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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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STUDY OF THE OCTUPOLE STATES OF EVEN-EVEN STRONGLY DEFORMED NUCLEI

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STUDY OF THE OCTUPOLE STATES OF EVEN-EVEN STRONGLY DEFORMED NUCLEI

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In investigating the collective, but non-rotational, excited states of even-even nuclei attention was first focussed on the quadrupole states with $\lambda = 2$ and $\mu = 0$ and 2 for which the most complete experimental data are available. The next term of expansion in multipoles of nucleon-nucleon interaction in a nucleus is a term corresponding to the octupole-octupole interaction. This interaction leads to collectivization of the excited states with $\lambda = 3$, $\mu = 0, 1, 2$ and 3 and negative parity. The experimental data on such states are systematized in ref./1/. They point out that in a number of cases the octupole states have a clear-cut collective nature. The octupole excited states were studied in refs./2,3/ both on the basis of the unified and the superfluid nuclear models. Main attention was paid to the investigation of the probabillties of electromagnetic transitions.

Calculations of the energies of the octupole states with $I\pi K = 1-0$ on the basis of the superfluid nuclear model were made in/4/ in the framework of the method of approximate second quantization. Close agreement has been obtained between the results of calculations and the corresponding experimental data. The investigation accounted for the lowering of the energies of the states with $K\pi = 0-$ in the isotopes of Th, U and Pu below the beta and gamma excited state energies.

The present paper is devoted to systematic investigation of the octupole excited states with $\lambda = 3$, $\mu = 0, 1, 2$ and 3 of even-even nuclei. The energies were calculated and the structure of these states were studied in both regions of strongly deformed nuclei on the basis of the superfluid nuclear model in the framework of the method of approximate second quantization.

1. Secular Equations and Wave Functions

The energies of the collective states will be calculated on the basis of the superfluid nuclear model. In this model the interaction Hamiltonian between nucleons in a nucleus is written in the form of three terms:

$$H = H_{av} + H_{pair} + H_{coll} \tag{1}$$

describing average nuclear field, interactions leading to pairing correlations of the superconductive type and interactions responsible for collective effects. The properties of even-even strongly deformed nuclei were studied in the model with the Hamiltonian $H_{av} + H_{pair}$. It is shown/5/ that this model rather well describes all the excited nuclear states but the quadrupole and, in some cases, the octupole states. In calculating the energies of the octupole collective states H_{coll} is taken in the form

$$H_{coll} = - \sum_{\mu=0,1,2,3} \{ \frac{\kappa_n^{(3)}}{2} Q^+_{3\mu}(n) Q_{3\mu}(n) + \frac{\kappa_p^{(3)}}{2} Q^+_{3\mu}(p) Q_{3\mu}(p) \quad (2)$$

$$+ \frac{\kappa_{np}^{(3)}}{2} (Q^+_{3\mu}(n) Q_{3\mu}(p) + Q^+_{3\mu}(p) Q_{3\mu}(n)) \}$$

where

$$Q_{3\mu}(n) = \sum_{ss' \atop \sigma\sigma'} f^{3\mu}_{ss'}(ss') a^+_{s\sigma} a_{s'\sigma}$$

$$f_{30} = r^3 Y_{30}, \quad f_{31} = \frac{r^3}{\sqrt{2}} (Y_{31} - Y_{3-1}) \quad (2^1)$$

$\kappa_n^{(3)}$, $\kappa_p^{(3)}$, $\kappa_{np}^{(3)}$ are the octupole-octupole interaction constants.

According to the superfluid nuclear model the wave functions of the collective states in the microscopic treatment are the superposition of wave functions of two-quasi-particle states of different type. The collective states are considered side by side with the two-quasi-particle ones, there being no restrictions to the collective state energies. In studying states with $\mu \neq 0$ in addition to the matrix elements $f^{3\mu}_{\sigma\sigma}(\rho_1 \rho_2) \equiv f^{\mu}(\rho_1\rho_2)$ where $K_1 \pm \mu = K_2$ the matrix elements $f^{3\mu}_{\sigma-\sigma}(\rho_1\rho_2) = \bar f^{\mu}(\rho_1\rho_2)$ with $K_1 + K_2 = \pm\mu$ are taken into account. Here K_1 and K_2 are the projections of moments on the nuclear symmetry axis, and ρ, ρ' are the quantum numbers characterising the average field levels both of neutron and proton systems. Further, by s we denote the quantum numbers of the neutron system states and by ν those of the proton ones.

