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PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS FROM COLLECTIVE TO GROUND STATES OF EVEN-EVEN STRONGLY DEFORMED NUCLEI

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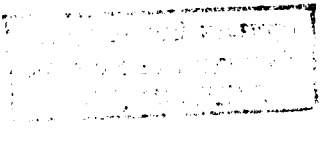

1. Introduction

The energies of quadrupole and octupole states of even-even deformed nuclei have recently been calculated[1,2]. It has been shown that the superfluid nuclear model, taking multipole-multipole interactions into account by the approximate second quantization method, gives a good explanation of the energy behaviour of these states. The structure of quadrupole and octupole states was investigated and these were shown to be collective levels in some nuclei and near to two-quasiparticle ones in others. Further investigations[3,4] proved that the blocking effect plays an important role in some cases and a method was developed, taking this into account. All the parameters of the model have been fixed in the mentioned papers and thus a basis was created for further applications.

In this paper the reduced probabilities of electromagnetic transitions between quadrupole and octupole states and the ground state of even-even deformed nuclei are calculated using the method of approximate second quantization in the superfluid nuclear model. The probabilities of E 2 transitions for γ -vibrations have been calculated by Marshalek and Rasmussen[5] and for β -vibration by Bes[6]. In this paper we calculate E2 transitions from γ -vibrational states and E3, E1 transitions from octupole states using a set of parameters determined in[1,2] and compare the results with the latest experimental data. An analogous calculation of E2 and E0 transitions for β -vibrational states is in preparation.

2. Transition Probabilities

We obtain the following expression for the reduced probability of a λ -multipole electrical transition between the phonon vacuum and a one-phonon state

$$B(E\lambda, I_i \to I_f) = \langle I_f \; \lambda \; K \; -K \mid I_i \; 0 \rangle^2 M^2 \qquad (1)$$

where M is the matrix element of the transition

$$M = e_p \sum_{\nu\nu'} \frac{f_{\nu\nu'} \cdot p_{\nu\nu'} \cdot U^2_{\nu\nu'}}{\epsilon_\nu + \epsilon_{\nu'} - \dfrac{\omega^2}{\epsilon_\nu + \epsilon_{\nu'}}} \cdot \frac{1 - 2\kappa_n X_n}{[(1-2\kappa_n X_n)^2 Y_n + 4\kappa_{np}^2 X_p^2 Y_p]^{1/2}} +$$

$$+ e_n \sum_{ss'} \frac{f_{ss'} p_{ss'} \cdot U^2_{ss'}}{\epsilon_s + \epsilon_{s'} - \dfrac{\omega^2}{\epsilon_s + \epsilon_{s'}}} \cdot \frac{1 - 2\kappa_p X_p}{[(1-2\kappa_p X_p)^2 Y_p + 4\kappa_{np}^2 X_n^2 Y_n]^{1/2}} \qquad (2)$$

Further we consider $\kappa_n = \kappa_p = \kappa_{np} \equiv \kappa$

Then we have

$$M = \frac{e_p \sum_{\nu\nu'} \dfrac{f_{\nu\nu'} \cdot p_{\nu\nu'} \cdot U^2_{\nu\nu'}}{\epsilon_\nu + \epsilon_{\nu'} - \dfrac{\omega^2}{\epsilon_\nu + \epsilon_{\nu'}}} + e_n \sum_{ss'} \dfrac{f_{ss'} \cdot p_{ss'} \cdot U^2_{ss'}}{\epsilon_s + \epsilon_{s'} - \dfrac{\omega^2}{\epsilon_s + \epsilon_{s'}}}}{[Y_n + Y_p]^{1/2}} \qquad (3)$$

here

$$X_n = \sum_{ss'} \frac{f^2_{ss'} U^2_{ss'}}{\epsilon_s + \epsilon_{s'} - \frac{\omega^2}{\epsilon_s + \epsilon_{s'}}} \qquad Y_n = \tfrac{1}{2} \frac{\partial X_n}{\partial \omega}$$

and $f_{ss'}$ is the quadrupole or octupole single-particle matrix element, $p_{ss'}$ the single-particle matrix element for the $E\lambda$ transition, $U_{ss'} = u_s v_{s'} + u_{s'} v_s$ ϵ_s energy of the quasiparticle in the state s, the index s, s' refers to the neutron system, ν, ν' to the proton one.

