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V.G. Pisarenko

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Recently a new method has been suggested by L.D. Soloviev to derive the sum rules from the assumption about the high-energy behaviour of the amplitude and unsabtracted dispersion relations for it.

The application of this method to the pion virtual photoproduction permits to derive some relations connecting the baryon magnetic moments with the isobar magnetic moments. These relations are in agreement with the $SU(6)$ -predictions.

The magnetic moment of a Σ - hyperon has been obtained in terms of the nucleon magnetic moment with the help of the dispersion sum rules method for $K\Sigma$ - photoproduction on nucleons in paper /3/.

In the present note the generalization /4/ of the above method is applied to the process of the virtual photoproduction of the strange and non-strange quasi-mesons on baryons. Making use of the $SU(3)$ -relations for the coupling constants we are able to calculate the hyperon magnetic moments in terms of the nucleon magnetic moment.

1. The Virtual Photoproduction of Quasi-Pions on Baryons.

Consider the quantity /4/:

$$T_{\mu} = \int d^4 x\, \theta(x_0)\, e^{-ikx} \langle p' | [J_{\mu}(x), \tilde{j}_{\alpha}(0)] | p \rangle , \tag{1.1}$$

where $\tilde{j}_{\alpha}(x)$ is the divergence of the axial current with the same quantum numbers as a pion , $J_{\mu}(x)$ is a vector current component, $|p\rangle$, $|p'\rangle$ are the states with the initial and final baryons of the mass m.

The quantity (1.1) has all the transformation properties of the amplitude of the pion virtual photoproduction on baryons. Therefore it can be expanded in the six gauge-invariant structure as follows:

$$T_{\mu} = - i \sum_{i=1}^{6} \bar{u}(p')\gamma_5 R^{i}_{\mu} L_i(s,t,k^2)\, U(p) \tag{1.2}$$

$$R_1 = p(p'\cdot k) - p'(p'k)$$

$$R_{2,3} = (p \pm p')(k\cdot\gamma) - \gamma((p \pm p')\cdot k) \qquad (1.3)$$

$$R_4 = (\gamma\cdot k)\gamma - \gamma(\gamma\cdot k)$$

$$R_5 = (p-p')k^2 - k(p-p')\cdot k$$

$$R_6 = -i(\gamma\cdot k)R_4$$

The isotopic structure of the amplitude L_6 has the form:

for photoproduction on N and Ξ:

$$L_6 = \delta_{\ell 3} L^{(v)}_{odd} + \tau_\ell L^{(s)}_{odd} + \tfrac{1}{2}[\tau_\ell, \tau_3] L^{(-)}, \qquad (1.4)$$

where ℓ is the isotopic index of the pion;

for photoproduction on Σ:

$$L_6 = \delta_{\ell 3}\delta_{jk} L^{(v)}_{odd} + \tfrac{1}{2}(\delta_{3j}\delta_{k\ell} + \delta_{3k}\delta_{j\ell})L' + \tfrac{1}{2}(\delta_{3j}\delta_{k\ell} - \delta_{3k}\delta_{j\ell})L^{(-)} + i\epsilon_{jk\ell} L^{(s)}_{odd},$$

where j, k are the isotopic indices of the initial and final Σ - hyperons;

for photoproduction on Λ:

$$L_6 = \delta_{\ell 3} L^{(v)}_{odd}. \qquad (1.6)$$

From the crossing symmetry properties of the amplitude we obtain:

$$L^{(s,v)}_{odd}(\nu) = - L^{(s,v)}_{odd}(-\nu), \qquad (1.7)$$

where $\nu = \frac{(p+p')k}{2m}$, k is the photon momentum.

Assume now the high energy behaviour of the amplitude L_6 which enables us to write down the unsubtracted dispersion relations both for the quantities $L_6(\nu)$ and $\nu\cdot L_6(\nu)$.

This assumptions and the crossing symmetry properties (1.7) yield:

$$\int_0^\infty \mathrm{Im}\, L^{(s,\nu)}_{odd}(\nu, t, k^2)\, d\nu = 0. \qquad (1.8)$$

Singling out the one-particle terms corresponding to a baryon and baryon isobar $J = \frac{3}{2}$ of the mass M and using the Goldberger-Treiman relation we obtain:

$$g_{BB\pi} M \mu'_{s,v}(B) - \frac{C_{B^*B}\, G^{(v,s)}_{B^*B}\, g_{B^*B\pi}}{6M} \times \qquad (1.9)$$

$$\times \left[2m^2 + Mm + \frac{m^3}{M} - \frac{m}{M}\mu^2 - 3t\right] = 0,$$

where $C_{\Xi^*\Xi} = C_{\Sigma^*\Sigma} = C_{\Sigma^*\Lambda} = 1$, $C_{N^*N} = \frac{4}{3}$

μ is the pion mass and the form factors $G^{(v,s)}_{B^*B}$ are defined as in paper /2/.