Basing on the variational principle, in the framework of the method of approximate second quantization we get a secular equation defining the energies ω_i of the excited states with $\lambda = 3$, $\mu = 0$, 1, 2 and 3 of even-even strongly deformed nuclei which is of the form

$$1 = 2\kappa_n^{(3)} \sum_{ss'} \frac{(f_\mu(ss')^2 + \bar f_\mu(ss')^2) U^2_{ss'}}{\epsilon(s)+\epsilon(s') - \frac{\omega_i^2}{\epsilon(s)+\epsilon(s')}} + 2\kappa_p^{(3)} \sum_{\nu\nu'} \frac{(f_\mu(\nu\nu')^2 + \bar f_\mu(\nu\nu')^2) U^2_{\nu\nu'}}{\epsilon(\nu)+\epsilon(\nu') - \frac{\omega_i^2}{\epsilon(\nu)+\epsilon(\nu')}} + \quad (3)$$

$$+ 4((\kappa_{np}^{(3)})^2 - \kappa_n^{(3)}\kappa_p^{(3)}) \sum_{ss'} \frac{(f_\mu(ss')^2 + \bar f_\mu(ss')^2) U^2_{ss'}}{\epsilon(s)+\epsilon(s') - \frac{\omega_i^2}{\epsilon(s)+\epsilon(s')}} \sum_{\nu\nu'} \frac{(f_\mu(\nu\nu')^2 + \bar f_\mu(\nu\nu')^2) U^2_{\nu\nu'}}{\epsilon(\nu)+\epsilon(\nu') - \frac{\omega_i^2}{\epsilon(\nu)+\epsilon(\nu')}}$$

the summation of $ss'(\nu\nu')$ being made over the average field single particle levels, $\epsilon(s) = \sqrt{C_n^2 + \{E(s) - \lambda_n\}^2}$, $\epsilon(\nu) = \sqrt{C_p^2 + \{E(\nu) - \lambda_p\}^2}$ $U_{ss'} = u_s v_{s'} + v_s u_{s'}$ the index i in ω_i denotes the first, second and so on roots of the secular equation. Notice that for $\mu = 0$ $\bar f(\rho\rho') = 0$.

In case $\kappa_n^{(3)} = \kappa_p^{(3)} = \kappa_{np}^{(3)} \equiv \kappa^{(3)}$, which we shall restrict to in what follows the secular equation takes simpler form, namely

$$\frac{1}{2\kappa} = \sum_{ss'} \frac{(f_\mu(ss')^2 + \bar f_\mu(ss')^2) U^2_{ss'}}{\epsilon(s)+\epsilon(s') - \frac{\omega_i^2}{\epsilon(s)+\epsilon(s')}} + \sum_{\nu\nu'} \frac{(f^2_\mu(\nu\nu') + \bar f_\mu(\nu\nu')^2) U^2_{\nu\nu'}}{\epsilon(\nu)+\epsilon(\nu') - \frac{\omega_i^2}{\epsilon(\nu)+\epsilon(\nu')}} \equiv F(\omega). \quad (4)$$

The wave function of the i-th octupole state is $Q_i^+ \Psi$ where the operator is Q_i

$$Q_i = \tfrac{1}{2} \{ \sum_{ss'} (\psi^i_{ss'} A(ss') - \phi^i_{ss'} A^+(ss') + \bar\psi^i_{ss'} \mathcal{A}(ss') - \bar\phi^i_{ss'} \mathcal{A}^+(ss'))$$

$$+ \sum_{\nu\nu'} (\psi^i_{\nu\nu'} A(\nu\nu') - \phi^i_{\nu\nu'} A^+(\nu\nu') + \bar\psi^i_{\nu\nu'} \mathcal{A}(\nu\nu') - \bar\phi^i_{\nu\nu'} \mathcal{A}^+(\nu\nu')) \} \quad (5)$$

where

$$Q_i \Psi = 0$$

in this case

$$A(\rho\rho') = \frac{1}{\sqrt{2}} \sum_\sigma \sigma a_{\rho'\sigma} a_{\rho-\sigma}, \quad \mathcal{A}(\rho\rho') = \frac{1}{\sqrt{2}} \sum_\sigma a^+_{\rho\sigma} a_{\rho'\sigma}$$

