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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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1. Introducion

It has been shown in refs.[1,2] that to describe correctly the energy dependence of s - and p - phase shifts of the pion-nucleon scattering it is necessary to take into account the pion-pion interaction. However, consideration has been carried out in a static approximation.

The effect of the relativistic corrections on the estimation of the pion-pion interaction in treating pion-nucleon scattering has been recently considered[3-7] in detail.

In papers[3,4] it has been shown that the formula CGLN for $\mathrm{Re}\, d_s^{(-)}$ does not describe experimental data even at energies not higher than 100 MeV. The Born term is especially sensitive to relativistic corrections. In refs.[5,6] the authors have used the Cini-Fubini method. Comparing their results for s -wave with the non-relativistic approximation the authors are led to the conclusion that the non-relativistic approximation can lead to an incorrect extraction of the pion-pion interaction. While for p -wave the relativistic corrections become remarkable (≈ 30%) only at energies 250 MeV.

The problem of pion-nucleon scattering at low energies is reviewed in ref.[7].

In the present paper we consider the $s^{(-)}$ - wave of the pion-nucleon scattering with account of the relativistic corrections and additional inclusion of the s-wave into the unitarity condition (in addition to the large p_{33} wave). The method presented in ref.[1,2] has been used. It differs from other approaches[3,6]: one employs the complex plane $\nu = q^2$ instead of s and one find an analytic expression for $\mathrm{Re}\, f_s^{(-)}$ satisfying the crossing symmetry relations instead of detailed calculation of the energy behaviour of phase shifts. The dispersion relations permit to determine the functional dependence of $\mathrm{Re}\, f_s^{(-)}$ on the parameters like scattering length, ρ - meson position, high-energy contribution and so on.

Already in the non-relativistic approximation this method has led to satisfactory agreement with the experimental data on s and p phase shifts of pion-nucleon scattering.

As long as the pion-pion contribution to pion-nucleon scattering obeys the symmetry conditions[2], it is sufficient to get information about the pion-pion contribution to s -wave to determine its contributions to other pion-nucleon scattering waves.

The comparison of the relativistic calculation of $s^{(-)}$ - phase shift with the non-relativistic one was made for the same values of the parameters (scattering length, pion-pion resonance position in $J=T=1$ state and pion-nucleon interaction constants).

2. Equation for $f_s^{(-)}$ Partial Wave and Comparison with the Non-Relativistic Calculation

In deriving equation for $f_s^{(-)}$ -wave use has been made of Eqs. (4.1)-(4.3) from ref.[1]. Expression for $\mathrm{Im}\, a(\nu,\pm 1)$ and $\mathrm{Im}\, B^{(-)}(\nu,\pm 1)$ were taken in the form:

$$\frac{\mathrm{Im}\, a(\nu,\pm 1)}{4\pi} = \frac{1}{4[K(\nu)+\nu]}\left[\frac{W+M}{E+M}\,\mathrm{Im}\, f_s^{(-)} + \left(\pm 3\frac{W+M}{E+M} + \frac{W-M}{E-M}\right)\mathrm{Im}\, f_{p3/2}^{(-)}\right] \qquad (2.1)$$

$$\frac{\mathrm{Im}\, B^{(-)}(\nu,\pm 1)}{4\pi} = \frac{\mathrm{Im}\, f_s^{(-)}}{E+M} + \left(\pm\frac{3}{E+M} - \frac{1}{E-M}\right)\mathrm{Im}\, f_{p3/2}^{(-)}; \qquad (2.2)$$

where

$$a(\nu,\pm 1) = \frac{A^{(-)}(\nu,\pm 1)}{s-\bar{s}}\ ; \quad K(\nu) = \sqrt{(\nu+1)(\nu+M^2)}\ ; \quad W = E+\omega$$

E, ω are the energies of nucleon and π -meson in the c.m.s. respectively, ν is the squared momentum of π meson (or nucleon in the c.m.s.), ± 1 are the values of the cosine of the forward or backward scattering angle.