It follows from (3) that the transition is enhanced for $\omega \to 0$ (strongly collectivized states) and for $\omega \to \epsilon_s + \epsilon_{s'}$ (two-quasi-particle state) we obtain the single-particle quantity $f_{ss'} U_{ss'}$. For E2 and E3 transitions from quadrupole and octupole states $f_{ss'} \equiv p_{ss'}$ and only positive quantities are summed in (3). The enhancement is maximal in this case and the accuracy of the probability determination corresponds to the accuracy with which the energy is calculated. The accuracy is worse for the other transitions (E1, E0) since asymptotically forbidden matrix elements start to play an important role.

We have neglected the interaction of vibrations and rotations in the derivation of formulae (2) and (3). The coupling parameters given in [7] show that errors in the absolute transition probabilities due to this effect are of the order 20-30% and are thus comparable with the accuracy of the method itself.

Effective charges figure in (2) and (3) in order to compensate the dependence on the arbitrary cut off of the sums. The interaction constants κ_n, κ_p, κ_{np} play the same role in the secular equation. Thus two "free" parameters are obtained for a given type of transition - the neutron and proton effective charges. Taking into account $\kappa_n = \kappa_p = \kappa_{np}$ we consider $e_p = e + e_{eff}$, $e_n = e_{eff}$ and e_{eff} is determined from experiments (if enough experimental data for a comparison exist). Note that $B(E\lambda)$ are given throughout in single-particle units $B(E\lambda)_{s.p.} = (2\lambda+1)\frac{1}{4\pi}\left(\frac{3}{3+\lambda} R_0^{\lambda}\right)^2 e^2 cm^{2\lambda}$.

3. Results and Discussion

γ-vibration; rare-earth region.

Most of the existing experimental data [8,9] refer to $B(E2, 0_g \to 2\gamma)$ in the rare-earth region. The best correspondence between experimental data and the theoretical values is obtained for $e_{eff} = 1e$. The results are illustrated in Table 1. and Fig. 1. The first curve on the figure is obtained by putting the experimental values of the energy ω into (3). The very good agreement shows that (3) gives a correct connection between the energy of the level and the transition probability. The errors in this case are no larger than the changes of B(E2) due to the interaction with rotations. The second curve is obtained for ω calculated theoretically with $\kappa^{(2)} = \frac{9,5}{A^{4/3}} \hbar\omega_0$. Since the agreement between the experimental and theoretical values of ω is satisfactory we also obtain a good agreement for B(E2) in this case. The probability of the E2 transition in Yb^{172} differs strongly from B(E2) in the neighbouring nuclei. According to our calculations the first state $K\pi = 2+$ in Yb^{172} is of the purely two-quasi-particle type with a forbidden E2 transition to the ground state. The following $K\pi = 2+$ state with the energy $\approx$ 2 MeV will be collectivized and $B(E2) = 2$. The possible inaccuracy of the method is naturally larger for this nucleus - the given values must be considered to be estimations of the order of magnitude.

γ-vibrations, actinide region

Only two experimental values of B(E2) are known in this region [6,10] for Th^{232} and U^{238}. The theoretical B(E2) for $e_{eff} = 1$ and $e_{eff} = 0.7$ are given in Table 2. We have taken somewhat more levels in (3) into account in the actinide region, so that e_{eff} can be expected to be $e_{eff} < 1e$. Too few experimental data exist for a serious comparison of the results. We list two values of B(E2) for U^{238}, Pu^{240}, Cm^{242}, in Table 2, one small, the second larger. Two near $K\pi = 2+$ states with given B(E2) are obtained for these nuclei, but the given model cannot predict their mutual position - a slight change of $K\pi = 2+$ or of the pole energy interchange these states.

Octupole vibrations

Only two experimental points exist [10] for the reduced probabilities B(E3) from octupole states; $B(E3) = 21$ for U^{238} and $B(E3) = 12$ for Th^{232}. However the determination of the number K for the state $I\pi = 1-$ in Th^{232} is not quite rigorous. Thus the comparison with experiment is somewhat doubtful. However there is one important quantitative fact the enhancement of B(E3). This is definitely 4 or 5 times larger for Th and U than the enhancement of B(E2). Table 2 shows that this fact is explained quite well and without introducing new parameters. For Th^{232} two $I\pi = 1-$ states are obtained in our model at the energy $\approx$ 1 MeV. The first state $I\pi K = 1-0$ has $B(E3) = 14.5$, the second $I\pi K = 1-1$ has $B(E3) = 1.2$ It is quite possible that the Coriolis interaction plays an important role in this case, leading to the mixing of states. If this is not so the theory cannot explain the value of B(E3) for the $I\pi K = 1-1$ state in Th^{232}.

