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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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ASYMPTOTIC RELATIONS BETWEEN CROSS SECTIONS WITH THE ELECTROMAGNETIC INTERACTION TAKEN INTO ACCOUNT

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Об асимптотических соотношениях между сечениями при учете электромагнитного взаимодействия.

Показано, что при учете сильного и электромагнитного взаимодействий отношение дифференциальных сечений рассеяния частиц и античастиц при высоких энергиях стремится к единице, если эти сечения измерены аппаратурой с одинаковой (достаточно хорошей) разрешающей способностью по энергии. Относительно поведения амплитуд при больших энергиях сделано предположение, аналогичное тому, которое было использовано в работе /1/ при рассмотрении сильного взаимодействия.

Работа издается только на английском языке.

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Asymptotic Relations between Cross Sections with the Electromagnetic Interaction Taken into Account.

It is shown that the ratio of the differential cross sections for particles and antiparticles with strong and electromagnetic interactions taken into account tends to one at high energy if both cross sections are measured with equal (high enough) energy resolutions. The assumption about high energy behaviour of the scattering amplitudes which is made here is similar to what has been assumed for strong interactions in paper /1/.

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In paper [1] the equality of the differential cross sections for particles and antiparticles at high energies has been established on the basis of general postulates of the local quantum field theory for strong interactions and under the assumption that at high energies the scattering amplitudes do not oscillate. In this treatement the electromagnetic interaction has not been taken into account. On the other hand, the present experimental data show that the difference between $\pi^- p$ and $\pi^+ p$ cross sections at 20 to 30 BeV is a few per cent. Can the electromagnetic interaction be responsible for this difference ? The purpose of this paper is to show that when the electromagnetic interaction is taken into account then the differential cross sections for particles and antiparticles at high energies are equal (their ratio is equal to 1) if both of them are measured with equal (high enough) energy resolutions. The electromagnetic interaction will be considered in the framework of the quantum electrodynamics and we shall make an assumption about the high energy behaviour of the scattering amplitudes which is similar to what has been assumed for strong interactions in paper [1].

Let us consider the processes (the bar designates the antiparticle)

$$a_1 + a_2 \to a_3 + a_4 \qquad (1)$$

$$\bar{a}_3 + a_2 \to \bar{a}_1 + a_4 \quad , \qquad (2)$$

where the particles interact through strong and electromagnetic (or only electromagnetic) interactions.

Let $p_i\,(p_i^2 = m_i^2)$ and $z_i\,e\,(e>0)$ be the momentum and the electric charge of the particle $a_i\ (i = 1,2,3,4)$ in process (1), $p_1(p_1^2 = m_3^2)$, $p_3(p_3^2 = m_1^2)$ and $p_i\ (i = 2,4)$ be the momenta of the particles $\bar{a}_3$, $\bar{a}_1$ and a_i respectively in process (2), $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$ and $u=(p_1-p_4)^2$.

Without the electromagnetic interaction the equality of the cross sections for processes (1) and (2) follows from the analytic properties of the four particle scattering amplitudes. When the electromagnetic interaction is included and process (1) involves charged particles, we cannot any longer confine ourselves to considering only four particle processes (or processes with any fixed number of particles) and must take into account an indefinite number of soft photons which are emitted in processes (1) and (2) and escape detection. The amplitudes of processes (1) and (2) without emission of the photons are equal to zero; this fact manifests itself in perturbation theory as infrared divergence. The usual way to avoid this difficulty is to introduce a fictitious photon mass λ into the photon propagator and to put $\lambda = 0$ only at the end of calculations when the cross section has been summed over all undetected photons. The disadvantage of this procedure is that the analytic properties of the amplitudes as functions of s and t may be different depending on whether $\lambda = 0$ or $\lambda \neq 0$. As we want to use these analytic properties we shall proceed in the following way. Let us introduce λ and denote by $T_\lambda(s,t)$ any of the invariant amplitudes for processes (1) and (2) without emission of photons. We shall consider the invariant amplitudes with definite crossing symmetry properties, which were used in paper [1]. The dependence of T_λ on the auxiliary parameter λ can be taken into account explicitly with the aid of the formula [2]

$$T_\lambda = \exp(F_\lambda)\, T'_\lambda \qquad (2)$$

where the factor $\exp(F_\lambda)$ contains infrared divergences at $\lambda \to 0$. It is very likely (though not rigorously proved [2]) that at $\lambda = 0$ T'_λ is finite in the physical region. Under this assumption we shall in the following consider the amplitude T'_λ at $\lambda = 0$ which will be denoted by T'.