here $a_{\rho\sigma}$ is the quasi-particle absorption operator. The functions $\psi^i_{\rho\rho'}$, $\phi^i_{\rho\rho'}$, $\bar\psi^i_{\rho\rho'}$, $\bar\phi^i_{\rho\rho'}$ are written in the form

$$\psi^i_{\rho\rho'} = \tfrac{1}{2}(g^i_{\rho\rho'} + w^i_{\rho\rho'}), \quad \bar\psi^i_{\rho\rho'} = \tfrac{1}{2}(\bar g^i_{\rho\rho'} + \bar w^i_{\rho\rho'})$$

$$\phi^i_{\rho\rho'} = \tfrac{1}{2}(g^i_{\rho\rho'} - w^i_{\rho\rho'}), \quad \bar\phi^i_{\rho\rho'} = \tfrac{1}{2}(\bar g^i_{\rho\rho'} - \bar w^i_{\rho\rho'}),$$

where

$$w^{i}_{\rho\rho'} = \frac{\omega_i}{\epsilon(\rho)+\epsilon(\rho')} g^{i}_{\rho\rho'} , \quad \bar{w}^{i}_{\rho\rho'} = \frac{\omega_i}{\epsilon(\rho)+\epsilon(\rho')} \bar{g}^{i}_{\rho\rho'} \qquad (6)$$

$$g^{i}_{\rho\rho'} = \frac{\sqrt{2}}{\sqrt{Y^i_n + \bar{Y}^i_n + Y^i_p + \bar{Y}^i_p}} \frac{f_\mu(\rho\rho') U_{\rho\rho'}}{\epsilon(\rho)+\epsilon(\rho') - \frac{\omega_i^2}{\epsilon(\rho)+\epsilon(\rho')}} \qquad (7)$$

$$\bar{g}^{i}_{\rho\rho'} = \frac{\sqrt{2}}{\sqrt{Y^i_n + \bar{Y}^i_n + Y^i_p + \bar{Y}^i_p}} \frac{\bar{f}_\mu(\rho\rho') U_{\rho\rho'}}{\epsilon(\rho)+\epsilon(\rho') - \frac{\omega_i^2}{\epsilon(\rho)+\epsilon(\rho')}}$$

in this case

$$Y^i_n + \bar{Y}^i_n = \sum_{ss'} \frac{(f_\mu(ss')^2 + \bar{f}_\mu(ss')^2) U^2_{ss'} (\epsilon(s)+\epsilon(s'))\omega_i}{[(\epsilon(s)+\epsilon(s'))^2 - \omega_i^2]^2}$$

After solving secular equations (3) or (4) we find the energies of the octupole states and the corresponding wave functions.

We discuss the particularities of eq. (4). As an example one gives in Fig. 1 the values of $F(\omega)$ as the function of ω for the states of U^{234} with $K\pi$ = 0-, 1-, 2-, 3-, (i.e. with μ = 0, 1, 2, and 3) The points of intersection of the curves $F(\omega)$ with the straight line $1/\kappa^{(3)}$ for each μ are the first and second roots of the corresponding secular equation. Fig. 1 gives the values of the first and second poles for μ = 0, 1, 2, 3. In those nuclei where the octupole-octupole interaction is effective it leads to the values of the first root being significantly smaller than that of the first pole and the curve $F(\omega)$ intersects the line $1/\kappa^{(3)}$ at a small angle. In this case the octupole state possesses the pronounced collective properties what is the case for $\mu=0$. If the state is not collective then the value of the first root ω_1 practically coincides with that of the first pole $\epsilon(\rho)+\epsilon(\rho')$ and the curve $F(\omega)$ intersects the line $1/\kappa^{(3)}$ at an angle close to 90°. S good example may be the case μ = 3 (Fig. 1). The values of the second roots lie between the values of the first and second poles. If the state corresponding to the second root is close to the two-quasi-particle one, then the value of the root is close to that of the second pole.