It has been shown in [2] that states with $K \neq 0$ are similar to two-quasi-particle ones. We have calculated B(E3) for these states, and as expected, the values B(E3) are near to unity. Since these calculations are only qualitative, we shall not give the results in this paper.

Table 1 shows the values of B(E3) for $I\pi K = 1-0$ states in the rare earth region ($e_{eff} = 1$). According to our calculations, no such enhancement of

the E3 transitions exists for the rare-earth elements as it does for Th, U, Pu. We have also calculated the reduced probabilities of E1 transitions for $I^{\pi}K = 1^{-}0$ states. The values of B(E1) change slowly from one nucleus to the other for all deformed nuclei and for $e_p = \frac{N}{A}e$, $e_n = -\frac{Z}{A}e$ we obtain $B(E1) = 4.10^{-2} - 8.10^{-2}$ single-particle units. This value is somewhat larger than that shown by Elbek/ 10/ to be the limit of the experimental resolution of B(E1). Although the error in the calculation of B(E1) is probably significant, the obtained result shows that it is important to perform the corresponding experiments.

The results of this paper show that the considered model gives a good description of the reduced probabilities of E2 and E3 electromagnetic transitions. This confirms the assumption, that the quantities which can be calculated using the approximate second quantization method are obtained in principle correctly.

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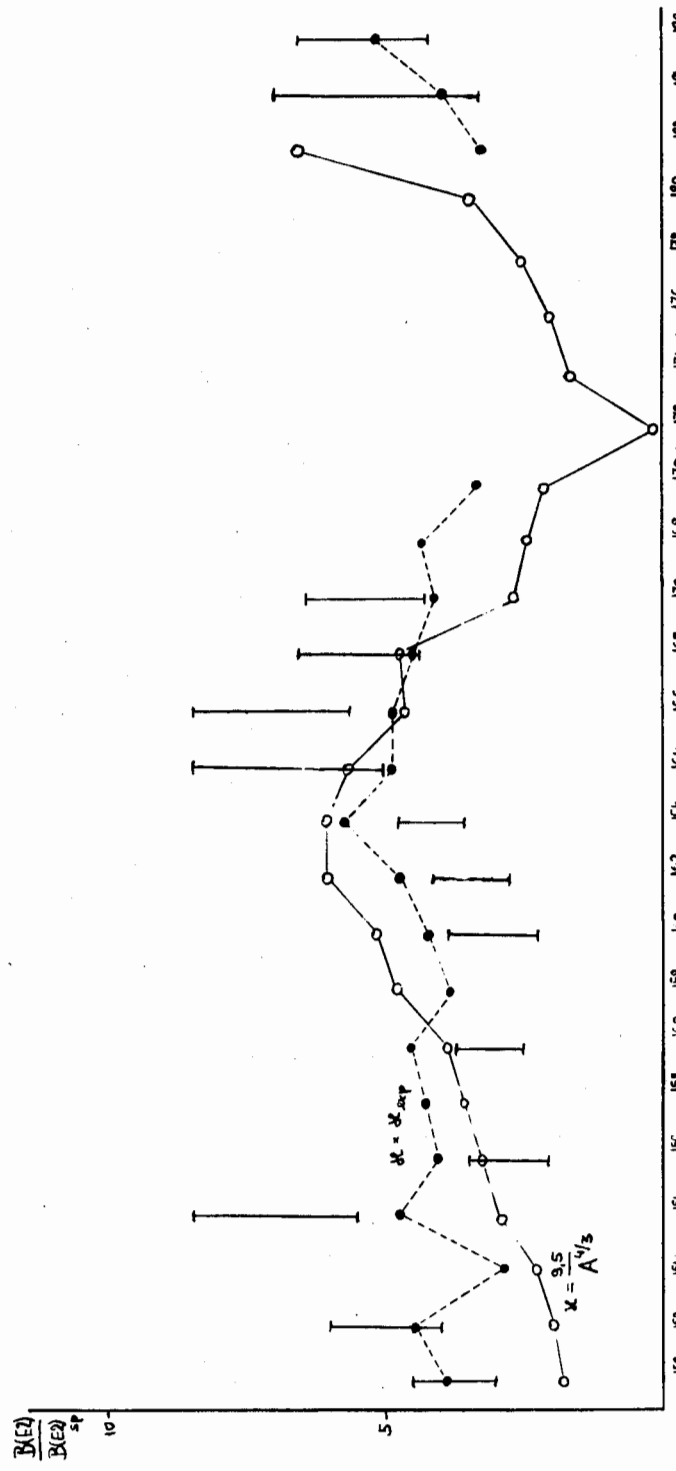

Fig. 1. $B(E2)_\gamma$ in the rare earth region .

