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1. Introduction

As is well known electron nucleon scattering experiments measure the electromagnetic form factors of the nucleon for spacelike momentum transfers. The form factors in this region of momentum transfer are real and to determine them it is sufficient to know the differential scattering cross section.

The experiments on the study of the annihilation processes

$$\bar{p} + p \to e^{+} + e^{-},$$
$$\bar{p} + p \to \mu^{+} + \mu^{-} \qquad (1)$$

are being presently made at CERN[1]. They yield the information about the proton electromagnetic form factors for timelike momentum transfers. It is easily seen from the unitarity of the S -matrix and the time reversal invariance that the form factors are complex in this region.

Measurement of the differential cross section for reaction (1) with unpolarized initial particles permits one to determine only the squares of the modules of the charge and magnetic form factors of the proton[2,3]. To determine their relative phase the experiments with polarized protons and antiprotons are necessary. It appears that such experiments become presently possible due to construction and successful application of a polarized proton target[4-6].

In the present paper we consider reactions (1) with polarized initial particles[x)]. The expressions for the cross sections will be obtained in the one-photon exchange approximation. We shall see that the measurements of the total cross sections for process (1) with polarized particles could provide information on the validity of this approximation.

2. Main Results

The diagram of processes (1) in the one-photon exchange approximation is given in Fig. 1

Here p and p' denote the four-momenta of the antiproton and the proton, q and q' are the momenta of the positron (μ^{+}-meson) and the electron (μ^{-}-meson)

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x) The cross sections of processes (1) when one of the initial particles is polarized have been obtained in refs.[2,3].

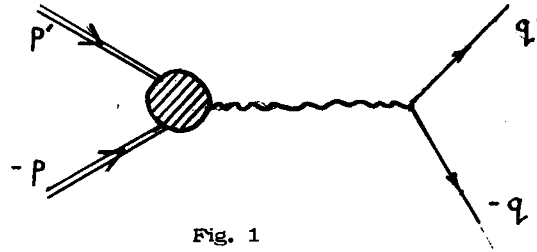

Fig. 1

The matrix element corresponding to this diagram is of the form

$$M(q\,q';p\,p') = -2\pi i\,\delta(q+q'-p-p')\left(\frac{m^2}{q_0\,q'_0}\right)^{1/2} \tag{2}$$

$$(-ie)\,\bar{u}_\ell(q')\,\gamma_\mu\,u_\ell(-q)\,\frac{1}{\kappa^2}\,\langle 0\,|\,j_\mu(0)\,|\,p\,p'\rangle\,.$$

Here m is the mass of the electron (μ -meson),

$q_0 = -iq_4\,,\quad q'_0 = -i\,q'_4\,,\qquad \kappa = p+p'$) and $j_\mu(0)$

is the current operator in the Heisenberg representation. The matrix element of this operator is

$$\langle 0\,|\,j_\mu(0)\,|\,p\,p'\rangle = i\,e\,\bar{u}(-p)\left[F_1(\kappa^2)\,\gamma_\mu + \frac{\mu}{2M}\,F_2(\kappa^2)\,\sigma_{\mu\nu}\,\kappa_\nu\right]u(p')\,\frac{1}{(2\pi)^3}\left(\frac{M^2}{p_0\,p'_0}\right)^{1/2} =$$

$$\frac{i\,e}{(2\pi)^3}\,\bar{u}(-p)\,\Gamma_\mu\,u(p')\,, \tag{3}$$

where M and μ are the mass and the anomalous magnetic moment of the proton, F_1 and F_2 are the Dirac and Pauli form factors ($F_1(0) = F_2(0) = 1$). The form factors F_1 and F_2 are connected with charge and magnetic form factors G_e and G_m by[7,8]

$$G_e(\kappa^2) = F_1(\kappa^2) - \frac{\kappa^2}{4M^2}\,\mu\,F_2(\kappa^2)\,,$$

$$G_m(\kappa^2) = F_1(\kappa^2) + \mu\,F_2(\kappa^2)\,. \tag{4}$$

Using (2) and (3) we get the following expression for the cross section of processes (1) in the general case of arbitrarily polarized initial particles:

$$d\sigma = \frac{4\,a^2\,m^2 M^2}{\sqrt{(p\cdot p')^2 - M^4}}\,\frac{1}{\kappa^4}\,\mathrm{Sp}\,\gamma_\mu\left[-\Lambda(-q)\right]\bar{\gamma}_\nu\,\Lambda(q')$$

$$\mathrm{Sp}\ \Gamma_\mu\ \rho'(p')\ \bar{\Gamma}_\nu\ \rho\ (-p)\ \delta\ (q + q' - p - p')\ \frac{d^3q}{q_0}\ \frac{d^3q'}{q_0'} \tag{5}$$

In this expression $\Lambda(q') = \frac{\hat{q}' + im}{2im}$ and $\Lambda(-q) = \frac{-\hat{q} + im}{2im}$ are the projection operators, a is the fine structure constant, and $\rho'(p')$ is the covariant density matrix[9] of the proton:

$$\rho'(p') = \Lambda(p')\ ½\,(1 + i\,\gamma_5\ \hat{\xi}')\,, \tag{6}$$

where $\Lambda(p') = \frac{\hat{p}' + iM}{2iM}$, ξ'_μ

is the 4-component pseudovector of the proton polarization (the mean value of the operator $i\gamma_5\gamma_\mu$). The matrix $\rho(-p)$ is connected with the density matrix of the antiproton $\rho_a(p)$ by:

$$\rho(-p) = -C\ \tilde{\rho}_a(p)\ C^{-1}, \tag{7}$$

where C is the charge conjugation matrix satisfying the conditions:

$$C\ \tilde{\gamma}_\mu\ C^{-1} = -\gamma_\mu\,, \qquad \tilde{C} = -C\,,\quad C^+C = 1\,. \tag{8}$$

where ~ denotes the transposition. If the four-vector of the antiproton polarization is denoted by ξ_μ then

$$\rho_a(p) = \Lambda(p)\ ½\ (1 + i\,\gamma_5\,\hat{\xi})\,. \tag{9}$$

From (8) and (9) we get:

$$\rho(-p) = -\Lambda(-p)\ ½\,(1 + i\,\gamma_5\,\hat{\xi})\,. \tag{10}$$

The differential annihilation cross section in the c.m. system from (3), (4), (5), (6), (10) is

$$\frac{d\sigma}{d\Omega} = \frac{a^2\beta}{16\,E\,|\vec{p}|}\ \{(2 - \beta^2\sin^2\theta)\,|G_m|^2 + (1 - \beta^2\cos^2\theta)\left(\frac{M}{E}\right)^2|G_e|^2 - \sin 2\theta\,\frac{M}{E}\,\beta^2\,\mathrm{Im}\,G_e G_m^* \times \tag{11}$$

$$\times\,[(\vec{n}\vec{\mathcal{P}}) + (\vec{n}\vec{\mathcal{P}}')] + 2\,\beta^2\cos^2\theta\left(\frac{M}{E}\,\mathrm{Re}\,G_e G_m^* - |G_m|^2\right)[(\vec{\mathcal{P}}\vec{k})(\vec{\mathcal{P}}'\vec{k}') + (\vec{\mathcal{P}}'\vec{k})(\vec{\mathcal{P}}\vec{k}')]$$

$$+\,2\,\beta^2|G_m|^2(\vec{\mathcal{P}}\,\vec{k}')(\vec{\mathcal{P}}'\vec{k}') + 2\,[-2\,\beta^2\cos^2\theta\,\frac{M}{E}\,\mathrm{Re}\,G_e\,G_m^*$$

$$+(1+\beta^2\cos^2\theta)|G_m|^2-(1-\beta^2\cos^2\theta)(\frac{M}{E})^2|G_e|^2](\vec{\mathcal{P}}\vec{k})(\vec{\mathcal{P}}'\vec{k})$$

$$+[(1-\beta^2\cos^2\theta)(\frac{M}{E})^2|G_e|^2-\beta^2\sin^2\theta|G_m|^2](\vec{\mathcal{P}}\vec{\mathcal{P}}')\} \cdot \qquad (11)$$

