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ON THE SMALL ANGLE SCATTERING OF CHARGED PARTICLES

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The formula which takes into account the difference in phases between nuclear and Coulomb scattering of charged particles and permits to derive from experiment the real part of the nuclear forward scattering amplitude was first obtained by Bethe in the framework of the non-relativistic quantum mechanics of extended particles/1/. From the point of view of experiments on high energy scattering of elementary particles it is of interest to derive the corresponding formula on the basis of the relativistic quantum field theory. We shall give a definition of the nuclear amplitude for scattering of charged particles and establish an exact optical theorem which relates this amplitude to the observed total cross section.

For the small angle scattering of charged particles the main contribution to the scattering amplitude is given by the diagrams 1 and 5 which correspond to nuclear and Coulomb scattering (dashed block denotes strong interactions). Let us consider these diagrams with radiative corrections. At the present accuracy of experiment it is sufficient to take into account only the corrections of order $\alpha = 1/137$, although this approximation is not of principle and the account of higher order corrections at small angles can be easily done/2,4/.

On the Figure some typical diagrams 2,3 and 4 for the radiative corrections to the nuclear scattering are represented. Among them the diagrams of the type 4 corresponding to the exchange of a photon between real charged particles give infrared divergences. These diagrams correspond to the long range Coulomb contribution to the nuclear amplitude. Singling out explicitly all the terms containing the infrared divergences we represent the nuclear amplitude with radiative corrections in the form

$$g_n\,(1 + F_\lambda)\,, \qquad (1)$$

where the gauge and Lorentz invariant factor F_λ includes all the infrared divergences and takes into account the long range Coulomb contribution to the nuclear amplitude/4/

$$F_\lambda = -\sum_{i<j} z_i\, a_i\, z_j\, a_j\, \frac{i\alpha}{8\pi^8} \int \frac{d^4k}{k^2-\lambda^2} \left(\frac{2\,a_i\,p_i - k}{2\,a_i\,p_i\,k - k^2} - \frac{2\,a_j\,p_j + k}{2\,a_j\,p_j\,k + k^2} \right)^2 . \qquad (2)$$

Here the summation is carried out over all charged particles at the beginning and the end of the reaction, z_i and p_i are the sign of the charge and the four momentum of a particle, $a_i = 1$ for the outgoing particle and -1 for the incoming one and λ is a small fictitious photon mass.

Eq. (1) defines an amplitude g_n which we shall call nuclear amplitude. It corresponds to the diagrams of the type 1,2,3 and to the finite part of the diagrams 4, differs from the purely nuclear amplitude (diagram 1) only by finite radiative corrections and has the same general properties as the purely nuclear amplitude, namely, it does not contain any divergences, is finite at zero scattering angle, gauge invariant, has the same crossing symmetry and, at least in the lower order perturbation theory, the same analytic properties/2,3/.

It is not difficult to show/2,3/, that the factor F_λ at zero scattering angle, i.e. at $t=(p_1-p_3)^2=0$ is equal to

$$F_\lambda = -i\eta \, \ln \frac{4p^2}{\lambda^2} + O\left(\alpha \frac{\ln p}{p^2}\right) , \tag{3}$$

where

$$\eta = \frac{z_1 z_2 \alpha}{v_L} , \tag{4}$$

v_L is the laboratory velocity of the incident particle and p is the modulus of the three momentum of a particle in the center-of-mass system. In what follows all the quantities without indices shall refer to the center-of-mass system. The function O does not contain infrared divergences, vanishes at high energies and can be omitted.

Consider now the one-photon diagram 1 with radiative corrections (diagrams of the type 5). It is sufficient to take into account only the part of their contribution which is most singular at small scattering angles

$$g_c\left(1 - i\eta \, \ln \frac{-t}{\lambda^2}\right) , \tag{5}$$

where g_c is the Coulomb scattering amplitude. At small angles

$$g = - \frac{2\eta}{p\,\theta^2} , \tag{6}$$

$$-t = p^2\theta^2 , \tag{7}$$

where θ is the scattering angle. We have neglected the terms of order $\alpha^2(-t)^{-1/2}$ and $\alpha^2 \ln(-t)$ in eq. (5) and the terms of order $\alpha(-t)^{1/2}$ and α in eq. (6) arising from the electromagnetic form factors of the particles in the one-photon diagram. They can be easily taken into account.

Adding eqs.(1) and (5) we get the whole scattering amplitude, in which all the infrared divergences are contained only in a phase factor.

As a result we have the differential cross section for small angle scattering

$$\frac{d\sigma}{d\Omega} = \left| g_c\left(1 - i\eta \, \ln \frac{-t}{4p^2}\right) + g_n \right|^2 = |g_c|^2 + |g_n|^2 + 2 g_c \left(g_{nR} + 2 g_{nI}\, \eta \, \ln \frac{2}{\theta}\right) , \tag{8}$$

where g_{nR} and g_{nI} denote the real and imaginary part of the nuclear amplitude g_n without changing spin orientations. Eq. (8) contains the factor 2 under the logarithm instead of $\theta_0 = 1.06/pa$ in the formula obtained by Bethe/1/, where a is the radius of nuclear interaction.