The value of the coupling constant $g_{N^*N\pi}$ can be obtained from experimental data on the width of the nucleon isobar decay $\Gamma = 120 + 1.5$ MeV. Using this value for $g_{N^*N\pi}$ we get from eq. (1.9) the value of the magnetic moment of the transmission $N^{*+} \to P + \gamma$ as follows:

$$\mu(N^{*+} \to P + \gamma) = \frac{2\sqrt{2}}{3}\, 1{,}28\, \mu(P)$$

which coincide with experimental value

$$\mu_{exp}(N^{*+} \to P + \gamma) = \frac{2\sqrt{2}}{3}(1{,}25 \pm 0{,}02)\,\mu(P).$$

As the other sum rules (1.9) it is necessary to remark that neither hyperon magnetic moments nor magnetic moments of the transitions $B^* \to B + \gamma$ are known experimentally.

However, the magnetic moment of the transitions $B^* \to B + \gamma$ can be connected with the experimentally known magnetic moment $\mu\,(N^{*+} \to P + \gamma)$ with the help of the $SU(3)$ -relations.

Then from eq. (1.9) for the forward scattering we obtain the following values of the hyperon magnetic moments x/:

x/ We use the values of the ratios $\frac{g_{BB\pi}}{g_{B^*B\pi}}$ which are obtained from the dispersion sum rules for the forward pion scattering on baryons /2,4/. These sum rules have the form:

$$\left(\frac{g_{BB\pi}}{g_{B^*B\pi}}\right)^2 = C_{B^*B}\left\{\frac{4m^2}{3} - \frac{(M^2 + m^2 - \mu^2)[(M-m)^2 - \mu^2]}{6M^2}\right\}; \qquad (1.10)$$

where: $C_{\Sigma^*\Sigma} = C_{\Sigma^*\Lambda} = C_{\Xi^*\Xi} = 1$, $C_{N^*N} = \frac{4}{3}$.

$$\mu(\Sigma^+) = 3{,}19 \frac{e}{2m_p}, \quad \mu(\Xi^0) = -2{,}66 \frac{e}{2m_p}$$

$$\mu(\Sigma^0) = 1{,}20 \frac{e}{2m_p}, \quad \mu(\Xi^-) = 0$$

$$\nu\mu'(\Sigma^-) = 0 \quad , \quad \mu'(\Sigma^0\Lambda) = 1{,}82 \frac{e}{2m_p}.$$

Now we shall calculate the hyperon magnetic moments from the virtual photoproduction of the strange mesons on baryons and compare it with the above results.

2. The Virtual Photoproduction of Quasi-Kaons on Baryons.

As in the previous section consider the quantity:

$$T_\mu = \int d^4x\, \theta(x_0) e^{-ikx} \langle p' | [J_\mu(x), \tilde{j}a(0)] | p \rangle , \tag{1.1}$$

where the divergence of the axial current $\tilde{j}_a(x)$ has the same quantum number as a K-meson.

The initial and final states $|p\rangle$ and $|p'\rangle$ correspond to the baryons the strangeness quantum number of which differs by unity.

Analogously to the previous case we get the sum rules:

$$\int_{-\infty}^{\infty} \mathrm{Im}\, L^{i}_{(6)}(\nu, t, k^2)\, d\nu = 0 . \tag{2.1}$$

We can not impose the restrictions of the crossing-symmetry on the amplitude in eq (2.1) because the strangeness of the initial baryon changes in the photoproduction of the strange mesons.

Consider the virtual photoproduction of K-mesons on nucleons:

$$\gamma + N \to K + \Sigma .$$

The isotopic structure of the amplitude L_6 is of the form:

$$L_6 = r_\rho L^{(s)} + \delta_{8\rho} L^{(v)} + \tfrac{1}{2}[r_\rho, r_8] L^{(-)} , \tag{2.2}$$

where ρ is the isotopic index of a Σ-hyperon. Again singling out in the sum rules (2.1) written for the amplitude $L^{(s)}$ the one-particle terms corresponding to N, Σ, Σ^* and using the Goldberger-Treiman relation, we get:

$$\mu'^{(s)}(\Sigma) + \mu'^{(s)}(N) - \frac{G^{(s)}_{\Sigma^*\Sigma}\, g_{\Sigma^* NK}}{g_{\Sigma Nk}} \times \mathfrak{M}(M_{N^*}, m_\Sigma, m_N) , \tag{2.3}$$

where

$$\mathfrak{M}(B^*, B_1, B_2) = \mathfrak{M}(M_1 m_1, m_2) = \frac{1}{6M}(2m_2^2 + Mm_2 + \frac{m_2^3}{M} - \frac{m_2}{M}\mu_k^2 + 3m_1^2 - 3m_1 m_2) ; \tag{2.4}$$

μ_k is the kaon mass.

The relation for $\mu'^{(v)}(\Sigma)$ can be obtained in the same way from the sum rules (2.1) written for the amplitude $L^{(-)}$.

