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STRUCTURE OF THE EXCITED STATES WITH $K\pi = 2+$ OF EVEN-EVEN DEFORMED NUCLEI

Many experimental and theoretical works are devoted to investigation of the collective $K\pi = 2+$ states (i.e. states with the momentum projection along the nuclear symmetry axis equal to 2 and positive parity). So, in [1,2] the energies of the $K\pi = 2+$ states and the probabilities of the B(E2) electromagnetic transitions are calculated on the basis of the superfluid nuclear model, taking into account quadrupole-quadrupole interactions. The present paper which is a continuation of paper [2] deals with the calculation of the energies of two most low-lying $K\pi = 2+$ states of even-even nuclei in the range $150 < A < 190$ and $228 \leq A \lesssim 254$, taking into account the blocking effect. The properties of these states are investigated and the relationship between the collective and two-quasi-particle structure of excited states is found.

In [2] using the variational principle in the framework of the method of approximate second quantization a secular equation is obtained which determines the excited $K\pi = 2+$ state energies ω_i. When the quadrupole-quadrupole interaction constants are equal, i.e. $\kappa_n^{(2)} = \kappa_p^{(2)} = \kappa_{np}^{(2)} \equiv \kappa$ the secular equation takes the form

$$1/\kappa = 2 \sum_{\rho\rho'} \frac{f(\rho\rho')^2 \; U^2_{\rho\rho'}}{\epsilon(\rho) + \epsilon(\rho') - \dfrac{\omega_i^2}{\epsilon(\rho) + \epsilon(\rho')}} \equiv F(\omega_i) \qquad (1)$$

where the summation of $\rho\rho'$ is performed over the average field levels, $f(\rho\rho')$ is the matrix element of the quadrupole momentum operator (the wave functions and the scheme of the one-particle levels of the Nilsson potential are used)

$$\epsilon(\rho) = \sqrt{C^2 + \{E(\rho) - \lambda\}^2}, \qquad U_{\rho\rho'} = u_\rho v_{\rho'} + u_{\rho'} v_\rho .$$

To improve the accuracy of calculation we take into account the blocking effect. It is very difficult to take into account the blocking effect in a rigorous way, therefore, we use here the following simplified method: the chemical potentials λ are determined from the condition of conservation, on the average, of the number of protons and neutrons in the $K\pi = 2+$ states, the values of poles $\epsilon(\rho) + \epsilon(\rho')$ in (1) are replaced by the two-quasi-particle state energies calculated in just the same way as in [3]. Energies calculated for different value of κ, the first two poles of (1) and the corresponding experimental data are presented in Fig. 1. The second roots of (1) are located between the values of the first and second poles. As long as in most cases the distances between these poles are not large then the energies ω_2 are mainly determined by the position of the appropriate poles. From Fig.1 it is seen that the energies of the first $K\pi = 2+$ states calculated for $\kappa = 10\,A^{-4/3}\hbar\omega_0^{\bullet}$ ($\kappa_{np} = \kappa$) are in sufficiently good agreement with corresponding experimental data in both regions of strongly deformed nuclei. The account of the blocking effect has led to an improvement of this agreement. Agreement between theory and experiment is improved if calculations in the range $150 \leq A \leq 186$ are made for $\kappa = 9{,}5\, A^{-4/3}\, \hbar\omega_0^{\circ}$, and in the range $228 \leq A \leq 254$ for $\kappa = 11\, A^{-4/3}\, \hbar\omega_0^{\circ}$.

We discuss the particularities of the solution of (1). For this in Fig2 we give the values of $F(\omega)$ for Pu^{240} and Cf^{250}. The points of intersection of the straight line $1/\kappa$ with the curve $F(\omega)$ are the roots of eq. (1). As is known the collective state wave function is a superposition of two-quasi-particle states of different kind. If a state possesses the very pronounced collective properties then the value of the root essentially differs from that of the nearest poles, and $F(\omega)$ intersects $1/\kappa$ at a small angle. If the state is practically two-quasi-particle one than the value of the root almost coincides with that of the pole and $F(\omega)$ intersects $1/\kappa$ at the right angle.

To answer the question with what weights the two-quasi-particle states enter the given collective state we use the normalization condition of the one-photon state wave functions. The study shows that the overwhelming majority of the lowest $K\pi = 2+$ states possesses collective properties and a large number of two-quasi-particle states contributies to them. Their structure is similar to that of the states of U^{234} given in Table 1, which contains the contribution of most important two-quasi-particle states[x)] to the first ω_1 and the second ω_2 states with $K\pi = 2+$. From Fig. 1 and Table 1 it is seen that the energy ω_1 and the structure obtained in the case $\kappa = 12\, A^{-4/3}\, \hbar\omega_0^{\circ}$ and $\kappa_{np} = 0{,}7\,\kappa$ are close to the case $\kappa = 10\, A^{-4/3}\, \hbar\omega^{\circ}$ and $\kappa_{np} = \kappa$. Thus, a decrease of κ_{np} as compared to κ_n and κ_p may be compensated by some increase of the latter.