The function F_λ for process (1) is equal to

$$F_{1\lambda} = -\sum_{i<j} z_i \theta_i z_j \theta_j F_{\lambda ij} , \qquad (4)$$

where $\theta_i = 1$ for outgoing particles and -1 for incoming ones,

$$F_{\lambda ij} = \frac{i\alpha}{8\pi^3} \int \frac{dk}{k^2-\lambda^2} \left(\frac{2\theta_i p_i - k}{2\theta_i p_i k - k^2} - \frac{2\theta_j p_j + k}{2\theta_j p_j k + k^2} \right)^2 , \qquad (5)$$

and α is the fine structure constant. For observable values of s $F_{\lambda ij}(s) = F_{\lambda ij}(s + i0)$. Integral representations for $F_{\lambda ij}$ were given in papers [2,3].

The function F_λ for process (2) can be obtained from equation (4) by the substitution

$$z_1 \rightleftarrows -z_3 , \quad m_1 \rightleftarrows m_3 . \qquad (6)$$

Let us define the functions in question for negative s as the causal limit for $Im\, s \to +0$. Then the functions $F_{i\lambda}(s,t)$ are crossing symmetric

$$F_{2\lambda}(s,t) = F^*_{1\lambda}(u,t) . \qquad (7)$$

From this and the continuity of T'_λ at $\lambda \to 0$ follows that the amplitudes T' have the same crossing symmetry properties as T_λ that is the properties which were considered in paper [1].

The analytic properties of the amplitudes T' are different from the analytic properties of the amplitudes in the theory of strong interactions [1]. In papers [3] there were considered the analytic properties of T' for photon, meson and fermion scattering in quantum electrodynamics. It was shown there that instead of the poles $(s-m^2)^{-1}$ and $(u-m^2)^{-1}$ which the amplitudes would have had in the case of a non-zero photon mass, the amplitudes T' have the so called infrared singularities, $\Phi(t)(s-m^2)^{-1+\beta(t)}$ and $\Phi(t)(u-m^2)^{-1+\beta(t)}$. If these singularities are subtracted from T', then at fixed $t<0$ in the fourth order perturbation theory the remaining functions can be represented as dispersion integrals over s along the real axis; they become infinite (in an integrable way) only at $s=m^2$ and $u=m^2$.

We can admit that the electromagnetic interaction in all orders in e does not violate the analyticity of T' in the upper half plane of s (at fixed t) and gives infinite singularities on the real axis (infrared singularities, or Coulomb poles for the charged particle scattering) only at a finite distance from the origin.

Let us assume now that the amplitudes T' satisfy the same conditions at $s\to\infty$ and fixed t as the amplitudes in the theory of strong interactions [1] that is: 1) T' is less than any exponent $e^{\epsilon|s|}$, $\epsilon>0$ at $s\to\infty$ in the upper half plane; 2) at $s\to\infty$ along the real axis there exist the finite limits

$$\lim_{s\to\infty}\frac{T_1'(s,t)}{\phi(s,t)}=V_1'(t)\,,\qquad \lim_{s\to\infty}\frac{T_2'^{*}(s,t)}{\phi(-s,t)}=V_2'(t)\quad {}^{*)} \tag{8}$$

among which at least one is not equal to zero. The function $\phi(s,t)$ here is an admissible function defined in [1]**. For example at $s\to\infty$ ϕ may behave as $s^{\mu(t)}(\ln s)^{\nu(t)}(\ln\ln s)^{\kappa(t)}\ldots$, where μ, ν, κ ... are real.

In this case, using the crossing symmetry conditions and applying Phragmén-Lindelöf's theorem with the contour going along the real axis for big $|s|$ and rounding the origin from above (to leave out the singularities of the amplitudes) we conclude that the limits (8) are equal to each other.