The account of the relativistic corrections and $\mathrm{Im}\, f_s^{(-)}$ in the unitarity condition yields to the following expression for the scattering length

$$a^{(-)} = \frac{2M}{M+1}\,\frac{f^2}{1-\frac{1}{4M^2}} + \frac{M}{M+1}\,\frac{1}{\pi}\int_0^\infty \frac{d\nu'}{\nu'}\left[\frac{1}{\omega'} + \frac{1}{E'}\right]\left\{\mathrm{Im}\, f_s^{(-)} + 2\,\mathrm{Im}\, f_{p3/2}^{(-)}\right\} \qquad (2.3)$$

This equation and analogous relations for the scattering wave lengths $a_1^{(-)}$ and $a_3^{(-)}$ obtained from equations for $\mathrm{Re}\, f_{p1/2}^{(-)}$ and $\mathrm{Re}\, f_{p3/2}^{(-)}$ has been used in deriving a final expression for $\mathrm{Re}\, f_s^{(-)}$. Using Eqs. (2.1)-(2.3) and dispersion relations (4.10), (4.2) from ref.[1] we get the followi g equation for $\mathrm{Re}\, f_s^{(-)}$:

$$\mathrm{Re}\, f_s^{(-)} = \frac{E+M}{4W}\sum_{i=1}^{5} f_i \qquad (2.4)$$

where

$$f_1 = 2\,\frac{M+1}{M}(W-M)\,a^{(-)}$$

$$f_2 = \frac{2\nu}{\pi} \int_0^\infty \frac{d\nu'}{\nu'(\nu'-\nu)} \frac{\omega E'}{a'} \, \mathrm{Im}\, f_s^{(-)}(\nu')$$

$$f_3 = [F(\nu)-1] \left\{ 2\frac{M+1}{M}(W-M)\, a^{(-)} - f^2 \frac{W-M}{M^2 - \frac{1}{4}} + \frac{1}{\pi}\int_0^\infty d\nu' \, \mathrm{Im}\, f^{(-)}_{p3/2} G(\nu') \right\}$$

$$f_4 = \frac{1}{\pi}\int_0^\infty d\nu' \, \mathrm{Im}\, f^{(-)}_{p3/2} \Big[\frac{\nu'}{a'(\omega+\omega')} \left(\frac{2}{\omega'} - \frac{1}{E'+M}\right) - \frac{\omega(a'+K')}{a'K'(E'+E)}\left(1 + \frac{\omega'(E'+M)}{\nu'}\right)$$

$$+ \frac{E'+M}{E'+E} \frac{1}{\nu' a'}\left(2\nu' + \omega\frac{\nu'-\nu}{E'+E}\right) - \frac{4}{a'} - \frac{3\omega'}{a'(E'+M)} + 3\frac{\omega'\nu'}{a'K'(E'+M)}\left(\frac{E'}{\omega+\omega'} + \frac{\omega}{E'+E}\right)$$

$$+ 3\frac{\omega\nu'}{a'K'(E'+E)} + \frac{AB}{K'} + G(\nu') \Big]$$

$$f_5 = f^2 (W-M)\left[\frac{1}{M^2\omega^2 - \frac{1}{4}} \left(1 - 8\nu M^2 \frac{a+K+\omega}{4a^2-1}\right) - \frac{1}{M^2 - \frac{1}{4}} \right]$$

$$a = K + \nu$$

$$A = 1 - \left(1 - \frac{1}{M^2}\right) \frac{\nu M^2}{(a'+a)^2} \frac{\left(1 + \frac{2a}{M^2+1}\right)\left(1 + \frac{2a'}{M^2+1}\right)}{1 + 2\frac{aa' + M^2}{(a+a')(M^2+1)}}$$

$$B = \frac{3}{E'+M}\left[\frac{a}{a'}(W'+M) + W - M\right] + \frac{E'+M}{\nu' a'}\left[\nu\omega' - \nu'\omega - \frac{\omega\omega'(\nu'-\nu)}{E'+E}\right].$$