The frequencies of the octupole oscillations ω_i are found by numerical solution of secular equations (3) with the aid of the electronic computer. The first root ω_1 is sought in the interval

$$0 < \omega_1 < \min_{ss', \nu\nu'} ((\epsilon(s)+\epsilon(s')), \quad (\epsilon(\nu)+\epsilon(\nu')))$$

halving successively the interval. The second and subsequent roots are sought between the successive poles in the righthand side of (3). The first root may be absent if $\kappa^{(3)} > \kappa^{(3)}_{max}$ what will happen, e.g., for μ = 0 (Fig. 1) if $1/\kappa^{(3)} < 1,6 \cdot 10^3$. The second and subsequent roots exist for any $\kappa^{(3)}$.

2. The Average Field Levels and the Values of $\kappa^{(3)}$

The calculations of the energies of the octupole states with $K\pi$ = 0-, 1-, 2-, 3- and the reduced probabilities of electromagnetic transitions were made in both regions of strongly deformed nuclei: $150 \leq A < 190$ and $228 \leq A \leq 254$. Use was made of the wave functions and the schemes of the Nilsson potential single-particle levels/6/. All the calculations in the region $150 \leq A < 190$ were performed with the wave functions at the deformation δ = 0,3 and in the region $228 \leq A \leq 254$ at δ = 0,2 and for the same scheme of the single-particle levels in the neutron (proton) system for each nucleus in each region. In order to clear up how strongly the results of calculations depend on the change in the wave functions with increasing deformation δ the energies ω_1 of the states with $K\pi = 0$ were calculated in the region $228 \leq A \leq 254$ with the wave functions at δ = 0,3 but with unchanged values of the energies $E(\rho)$ of the average field one-particle levels. The obtained values of ω differ little from those calculated with the wave functions for δ = 0,2 but with renormalized $\kappa^{(3)}$. To make the calculations most unambiguous the changes in the nuclear deformation were not taken into account, i.e. the same set of $E(\rho)$ was used for all nuclei in each region. Therefore near the boundaries of the regions of nuclei with a large deformation the calculations became somewhat worse, since the equilibrium deformation of nuclei changed, while the behaviour of the average field single-particle levels was unaffected.

As the average field one-particle levels we took the Nilsson scheme levels with the parameters rather close to the data in ref./7/. The energies of the average field one-particle levels (in units $\hbar\omega_0^{\circ}$) the correlation functions and the chemical potentials calculated in/8,9/ are given in Tables 1-4. The correctness of the location of the average field levels in the regions $61 < Z \leq 79$, $89 \leq N \leq 115$ and $87 \leq Z \leq 99$, $137 \leq N \leq 155$ is justified by the available experimental data on the single quasi particle levels of odd A nuclei. In addition to these levels account was taken of all levels of those subshells in which the location of, at least, one single-particle state was proved experimentally. As to the behaviour and the account of the other levels there is a certain arbitrariness related after all to the choice of the parameters of the Nilsson scheme.

The one-particles levels given in Tables 1,2 and 3 are fairly close to the levels used in /4/ in calculating the energies of states with $K\pi = 0^-$ while the neutron level scheme given in Table 4 differs by that the states 761↓ with $N = 153$ and 752↓ with $N = 157$ are replaced by the states 620↑ and 622↓ respectively. We note that in the present calculations we took into account greater number of the average field one-particle levels as compared to the calculations in /4/. Calculating the energies of the quadrupole states /10/ use has been made of the schemes of the levels given in Tables 1-4.

The calculated values of ω_i depend on the wave functions and the eigenvalues of the average field potential and on the octupole-octupole interaction constants. The terms in (4) corresponding to the particle and hole states for all considered nuclei with $|(E(\rho) - \lambda| \gg C$ and $|E(\rho') - \lambda| \gg C$ lead to the renormalization of $\kappa^{(3)}$. For example, when $\mu = 0$ the term corresponding to states 651↓ - 761↓ given in Table 4 leads only to the renormalization of $\kappa^{(3)}$ for all nuclei in the region $228 \leq A \leq 254$. The same terms in (4) which in some nuclei correspond to particle and hole states and in others only to particle (or hole) states lead to a change of ω_i in some nuclei as compared to others. For example, when $\mu = 0$ the term 660↑ - 530↑ (see Table 3) for the isotopes of *Th* corresponds to the matrix element between particle and hole states. This term gives a large contribution to the secular equation. For the isotopes of *Pu* this term 660↑ - 530↑ corresponds to the matrix element between hole states and in (4) it became far less important due to the factor $U^2_{\nu\nu'}$ as well as to increasing $\epsilon(\nu) + \epsilon(\nu')$. We notice that the average field single-particle levels which were disregarded in the calculations made, will not affect noticeably ω_i but will merely lead to the renormalization of $\kappa^{(3)}$.