Express $G^{(s,v)}_{\Sigma^*\Sigma}$ through the magnetic moment of the transition $N^{*+} \to P + \gamma$ and $g_{\Sigma^* NK}$ through $g_{N^* N\pi}$ with the help of the $SU(3)$-relations. Then from the sum rules for $\mu'^{(v)}(\Sigma)$, $\mu'^{(s)}(\Sigma)$ we get:

$$\mu(\Sigma^+) = 3{,}06 \frac{e}{2m_p} \tag{2.5}$$

$$\mu(\Sigma^0) = 1{,}00 \frac{e}{2m_p}$$

$$\mu'(\Sigma^-) = -0{,}26 \frac{e}{2m_p} .$$

Now we shall obtain the sum rules for the virtual photoproduction of the $K\Lambda$ on nucleons:

$$\gamma + N \to K + \Lambda .$$

The isotopic structure of the amplitude has the form:

$$L_6 = r_8 L . \tag{2.6}$$

The corresponding sum rule gives:

$$g_{nk\Lambda}\, \mu'_V(N) + g_{N\Sigma k} M' \mu'_v(\Sigma\ \Lambda) - G^{(v)}_{\Sigma^*\Lambda}\, g_{\Sigma^* NK} \cdot \mathfrak{M}(\Sigma^*, \Lambda, K). \tag{2.7}$$

Using the $SU(3)$-relations between the coupling constants for $\frac{D}{F} = 3$ we get from eq. (2.7):

$$\mu'(\Sigma\ \Lambda) = 1{,}52 \frac{e}{2m_p}. \tag{2.8}$$

Finally consider the virtual photoproduction of $K\Xi$ on Σ-hyperons:

$$\gamma + \Sigma \to K + \Xi .$$

The isotopic structure of the amplitude can be written as follows:

$$L_6 = \delta_{j3} L^{(v)} + r_3 L^{(s)} + \tfrac{1}{2}[r_3, r_j] L^{(-)} . \tag{2.9}$$

Then from the sum rules for the amplitudes $L^{(-)}$ and $L^{(s)}$ using the $SU(3)$-rela tions for the coupling constants we get:

$$\mu'(\Xi^-) = -0{,}08\,\frac{e}{2m_p}\,. \qquad (2.10)$$

Comparing eqs. (2.5), (2.8), (2.10) with eq.(1.10) we see that close results are obtained for the magnetic moments of the hyperons in the both cases.

The small difference in its values given by two methods of culculations can be explained in the following manner.

As is well-known, the SU(3)-relations which we use for the coupling constants are fulfilled only approximately.

Note, that all values of the hyperon magnetic moments obtained here do not contradict the SU(3) -predictions for them.

Remark also that the sum rules for the photoproduction of the K -mesons on Σ give a very small value for the $-\mu(\Xi^0)$ (in comparision with the SU(3) - prediction). But at the same time for $\mu'(\Xi^-)$ one gets a reasonable result. It can be explained as following.

The sum rules for $K\Xi$ -photoproduction on Σ connect $\mu'^{(s,v)}(\Xi)$ with $\mu'^{(s,v)}(\Sigma)$. The coupling constants in this sum rules were estimated with the help of the SU(3)-relations; $\mu'^{(s,v)}(\Sigma)$ are obtained from the SU(3)-relations for the coupling constants too. As a result, an inaccuracy in the estimation of the coupling constants due to the use of the SU(3)-relations gives rise to some distortion in the values of $\mu'^{(s,v)}(\Xi)$. Namely, the modules of their values decrease. Apparently, this distortion has a small effect on the difference $\mu'^{(s)}(\Xi)-\mu'^{(v)}(\Xi)=\mu'(\Xi^-)$, and at the same time the sum $\mu'^{(s)}(\Xi)+\mu'^{(v)}(\Xi)=\mu'(\Xi^0)$ becomes very small because of it.

In the sum rules (1.9) we can use for the coupling constants the values of $g_{B^*B\pi}$ culculated from the experimental width of the isobar decay. The coupling constants $g_{BB\pi}$ can be obtained from the SU(3) -relations together with the experimental value of the nucleon coupling constant $g_{NN\pi}=13{,}55$ for the value $D/F=3$. It yields for the Σ^+ magnetic moment the following value:

$$\mu(\Sigma^+) = 2{,}78\,\frac{e}{2m_p}$$

which is smaller than one given by eqs. (1.10), (2.5).

We remember that only the magnetic moment of Σ is known experimentally. Our result for it:

$$\mu(\Sigma^+) = (3{,}19 - 2{,}78)\,\frac{e}{2m_p}$$

is in agreement with the experimental value:

$$\mu_{exp}(\Sigma^+) = (4{,}3 \pm 1{,}5)\,\frac{e}{2m_p}\,,$$

and it is somewhat higher than the obtained recently /6/ experimental data:

$$\mu_{exp}(\Sigma^+) = (1{,}5 \pm 1{,}1)\,\frac{e}{2m_p}\,.$$

An inaccuracy connected with the use of the SU(3)-relations does not allow us to obtain more exact results.

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