$$+ \frac{a}{a'} - \frac{\nu}{\nu'} \frac{E'+M}{E+M}$$

$$G(\nu') = 6\frac{K\omega'}{K'\nu'(E'+M)} + 6\frac{W-M}{\nu'(E'+M)} - 4\frac{W-M}{\nu' E'} - 5\frac{\omega(\nu'-\nu)}{\nu' K'(E'+E)}$$

$$+ 3\frac{W-M}{K'(E'+M)} - \frac{\nu}{\nu'}\frac{(E'+M)}{K'(E+M)} - \frac{4\nu}{\nu'\omega'(E+M)} + \frac{\nu\omega}{\nu' K'(E+M)}$$

in which the sense of each term of (2.4) is clear from the definition.

In the limiting case $M\to\infty$ the functions f_i are equal to

$$f_1 = 2\,\omega\, a^{(-)}$$

$$f_2 = \frac{2\nu}{\pi}\int_0^\infty \frac{d\nu'}{\nu'\cdot(\nu'-\nu)}\,\frac{\omega}{\omega'}\,\mathrm{Im}\, f_s^{(-)}$$

$$f_3 = [F(\nu)-1]\; 2\,\omega\, a^{(-)} \tag{2.5}$$

$$f_4 = f_5 = 0$$

If, in addition, we neglect the contribution from $\mathrm{Im}\, f_s^{(-)}$ we obtain the non-relativistic approximation for $\mathrm{Re}\, f_s^{(-)}$ (see ref.[1])

$$\mathrm{Re}\, f_s^{(-)} = \omega\, a^{(-)}\, F(\nu) \tag{2.6}$$

Table 1 gives the values of each contribution of f_i separately to the expression for $\mathrm{Re}\, f_s^{(-)}$. The comparison with the non-relativistic expression (2.6) is also presented. Table 1 shows that the relativistic corrections are small in the considdered region of momenta $0\leq\nu\leq 3$. The relativistic corrections are largest in the resonance region $\eta\approx 1.5$. They change as desired the non-relativistic curve and make it closer to the experimental data (see Fig.1).

Table 1 representes the behaviour of the relativistic corrections to $\mathrm{Re}\, f_s^{(-)}$. The function f_2 associated with the account of $\mathrm{Im}\, f_s^{(-)}$ in $\mathrm{Im}\, A^{(-)}$ and $\mathrm{Im}\, B^{(-)}$ is small. Of importance is the compensation of functions f_4 and f_5 which leads to $\mathrm{Re}\, f_s^{(-)}$ being mainly determined by f_1 and f_3 i.e.

$$\mathrm{Re}\, f_s^{(-)} = \frac{E+M}{2M}\,\frac{M+1}{W}\,(W-M)\, a^{(-)}\, F(\nu) \tag{2.7}$$

Thus, the account of the relativistic corrections does not change practically the way in which the function depends $\mathrm{Re}\, f_s^{(-)}$ on the parameters $a^{(-)}$ and t_r and results in the appearance of some kinematic factor (see Table II).

Table III gives the comparison of relativistic and non-relativistic pion-pion contributions to $f_s^{(-)}$ -wave. The account of the relativistic corrections does not affect essentially the pion-pion interaction.

Conclusion

1. The calculation shows that in the used approach[1,2] the relativistic corrections are small through the whole energy region under consideration.

2. The account of the pion-pion interaction is essential to describe correctly the energy behaviour of $s^{(-)}$ and $p_{1/2}^{(-)}$ phase shifts of pion-nucleon scattering.

3. In more exact experimental determination of the pion-nucleon scattering phase shifts at energies up 300-400 MeV there is a reliable possibility to single out the pion-pion interaction and to determine its parameters (scattering wave length and resonance position).