The octupole-octupole interaction constant $\kappa^{(3)}$ is chosen to obtain the best agreement of the calculated energies for the states with $K\pi = 0-$ with the corresponding experimental data. The states with $K\pi = 0-$ are most strongly collectivized of all the octupole states and therefore their energies are most sensitive to $\kappa^{(3)}$. The constant $\kappa^{(3)}$ in the region $150 \leq A \leq 190$ was assumed to be $\kappa^{(3)} = 0{,}00101\, \hbar\omega_0^0$, in the region $228 \leq A \leq 254$

$\kappa^{(3)} = 0.00057\, \hbar\omega_0^0$. $\kappa^{(3)}$ was unaffected inside each region. If by analogy with the quadrupole states we assume that

$$\kappa^{(3)} = \frac{k^{(3)}}{A^{4/3}} \hbar\omega_0^0$$

then $k^{(3)}$ takes the following values $k^{(3)} = 0{,}8 - 1{,}0$. The value $k^{(3)} = 1$ is about 10-12 times as small as the quadrupole-quadrupole interaction constant $k^{(2)}$ for the identical systems of the average field level /10/. The value of $\kappa^{(3)}$ assumed by us is about 1.5 times as small as the values of $\kappa^{(3)}$ used in ref. /4/ due to a considerable increase of the number of terms in (4). The same values of $\kappa^{(3)}$ were taken in calculating the energies of all the octupole states with $K\pi$ = 0-, 1-, 2-, 3-.

Notice that we have made calculations for the most general case $\kappa_n^{(3)} \neq \kappa_p^{(3)} \neq \kappa_{np}^{(3)}$. However, the results of these calculations do not lead to a significant improvement of the agreement between theory and experiment as compared with the case $\kappa_n^{(3)} = \kappa_p^{(3)} = \kappa_{np}^{(3)} = \kappa^{(3)}$. The equal octupole-octupole interaction constants were used by us so the number of the parameters applied should be minimal.

The values of the matrix elements $f(ss')$ and $f(\nu\nu')$ change between 0.01 and 8 (in dimensionless units used in /6/) the most important of them for U^{234} are given in Tables 5-8 for all μ. To illustrate the decrease of the collective properties of the octupole states with increasing μ we give the sums of all the squared matrix elements used in our calculations. For the region $150 \leq A < 190$ these sums are $\mu = 0$ - 287, $\mu = 1$ - 165, $\mu = 2$ - 173, $\mu = 3$ - 143. For the region $228 \leq A \leq 254$ the sums of the squared matrix elements are $\mu = 0$ - 427, $\mu = 1$ - 230, $\mu = 2$ - 249 $\mu = 3$ - 173. The increase of these sums in the region $228 \leq A \leq 254$ as compared with $150 \leq A < 190$ is compensated by the decrease $\kappa^{(3)} \sim A^{-4/3}$.

In making calculations the conservation of the number of particles, on an average, was controlled, i.e. the Δn_i quantities were calculated

$$\Delta n_i = \langle Q_i \sum_{\rho\sigma} a^+_{\rho\sigma} a_{\rho\sigma} Q_i^+ \rangle - \langle \sum_{\rho\sigma} a^+_{\rho\sigma} a_{\rho\sigma} \rangle \qquad (8)$$

i.e. the difference of the number of neutrons (protons) in the excited and ground states. In most cases one has obtained $\Delta n_i \lesssim 0{,}2$, however, there are cases when $\Delta n_i \sim 1$. The values $\Delta n_i \sim 1$ occur when the root is close to the pole and the blocking effect should be taken into account and in case of very small ω_1 too.

3. Energies of the Octupole Excited States

The first and second roots of secular equations (4) for the states with $K\pi$ = 0-, 1-, 2-, 3- were calculated and the energies of the octupole excited states and the corresponding wave functions were found thereby. The calculated energies of the octupole states ω_1 and ω_2, the values of the first and second poles as well as the available experimental data are given in Figs. 2-9. The experimental values of the energies are denoted by the continuous line, the first and second poles by the dashed lines. The first and second roots of the secular equations are denoted by the dark circles joined by the straight lines.