The one-phonon collective state wave function turns into the two-quasi-particle state wave function when the root of the secular equation ω is very close to the pole. For the used values of κ for the lowest $K\pi = 2+$ states this occurs only when the matrix element $f(\rho\rho')$ corresponding to the first pole is very small. In this case the first or second state with $K\pi = 2+$ is two-quasi-particle one. If the straight line $1/\kappa$ intersects $F(\omega)$ first at the right angle, and then at a small one then the first state will be two-quasi-particle and the second one - collective. Such was the way in Yb^{172} where the contribution of the neutron state 512↑-521↓ (calculated with account of the blocking effect)

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x) By $Nn_z\Lambda\uparrow$ we denote the state $K\pi[Nn_z\Lambda]$ of the Nilsson potential with $K = \Lambda + \Sigma$ and by $Nn_z\Lambda\downarrow$ with $K = \Lambda - \Sigma$.

to the $K\pi = 2+$ state and of energy 1.468 MeV is 99.6%. The calculations made prove the correctness of the interpretation of this state given in [3] on the basis of the analysis of the beta decay of Tm^{172}.

If $1/\kappa$ intersects $F(\omega)$ first at an small angle and then at an obtuse one then the first state is collective and the second one - two-quasi-particle. Such is the way in Cf^{250} what is seen from Fig. 2 and Table 1. In U^{238} and Pu^{240} the situation is more complicated. A small change of κ or a displacement of the pole of the neutron state 622↑ - 631↓ leads to a change of the order of the collective and two-quasi-particle states. So, according to calculations without the account of the blocking effect [2] the first states in U^{238} and Pu^{240} are collective and the calculated probability of the electromagnetic transition agrees with experimental data [4]. In the present calculations for $\kappa = 11\, A^{-4/3}\hbar\omega_0^0$ the first state is two-quasi-particle and the second one-collective, yet, for $\kappa = 13\, A^{-4/3}\hbar\omega_0^0$ the first state is collective and the second one - two-quasi-particle. Experimental data on the Coulomb excitation of U^{238} [4] and on beta decay to Pu^{240} [5] point out that the observed $K\pi = 2+$ states are collective. To prove the correctness of the given statements about the relationship between collective and quasi-particle states it is necessary to study experimentally the structure of the two first states in Yb^{172}, U^{238}, Pu^{240} and Cf^{250}.

Thus, in the framework of the superfluid nuclear model a common description of two-quasi-particle and collective non-rotational states of deformed even nuclei is obtained. It is shown that the average nuclear field defines which of the low lying $K\pi = 2+$ states are collective and which are two-quasi-particle ones.

In conclusion I express my gratitude to N.Bogolubov and P.Vogel for interesting discussions and also to A.A.Korneichuk and G.Jungklaussen for performing numerical calculations.

