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E-1794
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## SYMMETRY OF STRONG INTERACTIONS

(Theoretical $\lambda$

| Rapporteur: | A. Salam |
| :--- | :--- |
|  | R.N. Faustov |
| Secretaries: | V.I.Ritus |
|  | B.N. Valuyev |

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This publication is of a preliminary character. To facilitate the rapid appearance of Reports, they are printed in the form as presented by Rapporteurs.
wy first task is to thank all the authors who have contributed to this session. The first slide (Fig.1) shows their names oThe Russian language häs a good phrase for such a slide: "Bratskaya Mogila" "the friendly communal grave". There is not a single important theoretical idea I shall report on today which has not been expressed by at least two groups of authors. It is not commonly recognized but in real sense Theoretical Physics has become as much of groupendeavour as Experimental Physics. If in mentioning names, I happen to omit some by inadvertance, I beg for your induagence.

After years of frustration and failure, it is always fun to report on a story of comparative sucpess. For even the most sceptical ones of us can not deny that the use of group-theoretic ideas has paid a handsome dividend to the symmetry physicist. My report shall naturally therefore have a strong group-theoretic bias.

I shall discuss -
First The successful tests of $\mathrm{SO}_{3}$ ( TO its failures I shall turn a blind eye.)

Second The composite models of elementary particles based on triplet models.

Third Group extensions and super-symetries like $\mathrm{SU}_{3} \times \mathrm{SU}_{3}$
Fourth Dynamical considerations.
I. Tests of the Unitary Symmetry

The eight - fold way ${ }^{(\text {F })}$ has to its credit a small but impressia ve number of suocessful tests. There are:

[^0](A) The existence of nearly pure multiplets containing 1, 8 and 10 particles of the same spin and parity. The positively identified nearly pure multiplets are the $0^{-}$and $1 / 2+$ octets and one $3 / 2^{+}$decuplet.

## (B) The Mass Formilae

Assuming that $\mathrm{SU}_{3}$ symmetry is broken and symmetry-breaking can be treated as a small perturbation one gets the well-known set of mass relations anong members of a given muitiplet. For
strong interaotion physics these appear amazingly well verified and constitute perhaps the most definitive support for unitary symmetry. As is well known the baryon octet and decuplet relations are setisfled to within $0.5 \%$, the scalar octet relation to $5 \%$. I shall not go into a detailed derivation of these, but it is important to say a word or two about which relations are better established theoretically than others ${ }^{7 F F}$. Write the interaction Lagrangian in the form

$$
L=L_{s}+L_{\mu s}+L_{E \mu}
$$

where (i) $L_{s}$ is the $\mathrm{SU}_{3}$ symmetric strong interaction for whoh particles of the same spin and parity form equal mass multiplets. As is well known these can be divided into submultiplets of either

I-spin or U-spin, and can be read off most easily from the
X) (see page 1).

The unitary group was first used in elementary particle physics "by S. Ogawa, Y. Ohnuki, M. Ikeda (Progr. Theor. Phys. 22, 715, (1959)) and Y. Yamaguchi (Progr. Theor. Phys. Suppl. (1960) 11,10 , 37) These authors correctly predicted the completion of the o multiplet (with $\eta^{0}$ ) though they followed Sakata in assigning baryons to a 3 -fold representation A. Salam and J.C. Ward (Nuovo Cip 10, 20, 419 (1961)) predicted existence or octets of ( $1^{-}$) (and (1 ${ }^{+}$)) gauge particles. The importance of spin one multiplets lies in the fact that the gauge particles must correspond to the regular repres sentation of a given symmetry group and therefore provide its "invariant" signature (in contrast to any other representations). The 8-fold way assignes not onfy 0 and 1 partioles to ootet repre-
sentation but also baryons $1 / 2$.
${ }^{3}$ FThe remarks that follow have been made (to my knowledge) by okun Akhiczer and Schwinger (papers submitted to this Conference) and in the critical form i have oresented them by P.T. Watchews and G. Feldman (Imperial College preprint 1964)
weight diagrams (see Pig. 1.)
(ii) $I_{\text {ms }}$ is the medium strong interactions which breaks unitary symmetry, but conserves I-spin and hypercharge $Y$. It produces the splitting between the isotopic submultiplets in a unitary multiplet.
(iii) $I_{\text {em }}$ is the electromagnetic interaction which breaks I-spin but conserves U-spin and hence charge $Q$ (whioh in 0-space plays the same role as hyperoharge in I-space). It induces the rsass splitting between the members of an I-spin multiplet. Since this infolves the emission and absorption of a photon, $\mathcal{L}_{\text {EM }}$ is of order

$$
\alpha=e^{2}=\frac{1}{137}
$$

Now LMS is a sealar in I-space. Thus in the absence of $L_{E M}$, but to any order in $L_{M S}$, all members of an I-spin multiplet have the same mass. Similarly, since $L_{E M}$ is a scalar in U-space, in the absence of $L_{M S}$ but to any order in $L_{E M}$, all members of a U-spin multiplet have the same mass. The general mass relations we are seeking are therefore those which are satisfied both by conservation of I-spin alone or by U-spin alone. These relations can be obtained very simply from the weight diagrams.

Consider any parallelogram of points in a weight diargam as illustrated in Fig. 2. If we neglect $I_{E M}$, to all orders in $I_{\text {MS }}$
$m(1)=m(2)$
$m(3)=m(4)$
If we neglect $L_{M S}$, to all orders in $I_{E M}$
$m(1)=m(4)$
$m(2)=m(3)$
(2)
clearly to all orders in $L_{\text {MS }}$ and to all orders in $I_{E M}$ (but neglecting all interference terms $L_{E M} \quad x \quad L_{M S}$ ) the one
relation which replaces (1) and (2) is

$$
m(1)-m(2)+m(3)-m(4)=0
$$

This is called parallelogram Law by Mathews and Feldman. They justify the neglect of $L_{E M} x \quad I_{M S}$ terms by remarking that experimentally, $I_{M S}$ appears to be 1/10 and $I_{E M} \sim 1 / 137$. The interference terms therefore are at least of order 10 ${ }^{-3}$. The parallelogram law should therefore provide some of the most accurate tests for unitary symmetry.

To take an example, for the decuplet we get from its three parallelograms,

$$
\begin{align*}
& N^{*-}-N^{* 0}+Y^{* 0}-Y^{*-}=0  \tag{3}\\
& N^{* 0}-N^{*+}+Y^{*+}-Y^{* 0}=0  \tag{4}\\
& Y^{*-}-Y^{* 0}+E^{* 0}-\Xi^{*=}=0 \tag{5}
\end{align*}
$$

At the Conference we have heard some evidence showing that (3) and (4) are verified.

