy to obtain an approximate expression for $R(s)$:

$$R(s) = \frac{64\pi\epsilon \ln s}{\sigma_{\text{tot}} \left(1 + \frac{\pi^2}{4 \ln^2 \frac{s}{\mu^2}} \right)} \quad (24)$$

To simplify the calculations this expression is considered everywhere at the point $s = s_d$. For particular values of $\sigma_{\text{tot}} = 50 \text{ mb}$ and $\epsilon = \frac{1}{20}$ the quantity $R(s)$ is

$$R(s) = 300\epsilon = 15. \quad (25)$$

VI. Calculation of the K^* -meson parameters. Conclusion

From eqs. (13), (20) and (21) it follows that in the problem the parameters Γ_1^1 , t_p , ϵ , t_d , s_0 , $R(s)$ are free. In reality neither of these quantities can assume arbitrary values. The width Γ_1^1 and the position of the resonance t_p of the ρ meson must be taken from experimental data. The slope of the Regge trajectory ϵ is chosen in limits $\frac{1}{10} > \epsilon > \frac{1}{50}$ which are reasonable from the physical point of view. The quantity $R(s)$ depending on σ_{tot} and ϵ lies in the limits $[200\epsilon - 350\epsilon]$ if it is assumed that $40 \text{ mb} < \sigma_{\text{tot}} < 80 \text{ mb}$. The quantity t_d on the one hand, is determined from the considerations to eq. (16), and on the other, it is defined as a boundary

from which the inelastic process contribution becomes essential. The position of the subtraction point s_0 (or the point of comparison) is determined basing on the following arguments^[4,7]: on the left its value is bounded by the expansion conditions for the functions $A_{\frac{1}{2}}^{\frac{1}{2}}(s,u)$ and $A_{\frac{1}{2}}^{\frac{1}{2}}(s,t)$ and on the right its values cannot lie higher than the physical process threshold. In our problem we choose it near the unphysical cut.

The self-consistent calculation of the K^* -resonance parameters consists in that for the given parameters Γ_1^1 , t_p , ϵ , t_d , s_0 , $R(s)$ some input values of the parameters $\Gamma_1^{\frac{1}{2}(\text{in})}$ and $s_p^{(\text{in})}$ were chosen as close as possible to the experimental data. The residues a_1 and a_2 (see eq. (21)) were determined according to this set of values and then eqs. (22) were solved. If the input values of the parameters $\Gamma_1^{\frac{1}{2}(\text{in})}$ and $s_p^{(\text{in})}$ coincided with those of $\Gamma_1^{\frac{1}{2}(\text{out})}$ and $s_p^{(\text{out})}$ then the problem was assumed to be solved finally. Since the problem was solved numerically and the obtained values of $\Gamma_1^{\frac{1}{2}(\text{out})}$ and $s_p^{(\text{out})}$ were equal to $\Gamma_1^{\frac{1}{2}(\text{in})}$ and $s_p^{(\text{in})}$ only approximately then they were recalculated according to the same formulas in order to check the convergence of the solution to the definite values.

In such a way we have found several solutions. We give here the most interesting cases. (See Table I).

All the above solutions are found taking into account ρ -mesic forces. Within the limits of the considered values of the parameters we have never succeeded in finding bootstrap solutions when the contribution from the ρ meson was absent. To improve the agreement between the calculated width $\Gamma_{K^*}^{\frac{1}{2}}$ and experimental data we need either to increase the slope of the Regge trajectory (comp. solutions 1 and 2) or to decrease the $s_p^{(\text{in})}$. There may exist two different points of comparison or two different slopes of the Regge trajectories and two different points of comparison giving the same solutions for s_p and $\Gamma_{K^*}^{\frac{1}{2}}$. The account of the inelasticity is very essential. To improve agreement with experimental data the quantity $R(s)$ must be taken in the interval $15 \leq R(s) \leq 25$. The solutions strongly depend on the choice of the point $s_0 = s_{\text{comp}}$. (see Fig. 3).

In the limits $-10 < s_0 < 20 \cdot 25$ and $s_0 < s < 50$ all the obtained solutions are unique.

So, satisfactory agreement with experimental data can be obtained even in the simplest one-channel problem, in determining the width especially. It is known

that the account of other channels effectively influences the decrease of the resonance width⁵. According to the Balaz method the effect of other channels on the solution occurs via the function $R(s)$.

The disadvantage of the method is the dependence of the solution on a large number of parameters. This leads to that side by side with solutions interesting from the physical point of view we can obtain solutions which are not connected with experimental data.

In conclusion I express my gratitude to M. Severinsky for useful discussions and to V. Nikitin for the aid in making numerical calculations on the electronic computer.