Let us now establish the optical theorem for the nuclear amplitude g_n. After introducing the photon mass λ the whole forward scattering amplitude g_λ satisfies the usual unitary condition

$$\mathrm{Im}\, g_\lambda = 4\pi W (2\pi)^4 \sum_n \frac{1}{m!} \prod_{i=1}^{n} \frac{d\vec{p}_{fi}}{(2\pi)^3\, 2p^0_{fi}} \sum_{spins} |\langle f | g_\lambda | p_1\, p_2 \rangle|^2 \delta\left(\sum_i p_{fi} - p_1 - p_2\right) , \tag{9}$$

where W is the total energy and m is the number of identical particles in the final state. Let us single out on both sides of eq. (9) all the terms which are singular as $\lambda \to 0$. Since the infrared divergences due to virtual and real soft photons cancel each other in the total cross section, the singular terms on the right-hand side of eq. (9) arise only from the integration of the Coulomb amplitude over small (and large for identical particles) angles. Noting that the imaginary part of the two-photon-exchange amplitude and the Coulomb cross-section in eq. (9) cancel each other, we write eq. (9) in the form

$$\mathrm{Im}\,[g_n (1 + F_\lambda)] = \frac{p}{4\pi}[\sigma(>\tilde{\theta}_{min}) - \sigma_c(>\tilde{\theta}_{min})] + \frac{p}{2\pi}\int_{\theta' < \tilde{\theta}_{min}} d\Omega' \, \mathrm{Re}\,[g_{c\lambda}(\theta')\, g_n(\theta')] , \tag{10}$$

where $\sigma(>\tilde{\theta}_{min})$ is the total cross section in which the elastic cross section is integrated over all angles greater than $\tilde{\theta}_{min}$, a small angle near the forward (and backward for identical particles) direction, $\sigma_c(>\tilde{\tilde{\theta}}_{min})$ is the same cross-section for Coulomb scattering and

$$g_{c\lambda} = \frac{2\eta p}{t-\lambda^2} \,. \qquad (11)$$

We see that all the λ dependent terms in eq. (10) cancel each other and we get the optical theorem for g_n

$$g_{nI} = \frac{p}{4\pi}\,[\sigma(>\tilde{\tilde{\theta}}_{min}) - \sigma_c(>\tilde{\tilde{\theta}}_{min})] + 2\, g_{nR}\,\eta\,\ln\frac{2}{\tilde{\tilde{\theta}}_{min}} \,. \qquad (12)$$

The Coulomb cross section is well known. For small $\tilde{\tilde{\theta}}_{min}$

$$\sigma_c(>\tilde{\tilde{\theta}}_{min}) = \frac{4\pi\eta^2}{p^2\,\tilde{\tilde{\theta}}^2_{min}} + 0\left(\frac{1}{\tilde{\tilde{\theta}}_{min}}\right). \qquad (13)$$

Thus eq. (12) contains only finite quantities and relates the imaginary and real part of the nuclear forward scattering amplitude (without changing spin orientations) to the observed cross section $\sigma(>\tilde{\tilde{\theta}}_{min})$.

The optical theorem (12) is correct up to terms of the order α. It is not difficult to take into account the next order terms. For this purpose it is necessary to replace in eqs. (1) and (10) $1+F_\lambda$ by $1+F_\lambda+\frac{1}{2}F^2_\lambda$ and $g_n(\theta')$ by $g_n(\theta')(1+F_\lambda)$ and take for $g_{c\lambda}$

$$g_{c\lambda} = 2\eta p\left(\frac{1}{t-\lambda^2} - \frac{i\eta}{\sqrt{t(t-4\lambda^2)}}\,\ln\frac{\sqrt{4\lambda^2-t}-\sqrt{-t}}{\sqrt{4\lambda^2-t}+\sqrt{-t}}\right). \qquad (14)$$

As a result we get

$$g_{nI} = \frac{p}{4\pi}\,[\sigma(>\tilde{\tilde{\theta}}_{min}) - \sigma_c(>\tilde{\tilde{\theta}}_{min})] + 2\left(g_{nR} + g_{nI}\,\eta\,\ln\frac{2}{\tilde{\tilde{\theta}}_{min}}\right)\eta\,\ln\frac{2}{\tilde{\tilde{\theta}}_{min}} \,. \qquad (15)$$

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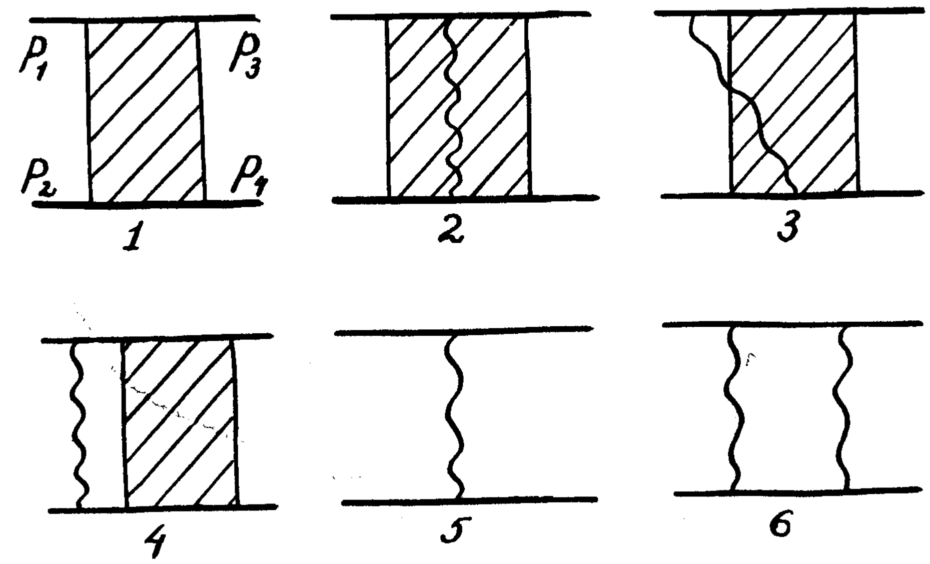

Fig.1