For the baryon octet, there are two particles which appear in the centre, $\Lambda$ and $\Sigma$. The parallelogram law therefore includes a term contatning the transition mass $m(\Lambda, \Sigma)$ which arises from remarking that in U-spece, the scalar combination is $\quad A_{k}=\frac{1}{2}\left(\sqrt{3} \Sigma^{0}+1^{0}\right) \quad$ while $\Sigma_{k}=\frac{1}{2}\left(\Sigma^{0}-\sqrt{3} \Lambda^{0}\right)$
is the third component of the vector with $n$ and $\Xi^{0}$ as the other two components. For the ootet there are altogether two parallelogram relations:

$$
\begin{align*}
& n-p+\Sigma^{t}-\Sigma^{0}+\sqrt{3}(\Sigma A)=0 \\
& \Sigma^{0}-\Sigma^{-}+\Sigma=\Sigma^{0}+\sqrt{3}(\Sigma A)=0 \tag{6}
\end{align*}
$$

Eliminating the trasition mass we get Coleman-Glashow 6-mass. relation

$$
\begin{equation*}
n-p+\Sigma^{+}-\Sigma^{-}+\Sigma-E^{0}=0 \tag{7}
\end{equation*}
$$

Including as it does $L$ MS to all orders, and with no restriction on the precise form of L MS, this is the best established theoretical relation in the subject. It should provide one of
the severest tests for unitary symmetry. With present evidence the relation in fact appears verified to within experimental accuracy ${ }^{\text {IF }}$.
3) So far we have retained in the computation of physical masses, terms like

$$
\mathcal{M}=\mu_{0}+\sum_{i, S}\left[\left(L_{H S}\right)^{T}+\left(L_{E M}\right)^{s}\right]
$$

but neglected the interference terms like ( $L_{M S} \times L_{E M}$ ). It is crucial to remark that no special form for $I_{\text {MS }}$. was assumed apart from the general requirement that it conserves I-spin and. hypercharge. The verification of (7) was therefore essentialily a verification of the statement that the photon is a scalar in U-space (and that $N, \Sigma, A$ and $\Xi$ ete form multiplets in U-space). We now for the first time assume apecial form for IMS which asserts that $I_{\text {MS }}$ transfoxms as the $I=0, I=0$ component of: an octet.

In U-space this implies that

$$
\begin{equation*}
L_{M S}=\frac{1}{2} u_{S}-\frac{\sqrt{3}}{2} u_{3} . \tag{8}
\end{equation*}
$$

Fo the first order in $J_{\text {MS }}$ (and all orders in $I_{E M}$ ) we therefore get for the mass-splittings an equal-spacing rule in U-space.

For the decuplet this reads

$$
\begin{equation*}
N^{*-}-Y^{*-}=Y^{*-}-\Xi^{*-}=\Xi^{*-}-\Omega^{-} \text {. } \tag{9}
\end{equation*}
$$

For the baryon octet

$$
\begin{equation*}
n-\Sigma_{u}=\Sigma_{u}-\Sigma^{0} \tag{11}
\end{equation*}
$$

or equivalently

$$
2\left(n+\Sigma^{0}\right)=3 \Lambda+\Sigma^{0}-2 \sqrt{3}(\Sigma \Lambda) .
$$

Eliminating ( $\Sigma \wedge$ ) transition mass from (6) and (11), we get

[^1]the mean-mass version of Gell-Mann-Okubo formula
\[

$$
\begin{equation*}
(n+p)+\left(E^{0}+\Sigma^{-}\right)=31+\left(\Sigma^{+}+\Sigma^{-}-\Sigma^{0}\right) \tag{12}
\end{equation*}
$$

\]

"This incorporates $L_{E M}$ to all orders but $L_{M S}$ to only the first. 'the interference $L_{E M} \quad x L_{M S}$ term of course is still not taken into account ${ }^{\text {IF }}$.

## (c) Model-dependent mass relations

In addition to these there are two other types of mass-relations which seem experimentally well-stablished. These are:
(1) Mixing relations between "impure" multiplets. An example is Schwinger's highly accurate quadratic relation between (mass) ${ }^{2}$ of $\phi, \rho, \omega$ and $K^{3}$ particles-:

$$
\begin{equation*}
(\varphi-\rho)(\omega-\rho)=\frac{4}{3}\left(K_{-\rho}^{*}\right)\left(\phi-\omega-2 K^{*}\right) \tag{13}
\end{equation*}
$$

(2) Intra-multiplet Relations

Examples are

$$
\begin{equation*}
K^{*}-\rho=K-t \tag{14}
\end{equation*}
$$

or the remarkable equality noted by Coleman and Glashow-:

$$
\begin{equation*}
a(8)=a(10), b(8)=b(10) \tag{15}
\end{equation*}
$$

Here $a$ and $b$ are the parameters in the standard okubo-Gell--Mann formula

$$
\begin{equation*}
\mu=\mu_{0}+a Y+b\left(I^{2}-\frac{1}{4} Y^{2}\right) \tag{16}
\end{equation*}
$$

and $a(8), b(8)$ refer to tie octet, and $a(10)$ and $b(10)$ to the decuplet. These relations differ from (3)-(12) in one very
important respect. Whereas (3)-(12) are general consequences of group-theoretic considerations, the mixing-relations or the in-tra-multiplet relations are consequences (at least so far as present derivations go) of specific dynamic models.
\% Unilike (7) there probably is no tremendous gain in writing the mean-mass form (12). This is because the negiect of
 of the higher oxders of. I EM.

## (3) Electromagnetic Mass differences

The same remarks applies to the detailed phenomenological calculation of elcctro-magnetic mass-differences (which agree with experiment to $0,5 \mathrm{MaV}$ ) carried out (and reported at the Conference) by Coleman and Glashow and by Marshak. I shall take up these model-dependent mass-relations later.

## (d) Magnetic Moments of Baryons

The next not so precise, test for $\mathrm{SU}_{3}$ comes from comparison of baryon magnetic moments. If photon is scalar in U-space and the symmetry-breaking term I MS is neglected, from the weight diagrams we get :
where

$$
\begin{align*}
& \mu_{p}=\mu_{\Sigma^{+}}, \\
& \mu_{\Sigma^{-}}=\mu_{\Sigma-},  \tag{17}\\
& \mu_{n}=\mu_{E_{0}}=\mu_{\Sigma_{u}},
\end{align*}
$$

$$
\mu_{\Sigma_{H}}=\frac{3}{4} \mu_{A}+\frac{1}{4} \mu_{20}-\frac{\sqrt{3}}{2} \mu_{\Sigma \Lambda}
$$

If it is assumed th et the electromagnetic current transforms Like

$$
\begin{equation*}
y_{3}+\frac{1}{\sqrt{3}} y_{8} \tag{18}
\end{equation*}
$$

we get the two additional relations

$$
\begin{equation*}
\mu_{n}=2 \mu_{A}=-2 \mu_{\Sigma} \tag{19}
\end{equation*}
$$

The new measurement of $\mu_{A}$ reported at this conference gi-
ven

$$
\mu_{A}=-0,66 \pm 0,35 \quad \text { (in } A \text { masons). }
$$

Considering the difficulties of precise measurement, this may possible be called agreement with theory at least in the sight

* As noted by Okubo, the inclusion of LMS to the first order unfortunately leaves only the equi-distance rule in U-space, $\mu_{A}-\mu_{\Sigma_{h}}=\mu_{\Sigma_{m}}-\mu_{00}$.
This is independent of assumption (18). The other relations (and in particular $\mu_{n}=2 \mu_{A}$ ) no longer hold.
of God I shall however comment on the precise significance of the result later.