Table 1

Quantities $B(E2)_\gamma = B(E2, 0_\gamma \to 2_\gamma)$; $B(E3) = B(E3, 0_\gamma \to 3\text{-}0)$

(all $B(E\lambda)$ in single particle units)

Nucleus	$B(E2)_\gamma$ exp	$B(E2)_\gamma$ $x = \frac{2.5}{A^{4/3}}$ $e_{eff} = 1$	$B(E2)_\gamma$ $x = x_{exp}$ $e_{eff} = 1$	$B(E3)$ $x = 0{,}00102$ $e_{eff} = 1$
Nd^{150}	3.6	1.8	3.9	4.0
Sm^{152}	5.0	2.0	4.5	3.2
Sm^{154}		2.3	2.9	5.8
Gd^{154}	7.0	2.9	4.8	3.4
Gd^{156}	2.8	3.3	4.1	5.1
Gd^{158}		3.6	4.3	6.5
Gd^{160}	3.4	3.9	4.6	6.0
Dy^{158}		4.8	3.9	4.5
Dy^{160}	3.2	5.2	4.3	5.0
Dy^{162}	3.4	6.1	4.8	4.5
Dy^{164}	4.1	6.1	5.8	5.0
Er^{164}	6.8	5.7	4.9	4.0
Er^{166}	7.3	4.8	4.9	4.0
Er^{168}	5.5	4.8	4.5	4.0
Er^{170}	4.9	2.7	4.2	4.3
Yb^{168}		2.5	4.4	4.0
Yb^{170}		2.2	3.4	3.8
Yb^{172}		0.02		3.9
Yb^{174}		1.8		3.7
Yb^{176}		2.2		1.7
Hf^{176}		2.3		3.5
Hf^{178}		2.6		4.0
Hf^{180}		3.5		3.8
W 180		4.2		3.6
W 182		6.6	3.3	3.0
W 184	5.5		4.0	2.8
W 186	5.5	6.1	5.2	2.2

Table 2

Quantities $B(E2)_\gamma = B(E2, 0_\gamma \to 2_\gamma)$, $B(E3, 0_\gamma \to 3\text{-}0)$

All $B(E\lambda)$ in single-particle units

Nucleus	$B(E2)_\gamma$ exp	$B(E2)_\gamma$ $x = \frac{11}{A^{4/3}}$ $e_{eff} = 1$	$B(E2)_\gamma$ $x = \frac{11}{A^{4/3}}$ $e_{eff} = 0{,}7$	$B(E3)$ exp	$B(E3)$ $x = 0{,}00053$ $e_{eff} = 1$
Th^{228}		10.0	6.5		29.5
Th^{230}		7.8	4.8		15.4
Th^{232}	3	6.0	4.0	12	14.5
Th^{234}		5.0	3.2		14.0
U^{230}		6.5	4.1		16.0
U^{232}		5.0	3.4		15.0
U^{234}		4.0	2.5		11.5
U^{236}		3.0	2.0		11.5
U^{238}	2	0.4(4.0)	0.4(2.5)	21	13
U^{240}		4.5	2.9		12.5
Pu^{236}		3.0	1.9		8.0
Pu^{238}		2.0	1.4		8.0
Pu^{240}		0.4(3.5)	0.4(2.2)		9.5
Pu^{242}		4.0	2.5		9.5
Pu^{244}		5.0	2.8		8.5
Cm^{242}		0.4(3.5)	0.4(2.2)		10.0
Cm^{244}		3.5	2.2		8.0
Cm^{246}		5.0	3.0		8.5
Cm^{248}		6.5	4.4		8.5
Cf^{248}		5.0	3.2		5.0
Cf^{250}		7.5	4.8		5.5
Cf^{252}		7.0	6.4		6.0
Fm^{250}		5.5	3.6		1.3
Fm^{252}		10.0	5.3		1.5
Fm^{254}		14.0	8.3		2.2