Here $\vec{k}$ and $\vec{k}'$ are the unit vectors in the directions of the antiproton and electron (μ^- -meson) momenta, θ is the angle between $\vec{k}'$ and $\vec{k}$, β is the electron(μ^- -meson) velocity, $\vec{n}=\frac{[\vec{k}\ \vec{k}']}{|[\vec{k}\ \vec{k}']|}$ is the normal to the reaction plane, E and $\vec{p}$ are the energy and the momentum of the antiproton, and $\vec{\mathcal{P}}$ and $\vec{\mathcal{P}}'$ are the antiproton and proton polarizations in their rest systems with the same orientation of the axes as the orientation of the axes of c.m. system. If, e.g., $(\xi^c, i\xi_0^c)$ is the value of the polarization four-vector in the c.m. system, then

$$\vec{\xi}^c=\vec{\mathcal{P}}+(\vec{\mathcal{P}}\vec{k})\vec{k}\frac{E-M}{M},$$

$$\xi_0^c=\frac{(\vec{\mathcal{P}}\vec{p})}{M}. \qquad (12)$$

The differential cross section (11) in the case of the unpolarized particle annihilation ($\vec{\mathcal{P}}=\vec{\mathcal{P}}'=0$) coincides with that obtained in ref.[2].

From (11) it is also seen that the asymmetry in the angular distribution of electrons, arising in the annihilation of polarized antiprotons with unpolarized protons ($\vec{\mathcal{P}}\neq 0$, $\vec{\mathcal{P}}'=0$) is equal to asymmetry of electrons when $\vec{\mathcal{P}}=0$, $\vec{\mathcal{P}}'\neq 0$ In refs.[2,3] it is stated that the asymmetries in these cases are equal in magnitude and opposite in sign. Notice that this difference is due to various definitions of the antiparticle polarization.

Integrating (11) over the directions of the electron (μ -meson) momentum we get the following expression for the total annihilatoin cross section:

$$\sigma=\frac{\pi a^2\beta}{12E|\vec{p}|}(3-\beta^2)\{2|G_m|^2+(\frac{M}{E})^2|G_e|^2$$

$$+(\frac{M}{E})^2|G_e|^2(\vec{\mathcal{P}}\vec{\mathcal{P}}')+2[|G_m|^2-(\frac{M}{E})^2|G_e|^2]\cdot(\vec{\mathcal{P}}\vec{k})(\vec{\mathcal{P}}'\vec{k})\} \qquad (13)$$

Finally, we give the expression for the differential annihilation cross section in lab. system:

$$\frac{d\sigma}{d\omega}=\frac{a^2}{2}\frac{1}{g^2(\theta_\Lambda)\ \epsilon\,|\vec{p}_\Lambda|}\{|G_m|^2+\frac{2\sin^2\theta_\Lambda}{g^2(\theta_\Lambda)}(\frac{M}{\epsilon})^2(|G_e|^2-\frac{\epsilon}{2M}|G_m|^2) \qquad (14)$$

$$-2\,\mathrm{Im}\,G_e G_m^* \frac{\cos\theta_\Lambda - a}{g^2(\theta_\Lambda)} \left(\frac{M}{\epsilon}\right)^2 \sin\theta_\Lambda [(\vec{n}_\Lambda \vec{\mathcal{P}}) + (\vec{n}_\Lambda \vec{\mathcal{P}}')] + 2\left(\frac{M}{\epsilon}\right)^2 \frac{1}{g^2(\theta_\Lambda)} \Big[\frac{\epsilon}{M} ((\cos\theta_\Lambda - a)\,\mathrm{Re}\,G_e G_m^*$$