## (e) Decay Widths

Next to the (essentially diagoaal) mass or magnetic-moment matrix elements, it is the simplest to include the effect of symmetry-breaking terms for the decay amplitudes $F\left(p_{1}^{2}, p_{2}^{2}, p_{3}^{2}\right)$

$$
\begin{aligned}
& A \rightarrow B+C \\
& \left(p_{1}\right) \quad\left(p_{2}, p_{3}\right)
\end{aligned}
$$

This has been done flar the decuplet decays $10 \rightarrow 8+8$ by V. Gupta and V. Singh and by C. Becchi, E. Eberle, G. Morpurgo These authors find $/ 7$ relations between 12 possible amplitudes. 'l'hese relations resemble Gell-Mannlokubo rules and have the form $2\left(N^{*} \rightarrow N_{\pi}\right)+2\left(\Xi^{*} \rightarrow \Sigma \pi\right)=3\left(Y_{1}^{*} \rightarrow A_{r}\right)+\left(Y_{1}^{*} \rightarrow \Sigma_{\pi}\right)$

Assuming that one may neglect the effect of relative mass differences in $F\left(m_{1}^{2}, m_{2}^{2}, m_{3}^{2}\right)$, an experimentwl comparison for the left and the right sides of (20) gives

$$
7.58 \pm 83(\mathrm{BeV})^{-1}=7.44 \pm 0.83(\mathrm{BeV})^{-1}
$$

## (f) Cross Seotion Relations

'the altimate test of unitary symmetry of course is the equality of reaction cross section. Now the reaction amplitude for a two-body process

$$
\begin{aligned}
& A+B \rightarrow C+D \\
& p_{1}, p_{2} \rightarrow p 3, p_{4}
\end{aligned}
$$

is a function of six invariants $F\left(p_{1}^{2}, p_{2}^{2}, p_{3}^{2}, p_{4}^{2},\left(p_{1}+p_{2}\right)^{2}\right.$, $\left.\left(p_{1}-p_{3}\right)^{2}\right)$.
To incorporate the effects of the symnetry breaking interaction is an arc still in its infancy. To see the drastic change which even a particl inclusion of symmetry breaking can produce, consider the example of reactions
(a) $\pi^{-}+\rho \rightarrow N^{*-}+\pi^{+}$
(e) $K^{-}+\Gamma \rightarrow Y_{1}^{+-}+\pi^{+}$
(c) $\pi^{-}+\mu \rightarrow Y_{1}^{*-}+K^{+}$
(d) $K+p \rightarrow \Xi^{*-}+k^{+}$
reported by Snow .
Using U-space methods, one can show that in the pure $\mathrm{Su}_{3}$ lImit $M_{a} / \sqrt{3}=-M_{b}=M_{c}=-M_{d}$

$$
(22)
$$

As Pig. 3 show, this is far from the experimental case. Inelusion of symmetry breaking to tine first order leaves just one relation between amplitudes

$$
\begin{equation*}
M_{a}+M_{b}=M_{c}+M_{d} \tag{23}
\end{equation*}
$$

Noting that (experimentally) $M_{b} \approx M_{d} \approx 0$ this amounts to checking if $\mathrm{Ma}_{\mathrm{a}} \approx \mathrm{M}_{\mathrm{c}}$, which from the data presented is not unreasonable.

I am here taking a highly optimistic view-point about presdiction of unitary symmetry regarding cross-sections equalities. The hope is that when one has learnt how to include symmetrybreaking properly, the tests would be more meaningful. The blant truth is that if these were the only posible tests of $\mathrm{SU}_{3}$, one would never at any rate at the present stage of the subject, have given much credence to unitary symmetry.

## Summarising

Unitary symmetry has a small but impressive list of successes, mainly in predicting mass relations. The successes are more imppressive than one has any right to expect. It has however no out sight failures. This is partly because unlike other sumetry proposals unitary symmetry does not forbid strong remotions otherwise allowed by I-spin and hypercharge conservation. The failures of unitary symmetry can reasonably be ascribed to our inability to include symatry-breaking except to the first order.

## II Composite Nodels and Unitary Triplets

The relative success of group theoretical models for unitary symmetry naturally leads one to examine its basic group-structure more closely. And here one immediately meets with a deep puzzle. iny does nature not employ the basic triplet representations of the unitary group, when from these elementary (spinor) representations one could compositely construct the tensor representations 1, 8, 10... etc. to which the physical particles seem to belong to? In other words winy has Sakata model failed? Or has it indeed failed; could it be that the fundamental Sakata-like triplets do exist not as tne pnysical entities $p, n$ and $\Lambda$ but in a different guise. During the last year a number of proposals have been made to employ the triplet representations. I shall examine some of the models. Even though some of these claim to oe dynamical in intent, the dynamics is of the most rudimentary character, the essential con'tent being sroup-theoretic.
(A) The Revolutionary Quark iodel

The most economical of all composite models is the Quark (or the Ace) model. Given the Bose multiplets, 1 and 8 , and the Fermi multiplets, 1,8 and 10 , find the one unit from which these multiplets, can be composed? The unique answer*) is a spin $1 / 2$ triplet $A=\left(\begin{array}{l}A_{1} \\ A_{2} \\ A_{3}\end{array}\right)$ where $A_{1}, A_{2}, A_{3}$ carry baryon number $B=1 / 3$ and with the other quantum number**)
*This is because
$3 \times 3^{*}=1+8$
$3 \times 3 \times 3=1+8+8+10$
${ }^{* *}$ ) In terms of the generators of $S U_{3} ; Y$ is defined as $Y=J_{8} / \sqrt{3}$ Thus $Q=J_{3}+\frac{1}{\sqrt{3}} J_{8}$ universally for all nadrons as well as for quarks.

|  | $I_{3}$ | $Y$ | $Q=I+Y / 2$ |
| :--- | :---: | :---: | :---: |
| $A_{1}$ | $1 / 2$ | $I / 3$ | $2 / 13$ |
| $A_{2}$ | $-1 / 2$ | $I / 3$ | $-1 / 3$ |
| $A_{3}$ | 0 | $-2 / 3$ | $-1 / 3$ |

This is essentially the Sakata triplet with a charge displace ment -1/3. Clearly the vorld of the quarks, $A_{1}, A_{2}, A_{3}$, if such exotic objects exist, is a world orthogonal to the world we are used to, in the sense that such particles could be created only in pairs from the known particles. Quarks would constitute new type of stable matter.
(B) Conservative Triplet Models

For most other models the fractional value of electric charge are too high a price to pay for the economy of having a single triplet. All known particles can be formed as composites either from two triplets*) or from one fermi triplet and a neutral singlet Now all triplets with integral cnarge fall basically into 2 categories:
(A) Sakata-like triplets

$$
S=\left(\begin{array}{l}
A_{1} \\
A_{2} \\
A_{3}
\end{array}\right) \quad\left(\begin{array}{l}
Q \\
0 \\
0
\end{array}\right) \quad \begin{aligned}
& Q=I_{3}+\frac{1}{2} Y+\frac{1}{2} C \\
& C=1
\end{aligned}
$$

(B) Lepton-like triplets

$$
I=\left(\begin{array}{l}
A_{1} \\
A_{2} \\
A_{3}
\end{array}\right) \quad\left(\begin{array}{c}
Q \\
-1 \\
-1
\end{array}\right) \quad \begin{aligned}
& Q=I_{3}+\frac{1}{2} Y+\frac{1}{3} C \\
& C=-2
\end{aligned}
$$

For both types of triplets, the integral charge requirement forces us to introduce a new quantum number C. This quantum number has been given different names by different people;

[^2]1. H.Baory, J.Nuyts, I.Van Hove, preprint CBRN (1964). (two Fermi triplets).
2. J. Schwinger, preprint; F.Gursey, T. Lee and M. Nauenberg, preprint (one Fermi and one Bose triplet).
"addivitive triality" by the Rochester group, "peculiarity" at GERN, "supercharge" by Okun. Personally I prefer the name given to.it by Glashow and Bjorken. They call it the "charm". note that $C=\langle\langle Q\rangle\rangle$. Por ordinary matter $C=0$.