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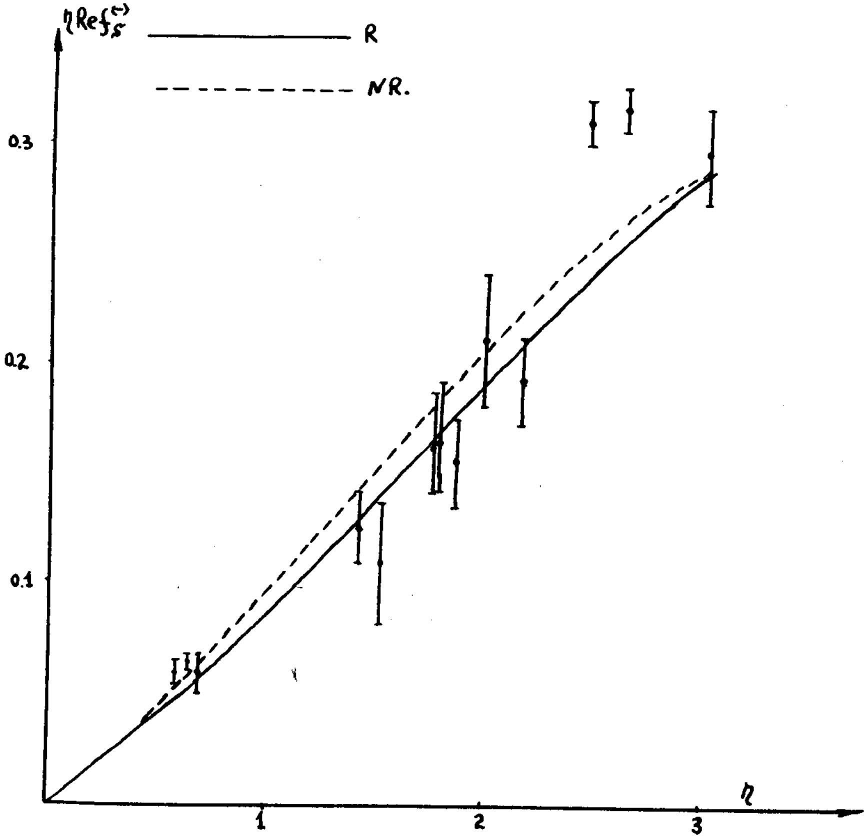

Fig. 1

Table I

$\eta \, Re \, f_s^{(-)}$ nonrelat	$\eta = \sqrt{\nu}$	f_1	f_2	f_3	f_4	f_5	$\eta \, Re \, f_s^{(-)}$ relativ	$\eta \, Re \, f_s^{(-)}$ (relativ) / $\eta \, Re \, f_s^{(-)}$ (nonrelativ)
0	0	0,I84	0	0	0	0	0	I
0,043	0,5	0,209	0,00049	-0,0I09	0,0I9	-0,0II4	0,0404	0,94
0,096	I	0,274	0,000I2	-0,0469	0,007	-0,027I	0,085I	0,89
0,206	2	0,465	0,0097	-0.I93	0,025	-0,0407	0,I90	0,92
0,288	3	0,699	0,0264	-0,409	0,040	-0,044I	0,287	I

Table II

η	$Re \, f_s^{(-)}$ (2.6)	$Re \, f_s^{(-)}$ (2.7)	(2.6) / (2.7)
0	0,08	0,08	I,00
0,5	0,086	0,086	I,00
I	0,096	0,096	I,00
2	0,I03	0,I00	I,03
3	0,096	0,89	I,08

Table III

η	$G_{\pi\pi}$ (nonrelativ.)	$G_{\pi\pi}$ (relativ.)	$G_{\pi\pi}$ (relativ.) / $G_{\pi\pi}$ (nonrelativ.)
0	0	0	I
0,5	-0,00384	-0,00467	I,2I
I	-0,0I74	-0,0I93	I,II
2	-0,0753	-0,07I8	0,952
3	-0,I57	-0,I37	0,873