$$-\cos\theta_\Lambda |G_m|^2)[(\vec{\mathcal{P}}\vec{k}_\Lambda)(\vec{\mathcal{P}}'\vec{k}'_\Lambda) + (\vec{\mathcal{P}}'\vec{k}_\Lambda)(\vec{\mathcal{P}}\vec{k}'_\Lambda)] + \frac{\epsilon}{M}|G_m|^2(\vec{\mathcal{P}}\vec{k}'_\Lambda)(\vec{\mathcal{P}}'\vec{k}'_\Lambda) + \sin^2\theta_\Lambda(|G_e|^2 - \frac{\epsilon}{2M}|G_m|^2)(\vec{\mathcal{P}}\vec{\mathcal{P}}')$$

$$+\left[\left(\frac{\epsilon}{M}\right)^2 (g(\theta_\Lambda) - \tfrac{1}{2}\sin^2\theta_\Lambda)|G_m|^2 - 2\sin^2\theta_\Lambda |G_e|^2 - 2\cos\theta_\Lambda(\cos\theta_\Lambda - a)\frac{\epsilon}{M}\mathrm{Re}\,G_e G_m^*\right](\vec{\mathcal{P}}\vec{k}_\Lambda)(\vec{\mathcal{P}}'\vec{k}_\Lambda)\Big]\Big\} \qquad (14)$$

Here $g(\theta_\Lambda) = 1 - a\cos\theta_\Lambda$ and $a = \frac{|\vec{p}_\Lambda|}{\epsilon}$, $\epsilon = E_\Lambda + M$, E_Λ and $\vec{p}_\Lambda$ are the laboratory energy and the momentum of the antiproton, $\vec{k}_\Lambda$ and $\vec{k}'_\Lambda$ are the unit vectors in the directions of the antiproton and electron (μ^- -meson) momenta, θ_Λ is the angle between $\vec{k}'_\Lambda$ and $\vec{k}_\Lambda$. In the above expression we have neglected the mass m .

3. Discussions

As is seen from the given formulas, the measurement of the differential cross section for the unpolarized particle annihilation makes it possible to determine only the modules of the charge and magnetic formfactors $|G_e|$ and $|G_m|$ The sine of the relative phase of form factors G_e and G_m can be determined from measurements of the left-right electron asymmetry when one of the initial particles is polarized. To determine the relative phase cosine (and thereby to determine the phase unambiguously) a study of antiproton-proton annihilation with both particles polarized is needed.

The expressions for the cross sections have been obtained by us in the one-photon exchange approximation. It is interesting to note that the measurements of the total cross sections with polarized particles allow one to test the validity of this assumption. In fact, as is easily seen from the considerations of invariance under space rotations and reflections the total cross section of any channel of the reaction with two polarized particles of spin 1/2 in the initial state is /10/

$$\sigma = \sigma_0 + \sigma_1 (\vec{\mathcal{P}}\vec{\mathcal{P}}') + \sigma_2 (\vec{\mathcal{P}}\vec{k})(\vec{\mathcal{P}}'\vec{k}'), \qquad (15)$$

where

$$\sigma_0 = \tfrac{1}{4}(\sigma_{t,0} + \sigma_s) + \tfrac{1}{2}\sigma_{t,1} ,$$

$$\sigma_1 = \tfrac{1}{4}(\sigma_{t,0} - \sigma_s), \qquad \sigma_2 = \tfrac{1}{2}(\sigma_{t,1} - \sigma_{t,0}),$$

and $\sigma_{t,0}$, $\sigma_{t,1}$ and σ_s are the total cross sections for the given channel of the reaction from the triplet state with the projections 0 and 1 and the singlet one, respectively. In the general case σ_0 , σ_1 and σ_2 are independent. However, in the one-photon exchange approximation the annihilation occurs only from $^3S_1$ and $^3D_1$ states of the proton-antiproton system. This means that in the one-photon approximation σ_0, σ_1 and σ_2 are connected by

$$\sigma_0 = \sigma_2 + 3\sigma_1 . \qquad (16)$$

The validity of this relation can be immediately verified by eq. (13) for the total cross section. The experimental check of eq. (16) is one of possible tests of applicability of the one-photon exchange approximation.

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