Following a classification given by Van Hove and Gell-Mann one may consider three distinct alternatives:
(1) The new quantum number $C$ is absolutely conserved. Since for ordinary matter $C=0$, the triplets then are a new type of stable matter. This case is as exciting as tine case of Quarks. Lee and Gürsey have speculated that it is this type of matter which constitutes the substance of the mysterious (Quasi Stellar) Radio Sources.

## (2) C is yiolated by weak interaotions.

In this case $C$ is closely paralleltohypercharge so far as its conservation is concerned and the triplets carry a new form of strangeness. The oharmed (or charming) particles can only be produced in pairs strongly, though they can decay singly into normal matter. On account of its analogy with leptons an attractive example of a composite theory is of all (hadronic) matter being built up from an L-type Fermi triplet along with a neutral singlet frcatonx $\left(\begin{array}{c}-1 \\ -\frac{1}{-1} \\ \frac{1}{0}\end{array}\right)$ fermion.
(3) Cis violated semi-strongly, though $\Delta\left(Y+\frac{2}{3} C\right)=0$ in order that $\Delta Q=0, \Delta I=0$. The "ciarming" particles can be created singly - though possibly less copiously
*) C defined above is $2 / 3$ times the number defined by Glashow \& Bjorken.
than those without charm. This model can be realised either*)
(a) through one S-type triplet + a neutral singlet
(b) or two triplets as in the models of Schwinger, Van Hove, Lee, Gurney and Nauenberg.
(c) Dynamical Predictions

Consider briefly some of the specific predictions of the various models. Their predictions are as a rule very similar.
(1) The Quark Model

Assuming that quarks are fairly heavy, $2 w e i g$ has built up a dynamical model of their binding to give the mass relations between the known $\mathrm{SU}_{3}$ multiplets. The model has the following characteristics:

1. The medium-strong symmetry-breaking is introduced by assuming that the masses of the basic quarks are different,

$$
\begin{align*}
m_{A_{3}} & >m_{A_{1}}=m_{A_{2}} \\
L_{M S} & =\left(m_{A_{3}}-m_{A_{1}}\right) A_{3}^{+} A_{3}
\end{align*}
$$

2. Since $M_{A_{1}}=M_{A_{2}}$, it immediately follows that the residual symmetry is of the $U_{2}$ group. This directly leads therefore to the following solution of the $\omega, \phi, \rho$ mixing problem: the physical particles (the eigen-states) have the transformation properties (corresponding to representations of $\mathrm{U}_{2}$ ):

$$
\begin{align*}
& \rho=\frac{1}{\sqrt{2}}\left(A_{1}^{+} A_{1}-A_{2}^{+} A_{2}\right) \\
& \omega=\frac{1}{\sqrt{2}}\left(A_{1}^{+} A_{1}+A_{2}^{+} A_{2}\right)  \tag{25}\\
& \phi=A_{3}^{+} A_{3}
\end{align*}
$$

The squared masses satisfy the two relations**)

[^3]\[

$$
\begin{aligned}
& \text { 1) } \rho=\omega \\
& \text { 2) } 2 \phi+\rho+\omega=4 K^{*}
\end{aligned}
$$
\]

3. Assuming that both $0^{-}$and $1^{-}$bosons bind from a quark and an anti-quark, and assuming that the binding ts independent of spin, one gets the relation

$$
k^{*}-\rho=m_{A_{3}}-m_{A_{1}}=k-\pi \quad(14 a)
$$

I hope this is not an unfair sample of the type of dynamical argument used in this and other composite models.

It is a type of argument calculated to send a self-respecting 5-matrix theoretist into fits of despair - despair because the results seem to have the sanction of nature. The most charitable thing one could say about these calculations is what Dr. Johnson once remarked about woman's preaching: "A woman preaching, Sir, is like a dog walking on his hind legs; it is not done well, but you are surprised to find it done at all."
(2) Schwinger's Field Theory of Matter

1. Starting with a dynamical analogy between leptonic interactions and strong interactions Schwinger introduces 2 sets of triplet fields to build compositely all known hadrons.

These are:
one Sakata-like Fermi triplet $\psi, B=1$
one Sakata-like Bose triplet $V, B=2$
2. The crucial assumption is made that at the most elemental level of dynamic theory, $\psi$ and $V$ transform as representations of two independent unitary groups

$$
\begin{aligned}
& \psi^{\prime}=u_{1} \psi, \\
& v^{\prime}=u_{2} v,
\end{aligned}
$$

$U_{1} \neq U_{2}$. We are thus dealing with a $\left(U_{3} \times U_{3}=W_{3}\right)$ group structure.

At this level there are 9 baryons $V \bar{\psi}$ corresponding to a $(3,3)^{*}$ representation of $\mathrm{m}_{3}$.
3. Mesons (with the group-structure $\bar{\psi} \psi$ transforming as $U \bar{\psi} \psi K^{-1}$ ) correspond to a reducible 9 fold $(9=1+8)$ representation of $\mathrm{U}_{3}$.
4. There are two symmetry breaking terms; one is introduced to split the 9 -fold of mesons into a singlet and an octet, the other, by-passes the $\mathrm{SU}_{3}$ structure leading directly from in to $\mathrm{U}_{2}$. The second interaction ( $L_{M S}=\bar{\psi}_{3}\left(\mathcal{F}_{i} V_{i}\right)$ ) is something of a tour-deforce. It is precisely the unaesthetic feature of bosons carrying two units of baryonic number which forces the on the theory this particular type of symmetry breaking. Note that in the second order $L_{M S} \times L_{M S}$ gives the effective interaction of zweig type

$$
\bar{\psi}_{3} \psi_{3}(\bar{\psi} V)\left(\bar{V}_{4}\right)
$$

Tne quadratic mass formula connecting $\phi, \rho, \omega$ and $K^{*}(\text { mass })^{2}$ mentioned earlier follows directly as the lowest order perturbation arising from the interplay of the two symmetry-breaking terms
Some further features of schwinger's model are the following:
(a) The decuplet $3 / 2^{+}$is part of some further feature of Schwinger's theory are the following: a $45^{\circ}$ - component multiplet which under symmetry-breaking splits as $45=8+10+27$ Glashow and Kleitman (Phys, Lett....) have given arguments for believing that the 27 - fold multiplet is likely to be fairly massive (2 BeV or more).
(b) If the symmetry - breaking terms are ignored, a number of processes are forbidden (compare the Sakata model). For example

$$
\begin{aligned}
& \pi^{+} \nrightarrow K^{1} \Sigma^{+} \\
& K^{-} n \not K^{0} E^{0} \\
& P \bar{P} \nrightarrow \bar{K}^{0} K^{0}
\end{aligned}
$$

Since the symmetry-breaking terms are assumed to be quantita-

[^4]tively enormous, this foraiddenness is perhaps irrelevant. ${ }^{\text {s }}$ )
(3) Groups of Rank Higher than 2

Given a new quantum number (C), a group theorist will immediately rush off to his copy of Dynkin and make an inventory of' all groups of rank higher than 2. Recall that the rank of a Lee group gives the number of its comuting generators - and therefore the number of conserved quantities it can accomodate. $\mathrm{SU}_{3}$ is a group of rank 2; it can accomodate two quantum numbers ( $I_{3}$ and $Y$ ). The next Fig. 4. snows Dynkin diagrams for some higher rank groups. Of groups of rank 3, the favourite ones are $S U_{4}$, and $S P_{6}$ (the symplectic group). The number of authors who have considered $\mathrm{SU}_{4}$ as a possible super symaetry of nature is legion.**)