Then we have at fixed t.

$$\lim_{s\to\infty}\frac{d\sigma_{1\lambda}'(s,t)}{d\sigma_{2\lambda}'(s,t)}=1\,, \tag{9}$$

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* When the processes in question are described by several invariant amplitudes some of the amplitudes may satisfy condition (8) after multiplying them by s or s^2. Then all the amplitudes contribute to the cross section at $s\to\infty$.

** $[\phi(s,t)]^{-1}$ at fixed t satisfies the conditions: 1) it is analytic in the upper half plane of s and less than any exponent $e^{\epsilon|s|}$, $\epsilon>0$ at $s\to\infty$ in the upper half plane; 2) it is continuous along the real axis for big s; 3) $\lim_{s\to\infty}\frac{\phi(s,t)}{\phi(-s,t)}=e^{-i\pi\mu}$ where $\mu(t)$ is real.

where the cross sections are formally defined by the amplitudes T'_1 and T'_2.

We shall use equation (9) in the following. It is worth noticing that this equation can be obtained under some other assumptions. Let us consider the amplitude T_λ for big enough λ. As a function of s it has analytic properties which are quite similar to the properties of the amplitudes in the strong interaction theory |1,4|. If at $s \to \infty$ $T_{1\lambda}$ and $T_{2\lambda}$ satisfy condition (8) (do not oscillate), then

$$\lim_{s\to\infty} \frac{d\sigma_{1\lambda}(s,t)}{d\sigma_{2\lambda}(s,t)} = 1 \tag{10}$$

where the cross sections $d\sigma_{i\lambda}$ are defined by $T_{i\lambda}$. The treatment up to this point is quite similar to that in strong interaction theory |1,4|.

It is not difficult to see (for instance, with the aid of a spectral representation for $F_{\lambda ij}$ |3|) that at fixed t

$$\lim_{s\to\infty} Re(F_{1\lambda} - F_{2\lambda}) = 0 . \tag{11}$$

From (3), (10) and (11) we can write

$$\lim_{s\to\infty} \frac{d\sigma'_{1\lambda}(s,t)}{d\sigma'_{2\lambda}(s,t)} = 1 , \tag{12}$$

where $d\sigma'_{i\lambda}$ is defined by $T'_{i\lambda}$.

To get from here equation (9) we must assume that in equation (12) the order of the limits at $s \to \infty$ and $\lambda \to 0$ can be changed. It is sufficient for this to assume that for the amplitudes T'_λ the limits (8) exist uniformly with respect to λ (for λ in a neighbourhood of zero), or that the amplitudes T'_λ (and function $\phi(s,t)$) are continuous at $\lambda \to 0$ uniformly with respect to s for sufficiently big positive s.

Let us consider now the physical processes with emission of an arbitrary number of soft, undetected photons. Let us suppose that the energy of these photons in the laboratory system is small enough:

$$p_1^0 + p_2^0 - p_3^0 - p_4^0 \le \epsilon\ ^{*)}, \tag{13}$$

where ϵ in general is much less than the electron mass. If for processes involving strong interaction we neglect the contribution of virtual electron-positron pairs, then ϵ is much less than the pion mass.

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* This condition can be written in a more general form:

$$(p_1 + p_2 - p_3 - p_4)c/\sqrt{c^2} \le \epsilon , \tag{13a}$$

where c is a timelike vector. Condition (13a) means that at fixed t in the coordinate system where $\vec{c} = 0$, the energy resolution of the experimental arrangement should be not worse than ϵ. In what follows we shall use the invariant condition (13a).

Let us introduce the photon mass λ and denote by $T_\lambda^{(n)}$ an amplitude of processes (1) or (2) with emission of n photons with the momenta k_i and the polarizations e_i . Under condition (13) we can write

$$T_\lambda^{(n)} = (\prod_{i=1}^{n} a_i e_i) \tilde{T}_\lambda , \tag{14}$$

where for process (1) /with substitution (6) for process (2)/

$$a_{i1} = e \sum_j z_j \theta_j p_j / p_j k_i . \tag{15}$$

The amplitude $\tilde{T}_\lambda$ differs from the amplitude T_λ in (3) only by the new momentum conservation law which contains now the momenta k_i . This difference can be neglected for small ϵ . Then the physical cross section for processes (1) or (2) with emission of undetected photons is equal to