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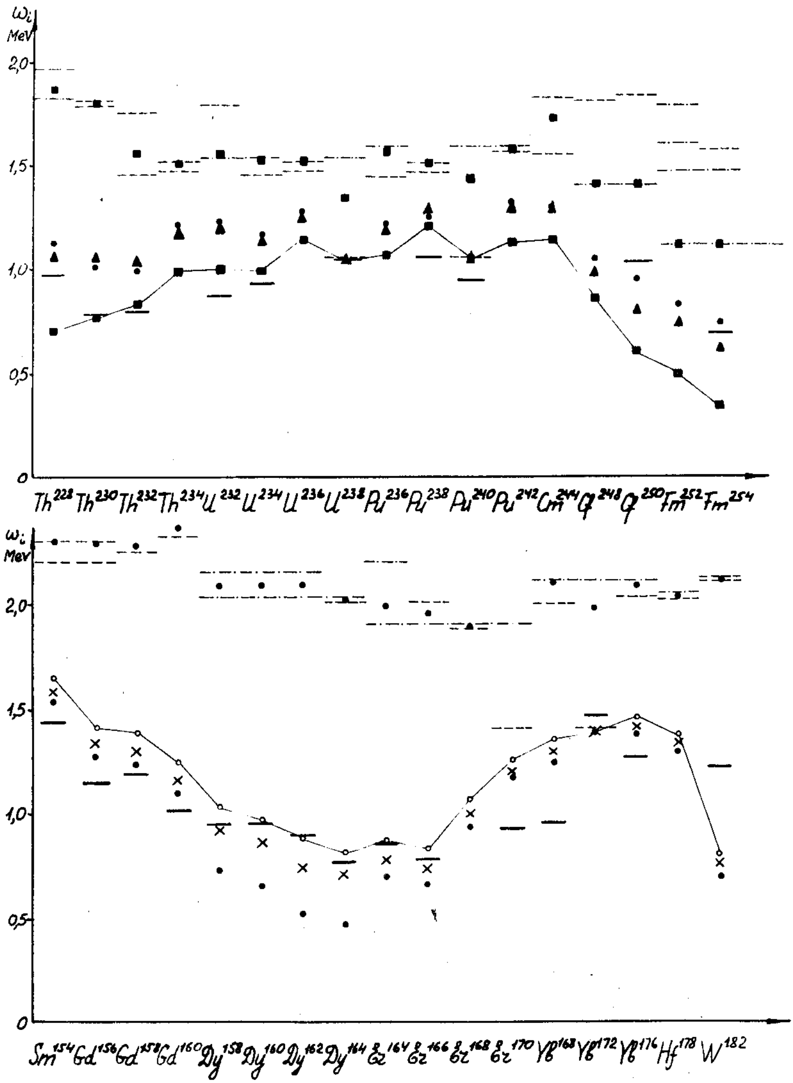

Fig.1. Energies of states with $K\pi = 2+$ for $\kappa = k\, A^{-4/3}\, \hbar\omega_0^0$

Notations: — experimental data

– – – neutron pole

—·—·— proton pole.