The symplectic group has only one set of votaries.***) The elementary representation of $\mathrm{SU}_{4}$ is a quartet (an $S$ or an I-type of triplet + a singlet); the corresponding representation for Sp6 has 6 components (one $S$ and one I-type triplet). ${ }^{* * * *) ~}$

[^5]$\mathrm{SU}_{4}$
As I stated earlier all $\mathrm{SU}_{4}$ models fall into 2 cathegories SU $_{4}$ Mark I
S-type Quartet
\[

\left($$
\begin{array}{l}
1 \\
0 \\
0 \\
0
\end{array}
$$\right)
\]

This allows for all three alternatives:
Either (I) C - absolute conservation
or (2) C - weak violation
or (3) C - semi atrong violation.
SU Mark II
I-type quartet

$$
\left(\begin{array}{r}
0 \\
-1 \\
-1 \\
\frac{-1}{0}
\end{array}\right)
$$

This allows only
Ether (1) C -absolute conservation
or (2) C - weak violation.
Some or the $\mathrm{SU}_{4}$ representations possess the following content:
(1) $0^{-}, \mathrm{I}^{-}=$adjoint representation, which in terms of $\mathrm{SU}_{3}$ multipleta decomposes as follows

$$
4 \times 4=1+15=8+3+3^{*}+1
$$

The submultipleta 3 and $3^{*}$ carry charm while for the singlet $C=0$. Clearly this singlet provides a natural place within the group atructure for a ninth boson, ( $\omega^{0}$ or the $/ \pi \eta /$ ).
(2) $1 / 2^{+}$fermions could belong to a $2 \theta$-Fold Representation which splits as

$$
20=8+6+3+3^{*}
$$

(3) $3 / 2^{+}$belongs to different $20^{\prime}$-Fold which would split as

$$
20^{\prime}=8+6+3+3
$$

The next 2 tables taken from Glashow \＆Bjorken；\＆Amati；
；Barry，Nuts，Prentki，illustrate，some of the simple mass assignments，assuming that the $\mathrm{SU}_{4}$ symmetry is broken in a＂na－ turel＂（Gell－Mann－Okubo－like）manner．
Fig. 5. Fig.6

Some people do not know when to stop．
（4）Tests for the Existence of Triplets．
If the＂charmed＂triplets do indeed exist，is there some indirect but recognisable effect they would produce which could constitute a test of their existence？

In so far as the chief distinguishing feature of the triplets is the additive term in the Gell－hann－Nishijima formula（ $C \neq 0$ ）

$$
\begin{equation*}
Q=J_{3}+\frac{J_{8}}{\sqrt{3}}+\frac{c}{3} \tag{27}
\end{equation*}
$$

the answer must lie within eleotromagnotis⿴囗⿰丨丨⿱一一⿻儿口一的 Nauenberg \＆Okïn for example have noted that the relation

$$
\begin{equation*}
\mu_{N}=2 \mu_{A} \tag{28}
\end{equation*}
$$

no longer holds if $C \neq 0$ ．（Note that for Quarks，$C=0$ ，so that Quarks do not produce any．＂indirectn electromagnetic effects）．

Now the violation of（28）xmactit certainly constitute a teat of the existence of the triplets．But this test has the drawback that the formula（28）is no longer valid（at present to an un－ predictable extent）also when the symmetry－breaking $\mathrm{I}_{\text {ms }}$ terms are included．Thus if $\mu_{N}-2 \mu_{A} \neq 0$ ；one would not know if this was the result of the presence of triplets or a consequence of the normal symmetry breaking mechanism．

A better test possibly is provided by the old chestnut， tine ratio

$$
R=\frac{\phi-\gamma \rightarrow \mu^{+}+\mu^{-}}{\omega \rightarrow \gamma \rightarrow \mu^{+}+\mu^{-}}
$$

Let us assume that the physical particles $\omega$ and $\phi$ rare mixcures of a pure "singlet" $\omega_{0}$ and an octet" $\phi_{0}$.

$$
\begin{align*}
& \omega=\phi_{0} \cos \theta_{s}+\omega_{0} \sin \theta_{s}  \tag{29}\\
& \omega=\phi_{0} \sin \theta_{s}+\omega_{0} \cos \theta_{s}
\end{align*}
$$

The angle $\theta_{S}$ can be determined from strong interactions al one (e. gas suggested by Sakurai by using the relation

$$
\Gamma_{\varphi \rightarrow K+K}=\cos ^{2} \theta \Gamma_{\varphi^{0} \rightarrow K+K}
$$

where $\Gamma_{Q^{\prime} \rightarrow k+K}$ is determined from $\Gamma_{\rho}$ and $\left.\Gamma_{k}^{*}\right)$. Now write

$$
\begin{equation*}
R=\left|\frac{\cos \theta+x \sin \theta}{-\sin \theta+x \cos \theta}\right|^{2}=\cot ^{2} \theta E M \tag{30}
\end{equation*}
$$

Clearly if $C=0, x \neq 0$ and $\theta_{\text {EM }} \neq \theta_{S}$. Conversely if $\theta_{E M} \neq \theta_{S}$,
and if the notions of unitary symmetry are correct there must exist triplets of integral charge.

If the triplets are very massive, in general $x$ will be small. There however are certain models (egg. Schwinger's) where irrespective of the mass of the triplets, $\theta_{E M}-\theta_{S}$ can be as large as $60^{\circ}$ in the exact $W_{3}$ limit.

## Summarising

The problems raised by the triplet models are highly significant and of the deepest relevance to the future of Physics. The Triplets may be stable; they exist either in the form of quarks or they may carry integral charge. In this case they define a new and a hitherto unsuspected regime of physical phenomena. The significance of this nev regime for Cosmology has been speculatedmay or may not concern us here today. We can not however fail to De fired by their significance.

## III. Group Extensions and Super-Symmetries

I now turn to what I consider as some of the most significant contributions to this Conference. This is the elegant study of the group Algebras connected with extensions of $\mathrm{SU}_{3}$. The study itself is not new. It was carried out in 1961 within the context of unitary symmetry by (see ref.1) M. Gell-mann, A. Salam and J.C. Ward and in terms of a four-field Fermi interaction by R. Marshak and S. Okubo ${ }^{\text {F }}$. It has naturally acquired renewed significance with the emergence of $\mathrm{SU}_{3}^{\mathrm{MEF}}$.