References

1. G.F. Chew and S. Frautschi, Phys. Rev. Lett. **7**, 394 (1961).
2. Д.В. Ширков, Доклад Теория пион-нуклонного взаимодействия в области энергий ниже 1 Гэв. Международная конференция по физике высоких энергий. Дубна, 5-15 августа, 1964 года.
3. R.H. Capps, Phys. Rev. **131**, 1307 (1963).
4. B. Diu, J.L. Gervais and H.R. Rubinstein, Phys. Lett., **4**, 280 (1963); Nuovo Cim. **31**, 27 (1964).
5. J.R. Fulco, G.L. Shaw, D.Y. Wong, ITP-139 (1964).
6. R.H. Capps, a) Phys. Rev. Lett., **10**, 32 (1963); b) Nuovo Cim. **30**, 340 (1963); c) Preprint ICTP-64/70.

Additional references on the other papers by Capps and on the papers of other authors connected with this topic are given in the present preprint.

7. L. Balaz, Phys. Rev. **128**, 1939 (1962); Phys. Rev. **132**, 867 (1963).
8. S. Rai, Choudhury and L.K. Pande, Phys. Rev. **135**, B1027 (1964).
9. B.M. Udgaonkar, M. Gell-Mann, Phys. Rev. Lett. **8**, 346 (1962).

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Table I

N sol.	ϵ	s_0	$R(s)$	t_r	Γ_1^1	$s^{(in)} - u^{(in)}$ (MeV) ⁺	s^{out} (MeV)	Γ_{K^*} (MeV)	$\Gamma_{K^*}^{1/2(in)}$	$\Gamma_{K^*}^{1/2(out)}$ (MeV)
1	1/20	1.3	310 ϵ	30	0.8	827	832	82.7	82.7	82.5
2	1/12	1.75	300 ϵ	30	0.8	817	815	52.4	52.4	51.4
3	1/20	1.62	350 ϵ	30	1.0	817	816	76.2	76.2	75.2
4	1/12	1.514	310 ϵ	30	3.0	803	803	61.0	61.0	61.0

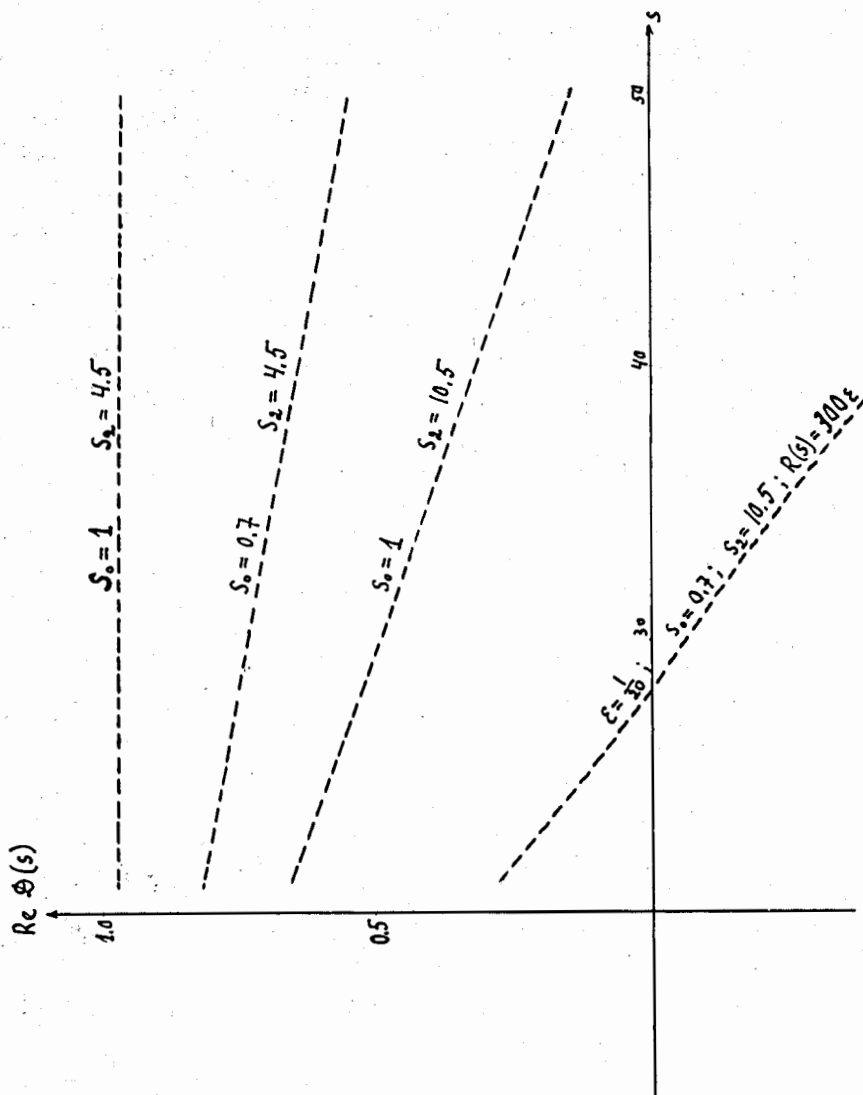


Fig. 3.

Characteristic behaviour of the curves $Re D(s)$ and their dependence on the $s = s_{comp}$ and s_2 (the value $s_1 = 10.5$ implies that the p -exchange forces are taken into account).