○ $k = 9,5$ $\kappa_{np} = \kappa$

• $k = 10$ $\kappa_{np} = \kappa$

■ $k = 11$ $\kappa_{np} = \kappa$

× $k = 11,5$ $\kappa_{np} = 0,7\,\kappa$

▲ $k = 12$ $\kappa_{np} = 0,7\,\kappa$

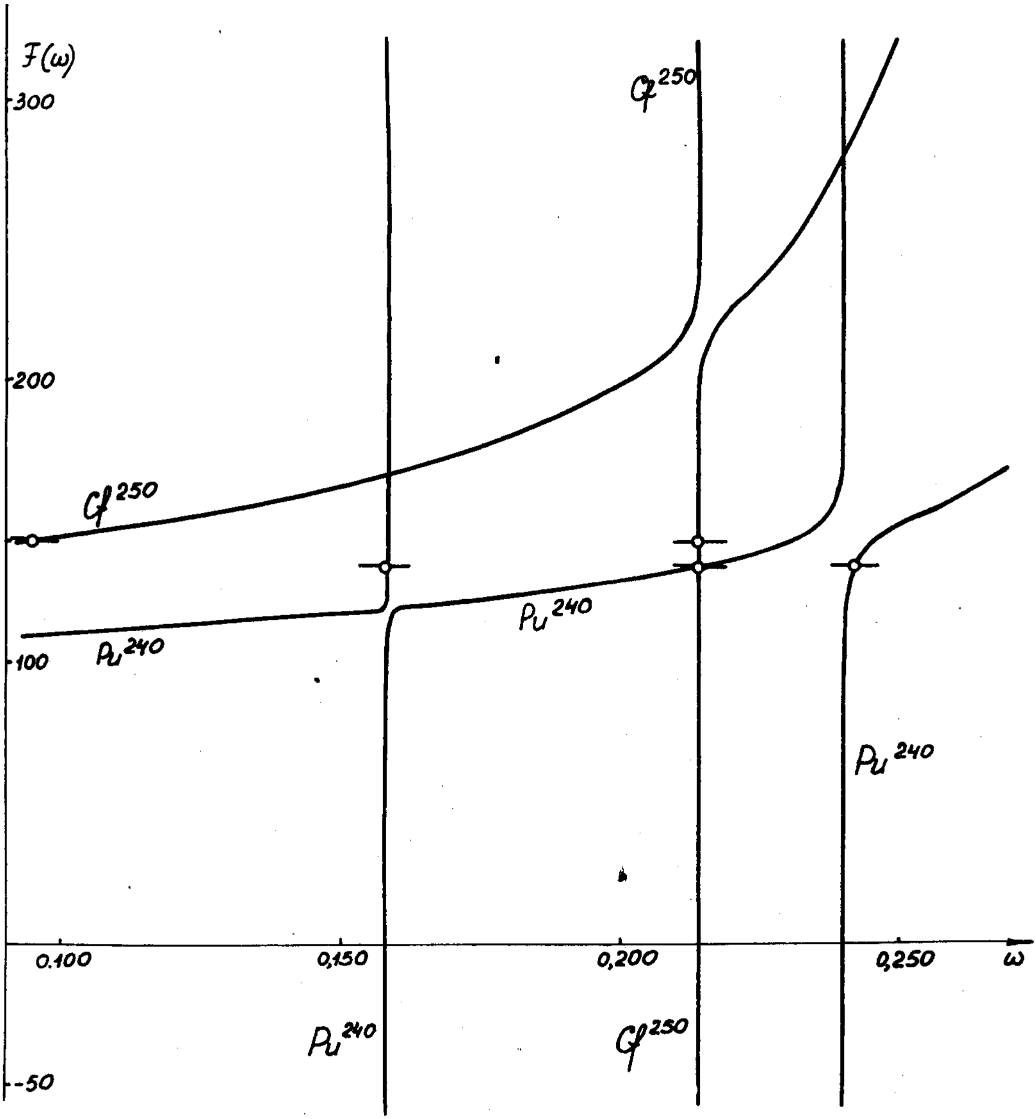

Fig.2. The behaviour of functions $F(\omega)$ of Pu^{240} and Cf^{250}
(—θ— denote $1/\kappa$ intersects $F(\omega)$)

TABLE I

Contribution of two-quasi-particle states to collective states with $K\pi = 2+$ (in percent) for $\varkappa = 11 A^{-4/3} \hbar \mathring{\omega}_0$ $(\varkappa_{np} = \varkappa)$

Configuration of two-quasi-particle states	$f(pp')$	U^{234} ω_1	U^{234} ω_1 *)	Pu^{238} ω_1	Pu^{238} ω_2	Pu^{240} ω_1	Pu^{240} ω_2	Cf^{250} ω_1	Cf^{250} ω_2
Neutron states									
633↓ - 631↓	-0,82	40,3	51,0	63,7	4,3	0,5	13,4	0,1	10^{-4}
622↑ - 631↓	-0,005	10^{-4}	10^{-4}	10^{-3}	95,3	94,9	10^{-3}	10^{-5}	10^{-8}
622↑ - 620↑	-1,67	0,8	0,5	2,5	10^{-3}	1,0	17,3	26,7	0,07
624↑ - 622↑	-1,42	0,2	0,2	0,6	10^{-3}	0,2	3,2	21,4	0,07
606↑ - 604↓	-1,84	1,2	0,8	0,8	10^{-3}	0,1	1,5	1,2	10^{-3}
743↑ - 761↑	-1,02	9,8	8,4	2,3	0,05	0,09	1,6	0,07	10^{-5}
734↑ - 752↑	-0,89	2,1	1,5	1,9	0,01	0,2	3,9	0,8	10^{-4}
785↑ - 743↑	-0,73	0,4	0,3	0,8	10^{-3}	0,1	2,3	1,2	10^{-3}
613↑ - 611↓	-2,04	0,04	0,03	0,05	10^{-4}	0,01	0,2	1,8	10^{-3}
631↓ + 631↑	-0,86	15,8	14,9	5,9	0,1	0,2	5,1	0,1	10^{-4}
622↓ + 620↑	-1,71	0,1	0,07	0,2	10^{-4}	0,06	0,7	18,2	0,05
Proton states									
523↓ - 541↓	-0,178	1,2	1,4	1,2	0,07	0,1	10,4	0,02	10^{-5}
523↓ - 521↓	-0,94	0,6	0,4	1,2	10^{-3}	0,2	2,5	3,5	10^{-3}
512↑ - 530↑	1,10	1,1	0,7	1,1	10^{-3}	0,1	2,1	1,8	10^{-3}
514↓ - 521↑	-0,098	10^{-3}	10^{-3}	10^{-3}	10^{-5}	10^{-3}	0,02	0,2	99,7
642↑ - 660↑	-0,96	2,5	1,9	1,0	10^{-3}	0,1	1,9	0,1	10^{-4}
633↑ - 651↑	-0,85	2,5	1,9	2,4	0,01	0,3	5,0	1,4	10^{-3}
532↓ + 530↑	0,74	2,9	2,4	0,7	10^{-3}	0,08	1,3	0,07	10^{-4}
521↓ + 530↑	0,60	2,4	1,9	3,6	0,2	0,4	10,0	1,2	10^{-3}
521↓ + 521↑	1,23	0,3	0,2	0,6	10^{-3}	0,07	1,1	9,5	0,02

*) for $\varkappa_{np} = 0,7 \varkappa$, $\varkappa = 12 A^{-4/3} \hbar \mathring{\omega}_0$