The story starts with what Gell-Mann called F and D couplings and $F$ and $D$ (Currents.) Consider the interaction of pacudoscalar mesons with baryons. Write the conventional $3 \times 3$ matrix for the baryons.

$$
\begin{aligned}
& \text { and similarly for the mesons } M \text {. } \\
& B=\left(\begin{array}{ccc}
\frac{Y^{0}}{\sqrt{3}}+\frac{\Lambda^{0}}{\sqrt{6}}+\frac{\Sigma^{0}}{\sqrt{2}} & \sum^{0} & p \\
\Sigma^{0} & \frac{Y^{0}}{\sqrt{3}}+\frac{\Lambda^{0}}{\sqrt{6}}-\sum^{0} & n \\
\Xi^{2} & n \\
\text { for the mesons } M . & -\sqrt{2}^{0} & \frac{Y^{0}}{\sqrt{3}}-\frac{2 \Lambda^{0}}{\sqrt{6}}
\end{array}\right)((3))
\end{aligned}
$$

The three field interaction can be written either in the forma:
Tr. B+ BM
or in the form-:
Tr. $B^{-\quad} \quad B$

[^6]These are the only two ways of multiplying three matrices within the trace operation. Now with Gell-Mann one can define the (syametric) and anti-symmetric combinations the above two (couplings) as follows-:
$\operatorname{Tr} B^{+}(B M+M B)=\operatorname{Tr}_{f} B^{+}\{B, M\}$; The so called D-coupling. and $\operatorname{Tr} \mathrm{B}^{+}(\mathrm{BN}-\mathrm{Mi} \mathrm{B})=\operatorname{Tr} \mathrm{B}^{+}[\mathrm{B}, \mathrm{N}]$; The so called F-coupling. One of the important fundamental parameters in the theory is P/D ratio.

One way to remember the distinction of $F$ and $D$ is to remark that for $F$ couplings there is no $\mathbb{S} \rightarrow \Lambda+\pi$ transition, for pure $D$ case there is no $\Sigma \rightarrow \sum+\boldsymbol{J}$ transition. The vector couplings of $\rho, K^{3,}, \varphi$ and $\omega$ are conventionally assumed as pure F. For $\mathcal{T}$-mesons, however, hyperfragment binding clearly calls for non-zero $D\left(g_{\pi \wedge \Sigma} \neq 0\right)$. The dymanical calculations of hartin and Wali and others go even further and show that not only must the D-coupling exist for pseudosoalar mesons, they must predominate ( $F / D \approx 1 / 3$ ). The same story seems to repeat itself for weak, interactions, Treiman will tell you to-marrow. The $\gamma_{5}^{\text {-currents (axial. }}$ veotors) appear predominantly $D$, the vector ourrents are $F$.

The question arises; within the unitary symmetry scheme, what is the origin of $F$ and $D$ couplings; or if we consider vector particles - what is the origin of two types of distinct currents $F$ and $D$ ?

The unique answer 1108 in the group extension $\mathrm{SU}_{3} \times \mathrm{SU}_{3}$. Consider the two unitary triplets $A$ and $B$ transforms as

$$
\begin{aligned}
& A^{1}=U_{1} A \\
& B^{1}=U_{2}^{\# H} B
\end{aligned}
$$

If the known 9 - Folds, e.5. the baryon nonets are formed as

$$
=A B^{T}=\left(\begin{array}{l}
A_{1} \\
A_{2} \\
A_{3}
\end{array}\right)\left(\begin{array}{lll}
B_{1} & B_{2} & \left.B_{3}\right)
\end{array}\right.
$$

transforms as

$$
\begin{equation*}
\psi^{\prime}=U_{1} \psi u^{-1} \tag{32}
\end{equation*}
$$

As stressed earlier (in connection with Schuvinges's Field theory of matter), $\psi$ belongs to the ( $3,3^{3}$ ) representation of sw . provided $U_{1} \neq U_{2}$. If $U_{1}=U_{2}$ is.

$$
\psi^{\prime}=U_{1} \psi U_{1}^{-1}
$$

we are dealing with the (reducible) g-fold representation of $\mathrm{SU}_{3}$ alone.

Now there is a standard procedure for generating conserved currents - the so-called gauge procedure corresponding to any given transformation. In its essentials, the procedure $1 s$ to write the transformation concerned infinitesimally eeg. write

$$
u_{1}=1+i x, \quad u_{2}=1+i x
$$

- where $X$ and $I$ are hermitian $3 \times 3$ matrices.

The transformation (32) reads

$$
\psi^{\prime}=(1+i X)_{\psi}(1-i Y)=\psi+i(X \psi-\psi Y) .
$$

Likewise $\quad \partial_{\mu} \psi^{\prime}=\partial_{\mu} \psi+i\left(x_{\mu} \psi-\psi y_{\mu}\right)$.
The free energy $\bar{\psi} \partial_{\mu} \partial_{\mu} \psi \quad$ therefore transforms to

$$
\bar{\psi} \delta_{\mu} \partial_{\mu} \psi+i \bar{\psi} \gamma_{\mu}\left(x_{\mu} \psi-\psi \xi\right)
$$

The extra terms generated by this procedure represent the coupling of spin one objects $X_{\mu}$ and $X_{\mu}$. with the baryon - currents. Rewriting these we get:

$$
\begin{aligned}
\bar{\psi} \gamma_{\mu} X_{\mu} \psi-\bar{\psi} \gamma_{\mu} \psi Y_{\mu} & =\frac{1}{2} \bar{\psi} \gamma_{\mu}\left[X_{\mu}+Y_{\mu}, \psi\right]+\frac{1}{2} \bar{\psi} \gamma_{\mu}\left\{X_{\mu}-Y_{\mu}, \psi\right\}= \\
& =F^{v}+g^{v}
\end{aligned}
$$

Starting therefore with (32) we see that we have generated natrally both $F^{V}$ as well as $D^{V}$ currents. If we had specialised to the case $U_{1}=U_{2}$ (i.e. $X=Y$ ), we would have generated only the Algebra corresponding to $F^{V}$ alone.

It is easy to check that the commutation relations of $P^{\gamma}$ and $D^{\nabla}$ are as follows:

$$
\begin{aligned}
& {\left[F_{i}, F_{j}\right]=i f_{i j k} F_{k}} \\
& {\left[F_{i}, D_{j}\right]=i f_{i j k} D_{k}} \\
& {\left[D_{i}, D_{j}\right]=i f_{i j k} F_{k}}
\end{aligned}
$$

Now so far we have no axial vector currents (or the corresponding ps. $\gamma_{5}$ coupling). But we know these exist; in fact that for the $\gamma_{5}$ case they predominate. To generate these the standard procedure once again is to consider in the zero baryon mass limit, the two-component entities

$$
\begin{aligned}
& \psi_{L}=\frac{1+\gamma_{S}}{2} \psi \\
& \psi_{R}=\frac{1-\gamma_{S}}{2} \psi, \\
& \psi=\psi_{L}+\psi_{R}
\end{aligned}
$$

One can now make 4 independent transformations

$$
\begin{aligned}
& B_{L}^{\prime}=U_{1} B_{L} U_{2}^{-1} \\
& B_{R}^{\prime}=U_{3} B_{R} U_{4}^{-1}
\end{aligned}
$$

Clearly one will now generate 4 types of currents,

$$
\begin{aligned}
& F^{V} \\
& F^{A} \\
& D^{V} \\
& D^{A}
\end{aligned}
$$

**) Note that each set contains 8 conserved currents (conserved in the limit $m_{s} \rightarrow 0$ ) so that the overall Algebra generated by these 32 currents, with the commutation relations.

$$
\begin{aligned}
& {\left[F_{i}^{V, A}, F_{j}^{V, A}\right]=i f_{i j k} F_{k}^{V},\left[F_{i}^{V, A}, F_{j}^{A, V}\right]=i f_{i j k} F_{k}^{A},} \\
& {\left[F_{i, A}^{V,}, D_{j}^{V, A}\right]=i f_{i j k} D_{k}^{V}, \quad\left[F_{i}^{V, A}, g^{A, V}\right]=i f_{i j k} D_{k}^{A},} \\
& {\left[D_{i}^{V, A}, D_{j}^{V A}\right]=i f_{i j k} F_{k}^{V},\left[D_{i}^{V, A}, D_{j}^{A, V}\right]=i f_{i j k} F_{K}^{A}} \\
& \text { (the footnote is continued on page 26, })
\end{aligned}
$$

In its widest form, then; and essuming that possibly correspone ing to these currents there also might exist physical partioles we maj have a total of sixteen $1^{-}$and sixteen $1^{+}$partioles.