$$d\sigma(\epsilon) = \lim_{\lambda\to 0} \sum_{n=0}^{\infty} \frac{1}{n!} \int\dots\int (\prod_{i=0}^{n} \frac{d\vec{k}_i(-a_i^2)}{(2\pi)^3 2k_i^0}) \exp(2\mathrm{Re}F_\lambda) d\sigma'_\lambda , \tag{16}$$

where the region of integration is defined by the condition $\sum_{i=1}^{n} k_i c/\sqrt{c^2} \leq \epsilon$. Summing here over n |5| and taking the limit at $\lambda \to 0$ we get the following result

$$d\sigma(\epsilon) = (\frac{2\epsilon}{\mu})^B \Psi \, d\sigma' , \tag{17}$$

where μ is an arbitrary mass ($d\sigma$ does not depend on μ). For process (1) /with substitution (6) for process (2)/ we have

$$B = \sum_{i<j} z_i \theta_i z_j \theta_j b_{ij} , \tag{18}$$

$$b_{ij} = \frac{\alpha}{\pi} (2 - \frac{1}{a} \ln \frac{1+a}{1-a}) ; \quad a = (1 - \frac{m_i^2 m_j^2}{(p_i p_j)^2})^{1/2} ; \tag{19}$$

$$\Psi = [\Gamma(1+B)]^{-1} \exp(-CB - D - 2\mathrm{Re}G) , \tag{20}$$

where Γ is the Γ function and C is the Euler constant. The expressions for D and G have the form of equation (18) where

$$D_{ij} = d_{ii} - 2d_{ij} + d_{jj} ; \; G_{ij} = g_{ii} - 2g_{ij} + g_{jj} ; \tag{21}$$

$$d_{ij} = \frac{\alpha}{\pi} p_i p_j \int_0^1 \frac{dx}{p_x^2} \frac{1}{2h} \ln \frac{1+h}{1-h} ; \tag{22}$$

$$h = \left(1 - \frac{p_x^2 c^2}{(p_x c)^2}\right)^{1/2}, \quad p_x = p_i x + p_j (1-x); \tag{23}$$

$$\delta_{ij} = \frac{a}{4\pi} \int_0^1 dx \left(\frac{-\theta_i \theta_j p_i p_j}{\tilde{p}_x^2} - \tfrac{1}{2} \right) \ln \frac{\tilde{p}_x^2}{\mu^2}; \quad \tilde{p}_x^2 = (\theta_i p_i x + \theta_j p_j (1-x))^2 - i0 . \tag{24}$$

At $s \to \infty$ and fixed t the functions B and Ψ for processes (1) and (2) are equal

$$B_2 = B_1 + O\left(\frac{1}{s}\right); \quad \Psi_2 = \Psi_1 + O\left(\frac{\ln s}{s}\right). \tag{25}$$

From (9), (17) and (25) we conclude that when the electromagnetic interaction is taken into account then the ratio of the differential cross sections for processes (1) and (2) at $s \to \infty$ and fixed t is equal to

$$\frac{d\sigma_1(\epsilon_1)}{d\sigma_2(\epsilon_2)} = \left(\frac{\epsilon_1}{\epsilon_2}\right)^b . \tag{26}$$

The ratio of the cross sections depends on the ratio of the energy resolutions and is equal to 1 when the energy resolutions of both experiments coincide.

The exponent b is equal to

$$b = \sum_{i<j} z_i \theta_i z_j \theta_j b_{ij} , \tag{27}$$

where for ij = 12, 14, 34 and 32

$$b_{ij} = \frac{2a}{\pi}\left(1 - \ln \frac{s}{m_i m_j}\right) \tag{28}$$

and for ij = 13 and 24 b_{ij} is determined by equation (19).

The exponent b does not depend on energy s for the processes in which $z_1 = z_3$, for instance for processes (1) such as $\pi^+ + p \to \pi^+ + p$, $K^+ + p \to K^+ + p$, $\pi^+ + p \to K^+ + \Sigma^+$, $p + p \to p + p$, $\Sigma^+ + p \to \Sigma^+ + p$ and $\Sigma^+ + p \to p + \Sigma^+$.