Now it is possible (and indeed quite probable) that nature does not use the generous freedom afforded by all the possibilities listed above. An attraotive restricted special case is the following-:

$$
\left.\begin{array}{rr}
B_{L}=U_{1} B_{L} U_{2}^{-1} \\
B_{R}=U_{2} B_{R} U_{1}^{-1} & \text { (i.e. take } U_{1}=U_{4} \\
U_{3}=U_{2}
\end{array}\right)
$$

In this case there are only $F^{\nabla}$ and $D^{A}$ currents.
(1) It is an attractivo hypothesis (forced upon us by the existence of $D$ currents and their dominance for the $\gamma_{5}$ case) that there is possibly in nature a super-symmetry corresponding to $\mathrm{SU}_{3} \times \mathrm{SU}_{3}$. The baryon nine-fold belongs to the representation,

$$
\left(3,3^{3 i}\right)_{L}+\left(3^{3 F}, 3\right)_{R}
$$

(2) The symmetry exists in the limit $m_{0}=0$
(3) There may exists a normal octet of $1^{-}(C=-1)$ and a normal $\left(C=+1\right.$ ) octet of $1^{+}$particles, correspondind to $(1,8) \pm(8,1)$ representations.
(4) In addition to these $1^{-}$and $1^{+}$particles, there may exist $\left(\mathrm{O}^{+}\right)$and ( $\mathrm{O}^{-}$) mesons. These spin zero entities may belong either (like baryons) to the nonet representation ( $3,3^{\text {if }} \mathbf{~} \pm\left(3,3^{\text {min }}\right)$

## (see page 25)

in the Algebra of $\mathrm{SU}_{3} \times \mathrm{SU}_{3} \times \mathrm{SU}_{3} \times \mathrm{SU}_{3}$. There are of course in addition $4 \mathrm{SU}_{3}$ singlets making a total or 36 entities reminiscent of $\mathrm{SU}_{6}$ ).
$(C=+1)$ or like vector particles correspond to $(1,8) \pm(8,1)$
(with $C=1$ for $0^{-}$and -1 for $0^{+}$).
(6) What happens to the symmetry when the baryon mass is turned on.

Gell-Nann computins in the lowest order shows that the baryon nonet then splits into a singlet and an octet, with

$$
m_{\text {singlet }}=-2 m_{\text {octet }}
$$

Interpreting the negative mass particle as one with opposite parity the fussi prediction of this higher symmetry group is that the 9-th baryon may be twice as heavy as the octet but with spinparity $1 / 2^{-}$.
(7) For the scalar and pseudosealar meson (mass) ${ }^{2}$ spectrum, Gell-Mann and Marshak et al obtain for the $\left(3,3^{3}\right) \pm\left(3^{x}, 3\right)$ the followins results

$$
\begin{aligned}
& \mu^{2}+2 \Delta=O^{-}(1) \\
& \mu^{2}+\Delta=O^{+}(8) \\
& \mu^{2} \cdots O^{-}(8) \\
& \mu^{2}-\Delta \cdots o^{+}(1) \\
& \mu^{2}-2 \Delta \cdots
\end{aligned}
$$

With the inclusion of Gell-wann-Okubo type of symetrybreaking, and assuming that the now ubiquitous $K=730 \mathrm{mev}$
is indeed the "strange" number of the $O^{+}$octet; one predicts. ${ }^{3 I}$

$$
\begin{aligned}
K^{I}=K= & 730 \mathrm{Mev}(\text { input }) \\
\pi^{\prime}=\quad & 560 \mathrm{MeV}(G=-1 ; \text { decay modes } 2 \pi+\gamma \\
& \text { to order } \alpha, 2 \pi+2 \gamma, 2 \gamma, 2 \pi \\
& \text { to order } \left.\alpha^{2}\right)
\end{aligned}
$$

$$
\eta^{\prime}=\quad 770 \mathrm{Mev}
$$

파 For the $C=-1$ (abnormal) case, Marshal et al give the following values.

$$
\begin{aligned}
K^{l} & =688 \mathrm{Mev} \\
\eta^{\prime} & =630 \mathrm{Mev} \\
\pi^{\prime} & =837 \mathrm{Mev}
\end{aligned}
$$

The "abnormal" case was first considered by Nambu and Sakurai (Phys. Rev. Letters 11,42 (1963)) who showed that the production and decay rates of a C = -l octet are highly suppressed).

If the $0^{+}$and $1^{+}$objects exist, where are they? $* *$
To my mind, this is one of the deeper mysteries of the situston. Personally $I$ have no doubt in my mind the extended Algebra $\mathrm{SU}_{3} \times \mathrm{SU}_{3}$ has something to do with nature. That corresponding to each component of the Algebra, there exists a physical particle which XW There appears to be a fair sprinkling of $1^{+}$entities all over the mass spectrum there are enough possible suspects even to make an octet and a singlet (e.g. $\phi^{\prime}=1415, \quad \omega^{\prime}=980$, $K^{l x}=1320, \quad \rho^{\prime}=1220 \mathrm{Mev}$ seem to satisfy $\left.\quad 2 \phi^{\prime}+p^{\prime}+\omega^{\prime}=4 K^{*}, \quad K^{\prime}-\pi^{\prime} \approx K^{*}-\rho \approx K^{\prime *}-\rho^{\prime}\right)$ but the multiplet appears to possess the wrong $C$-parity, $C=-1$ ).
is an extrapolation from the existence of $1^{-}$and $0^{-}$particles. It . is possible that this extrapolation is not wholly warranted at least in the simple form it has been used so far.

## IV. Dynamical Models

In so far as dynamical models are relevant to my material, these fall into two classes. Firstly are the models which start conservatively with an eightfold of baryons and mesons and using the methods of Smatrix theory (and assuming trilinear couplines) predict the existence of the lofold (or the lack of binding for some other multiplets). This of oourse is good Physics. Its crowning achievement is in the work of Wali and warnock who show that a broken octet (broken in the sense that the masses satisfy the G-K-0 mass relation) leads dyamically to a broken decuplet (again broken in the sense of equal mass spacing).

Second
The next degree of sophistication is to seek to establish the existence of the starting 8-fold itself from the reciprocal
self consistency of a Bootstrap. this would provide a "dynamical oriEin" for the observed symmetries. The still higher sophistication is to look for a spontaneous breakdown of the symmetry within the, stability and the over-riding uniqueness postulates of the Bootstrap approach.

The Bootatrap ideamtraced recently by Lovelace at Imperial College - to Baron Kunchausen ${ }^{\text {Wín }}$, is an oxtremely attraotive idea. It is basically the idea that the physical universe is unique and the uniqueness demand coupled with analytioity and unitarity is sufficient to predict the observed features of the Universe including its symmetries.

I think both in theology and cosmology, from the very nature of these disciplines one always looks at the problem of the structure of the Universe in this light. for elementary particle theory, however, this type of thinking is new, deep and potent, I believe among natural philosophers Voltaire was the first to voice something similar this. Voltaire attributed to Leibuitz the principle that we live in the best of all possible worlds. The modern theoretical physicist seems to go beyond Lebnitz in asserting that we live not only in the best of all possible worlds - but in the only possible world. In lighter moments I sometimes wonder if the principle does not have the ring of the comfortins thought with which Dr. Panslos made life worth enduring for honest Candide.

[^7]This was the occasion of the famous Lisbon earthquake when 30,000 persons lost their lives. Let me quote from the famous Doctor. "Candide there is no effect without cause and in this best of all possible worlds everything is necessarily for the best, a rolcano at $F$ Lisbon it could not be anywhere else, for it is 1mpossible that things be not where they are- and all 1 s welln Let me. aummarise the situation as I see it.

I do not know who first used the word strange particles to characterise some of the mosteraiting objects one has discovered in Physics. Perhaps the smallest measure of change that has come over the subject during the last year is that strange particles are strange no more - and that the strangeness quantum number is as little or as much strange as isotopic spin or electric charge.

There is a suspicion that there might exist still higher symmetry - with $\mathrm{SU}_{3}$ as possibly an important link in the symmetry chain. There may be a new quantum number, it may be connected with the existence of triplets of integral charge. These triplets (the Sakatons in a completely nes guise) at their most exciting, may be a new form of Matter. It is a prospect before which imagination reels.

But with all this optimism there is also mixed a feeling of awe - awe at the magnitude of our ignorance.

We do not know what dynamical mechanism gives this tremendous stability, to the mass calculations. Is it that there are very heavy basic triplets, with masses of several Bev binding fieroely and defining a mass scale before which the baryon mass differences are but a small perturbation. Notwithstanding the
heroic efforts of the bootstrap physicist, we do not quite yet understand where the origin of the symetries lies. Or is it that this question is as futile as asking why spaceutine has dimensionalr ty four? The discovery of the symmetry group of strong interactions was an achievement but when one thinks of the problems that remain one wonders if this was perhaps not the last of the relatively simpler problems. Somehow pertaps the harder tasks remain - the deeper, the more challagging understandings have yet to come. Before $I$ close $I$ have one more debt to pay. In $1962, V$. Weisskopf summed up the spirit of the CERN Conference with Pyramids. Fig. 7.

During 1963 the major item of news the unfortunate demise of the Regge Pole uodel. The next slide presented at the Stanford Conference capture the spirit of 1963. Fig. B. Since then the Pyramids have become something of a tradition.

The apprehensive fears of 1964-perhaps somewhat exaggerated are shown in the next slide. Pig. 9.

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36. C.Zemech.
37.N.Gell-Mann.
38.J.Schwinger.
```

P1g. 1.

$m(1)-m(2)+m(3)-m(4)=0$

Fig. 2.


Fig. 3.



Pig. 5.

| (b) | resonances (oqual spacing lav) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | decuplet | sextet | triplot | singlet |
| T-0 | $\Omega^{-1672}$ | $\Omega^{-1} 1950$ | $\Omega^{\prime} \quad 2288$ | $\mathrm{SL}^{-1 / 2} 2506$ |
| $7-\frac{1}{2}$ | E*1527 | $\square^{\circ \prime 1} 1805$ | $\underbrace{\prime \prime \prime} 2003$ |  |
| $T=1$ | $r_{1}^{0} 1382$ | $\mathrm{I}_{1}^{\text {1 }} 1660$ |  |  |
| $T=\frac{3}{2}$ | $N^{*} 12 \times 7$ |  |  |  |
| Theso numbere have been corfited by condidering $Y_{1}^{\prime}(1,60)$ as the fecuilar $Y_{i}^{* \prime}$ jarticle |  |  |  |  |

71g. 6.


Fig. 7.


If thes is what thonk in is, let's cover it up and forget it ${ }^{\circ}$

Fig. 8.


I hope this structure holds till the next oonference.
Нядедсs, что соорухение продерхится до следурдея хонференияи
Pig.9.


[^0]:    Fir 1. J. Ne' eman, Nucl. Phys. 26, 222 (1961)
    2. M. Gell-Mann, Phys. Nev. 125, 1067 (1962)
    (to be continued on page 2 )

[^1]:    R.Dalitz (Phys. Le才t $\frac{5}{\Sigma}, 53(1963)$ ) as used (6) directly to compute the transition aess ( $\Lambda \Sigma$ ) and compared the resuit with that ${ }^{\circ} \mathrm{ob}$ tained from a study of the binding of mirror hypernuclei $\mathrm{He}^{4}$ and $H^{4}$. The agreement is not unsatisfactory.

[^2]:    *) Two triplets models have been considered by the following:

[^3]:    *) For an L-type triplet + a singlet one could not simultaneously conserve $Y$ and violate $C$.
    **) Relation (2) is a consequence of last order symmetry breaking. (1) and (2) have also been derived by Lee, Gursey \& Nauenberg.

[^4]:    *) In effect this is tantamount to giving the meson singlets a base mass different from the octets.

[^5]:    *) Liy personal view is that the most significant part of Schwingeris ticory is not so much its dynamical content but the introduction and the insistence upon the wider group-theoretic structure $\mathrm{U}_{3} \mathrm{xU}_{3}$. I know Schwinger disagrees with me. I s.all however return to this topic later.
    **) 1. S.L. Glashow and Bjorken. Preprint. Phys. Lett, 11, 255 (1964)
    2. V.V. Vladimirsky, $\mathrm{SU}_{4}$-symmetry. preprint.
    3. P. Tarjanne and V.L. ${ }^{4}$ replitz, Phys. Rev. Lett. II, 447 (1963)
    4. $\because$. Krolikowski, Nucl. Phys. (to be published).
    5. I. Cohen, $\mathrm{SU}_{4}$ model of particles and resonances. Preprint, 1964.
    6. Y. Hara. Fhys. Rev. 134, B 701 (1964).
    7. Z. Maki and Y. Ohnuki. Quartet Scheme for Elementary Particles. Freprint (1964).
    8. F. Hama, K. Fiatumoto and S. Tanaka. Broken U(4)-Symmetry in Barion-1.eson System. (Contributed paper).
    
    ***) H.Bacry, J.Nuyts, L.Van Hove, preprint CERN. ;
    A theory based on Sp6 has certain similarities with Schmingers the ory. In particular these authors aiso derive the $\rho, \phi, \omega, K^{*}$ quadra-
    tic formula: $(\omega-\rho)(\phi-\rho)=\frac{4}{3}\left(K^{*}-\rho\right)\left(\phi+\omega-2 K^{*}\right)$. tic formula: $(\omega-\rho)(\phi-\rho)=\frac{4}{3}\left(K^{*}-\rho\right)\left(\phi+\omega-2 K^{*}\right)$.
    ****) Notice that the adjoint representation of $\mathrm{SU}_{4}$ (to which must velons spin one particles) contains 15 components; the adjoint representation of Sp6 is ricier and admits of 21 ( $1^{-}$) entities.

[^6]:    ${ }^{3}$ R. Marshak and S. Okubo, Nuovo Cim. 12, 1226 (1961) Hz:

    Works on this topic was reported at the Conference by

    1. Y. Nambu and P.G.O. Freund
    2. i. Gell-iann
    3. R. Marshak, N. Mukunda and S. Okubo
    4. A. Salam and J.C. Ward.
[^7]:    KThe Baron lifted himself out of a swamp by his bootstraps. History narrats that the Baron's achievement was not appreciated by his contemporaries