In this case b is equal to

$$b = -b_{13}(t) - b_{24}(t) . \tag{29}$$

In particular, for the elastic pion proton scattering at $m^2 \ll |t| \lesssim M^2$ (m is the pion mass, M is the nucleon mass)

$$b \approx \frac{2a}{\pi} \ln \frac{|t|}{m^2} . \tag{30}$$

For $|t| \approx 1$ BeV ^{2} and $\epsilon_-/\epsilon_+ = 3$ $d\sigma_-/d\sigma_+ \to 1{,}02$.

For $t \to 0$ the functions b_{ij} in (29) go to zero if $m_i = m_j$. If $m_i > m_j$, then

$$b_{1l}(0) = \frac{a}{\pi}\left(2 - \frac{m_1+m_l}{m_1-m_l}\ln\frac{m_1}{m_l}\right). \qquad (31)$$

The exponent b grows logarithmically at $s\to\infty$ if $\epsilon_1 \neq \epsilon_2$. In this case the ratio of the cross sections becomes sensitive to the ratio of the energy resolutions. For instance

$$b = \frac{2a}{\pi}\ln\frac{s}{m_1 m_2} \qquad (32)$$

for processes (1) such as $\pi^- + p \to K^0 + \Lambda$, $K^- + p \to K^0 + \Xi^0$, $\Sigma^- + p \to \Lambda + n$ and $\Sigma^- + p \to n + \Lambda$. The same is true for processes like

$$\bar{p} + p \to \pi^+ + \pi^- \qquad (1a)$$

$$\pi^- + p \to p + \pi^- . \qquad (2a)$$

Let us suppose that at $s\to\infty$ and fixed t (which correspond now to backward $\pi^- p$ scattering in c.m.s.) the amplitudes of these processes do not oscillate or oscillate in some special way, for instance according to a single fermion Regge pole contribution. Taking into account the electromagnetic interaction we get that at $s\to\infty$

$$\frac{d\sigma_1(\epsilon_1)}{d\sigma_2(\epsilon_2)} = \tfrac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2}\right)^b, \qquad (33)$$

where

$$b = \frac{4a}{\pi}\ln\frac{s}{Mm} . \qquad (34)$$

From the point of view of the present experimental accuracy ratio (26) for available energies is not far from 1 if the energy resolutions in processes (1) and (2) are approximately equal.

For a more precise definition of this ratio it is necessary to know accurately the energy resolutions. For this purpose detection of emitted photons (for instance, in bubble chambers) may be useful. Let us consider in this connection the spectra of soft photons emitted in processes (1) and (2) at high energies. Let us pick out the events in which condition (13) is fulfilled and n (= 0,1,2...) photons with the momenta k_i and the energy in the interval

$$\delta \le \sum_{i=1}^{n} k_i^0 \le \epsilon \qquad *) \qquad (35)$$

are emitted. The events in which the photon energy is less than δ are taken as "elastic" ones not depending on whether such photons can be detected or not. From (14) we get that the differential cross section of such n photon processes after summation over photon polarizations and integration over photon angles is equal to

$$d\sigma(\delta, k_1, \dots, k_n) = B^n\left(\prod_{i=1}^{n}\frac{dk_i^0}{k_i^0}\right) d\sigma(\delta), \qquad (36)$$

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*) For the processes with strong interaction this interval practically begins from several MeV and more and ends about 100 MeV.

where $d\sigma(\delta)$ is the "elastic" cross section given by formula (16) and B is given by equation (18). From (25) we conclude that at $s \to \infty$ and fixed t the spectra of soft photons emitted in processes (1) and (2) coincide.

Integrating equation (36) over the photon spectrum in the interval

$$\delta \leq \sum_{i=1}^{n} k_i^0 \leq \Delta \leq \epsilon \ , \qquad (37)$$

where the upper bound Δ is arbitraty (but less than ϵ) and summing over all the detected photons ($n = 0,1,\ldots$) we get the cross section $d\sigma(\delta,\Delta)$. For the processes (1) and (2) at $s \to \infty$ and fixed t

$$\frac{d\sigma_1(\delta,\Delta)}{d\sigma_2(\delta,\Delta)} \to 1 \qquad (38)$$

where the bounds δ and Δ of the photon spectrum can be accurately fixed.

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