The first roots of secular equations (4) for the case $\mu = 0$ in both regions of strongly deformed nuclei lie far lower than the energies of the first poles. The discussion of the location of the energies of the states with $K\pi = 0-$ and the comparison between theory and experiment has been made in ref./4/. The agreement between theory and experiment is in this case rather good. It should be noted that a fairly important result is the explanation of the lowering of the states with $K\pi = 0-$ below the energies of the beta and gamma vibrational states in some isotopes of Th, U and Pu. Note that the changes made, as compared with/4/, in the neutron scheme of single particle levels and the large increase of the general number of terms in (4) do not lead to any significant changes in the energies of the states with $K\pi = 0-$. This shows that the average field single-particle levels which are disregarded by us will not change all the more the quantity ω_1 and if they will be taken into account this will lead to the renormalization of $\kappa^{(3)}$.

If the octupole state in its structure is close to the two-particle one then the blocking effect is of importance and its influence should be taken into account. In calculating the energies of the states with $K\pi$ = 1-, 2-, 3-, secular equations (4) were solved and the blocking effect was not taken into account. If the first root is close to the first pole then the influence of the blocking effect on the given octupole state is equal to the lowering of the energy of the appropriate two-quasi-particle state due to the blocking effect. In this case additional calculations are not needed and use should be made of the results obtained in/8,9,11/. In Fig. 4-9 the arrows denote the lowering of the octupole state energies due to the change in the values of the secular equation poles due to blocking effect.

The comparison of the calculated energies of the lowest states with the corresponding experimental data shows that if the blocking effect is taken into account then the agreement between calculations and experiment is rather good. It should be noted that there are at present not many experimental data on the states with $K\pi$ = 1-, 2-, 3- and a significant growth of the experimental material is very desirable. The energies of the first octupole states with $K\pi$ = 1-, 2- are lowered by 100-300 KeV as compared to the first poles. The values of ω_1 for the states with $K\pi$ = 3- are practically the same as the energies of the first poles. Therefore in most cases the effect of the octupole-octupole interactions on the energies of states with $K\pi$ = 1- and 2- should be taken into account while this effect should be neglected for states with $K\pi$ = 3-.

It is necessary to note that the correctness of the results of our calculations is proved not only by the available experimental data on octupole states but also by the experimental evidence for the fact, that up to certain energies in some nuclei there are no octupole states with a given $K\pi$. We consider, e.g., states with $K\pi$ = 2- in the region $150 \leq A \leq 186$. In the isotopes of Dy and W state with $K\pi$ = 2- is the lowest proton two-quasi-particle state whose energy is somewhat lowered by the octupole-octupole interactions. In the nuclei Dy^{160} and W^{182} best investigated experimentally, states with $K\pi$ = 2- are found, there is evidence for the existence of such a state in W^{184}. These data are in close agreement with calculations. The spectra of other isotopes of Dy and W are badly investigated. On the other hand, the available experimental data point out that in Yb^{172}, e.g., there are no levels with $K\pi$ = 2- lower than (1.7 - 1.8) MeV, which agrees also with our calculations.

Figs. 2-9 show that the values of the second roots ω_2 lie between the values of the first and second poles. If the first root of (4) is close to the first pole then the second one is usually close to the second pole. If the first root is lowered by 100 KeV and more with respect to the first pole, then the second one may practically lie at any point between the first and second poles and near the first pole, too.

4. Structure of Octupole Excited States

In considering the behaviour of the functions $F(\omega)$ and the energies of the octupole excited states it has been noted that states whose energies are far lower than the first poles possess the pronounced collective properties, while states whose energy coincide with the poles are, with sufficiently good accuracy, two-quasi-particle states. Now we look at the study of the structure of octupole states the other way round. Let us consider the contribution of each separate term in the secular equation. These terms of eq. (4) are denoted by

$$X_i(\rho\rho') = 2\kappa^{(3)} \frac{(f(\rho\rho')^2 + \bar{f}(\rho\rho')^2)\, U^2_{\rho\rho'}}{\epsilon(\rho) + \epsilon(\rho') - \dfrac{\omega_i^2}{\epsilon(\rho) + \epsilon(\rho')}} \qquad (9)$$

Further we shall treat the problem as to with what weights separate two-particle states enter the given collective state. To this end we shall use the normalization condition of the collective state $Q_i^+ \Psi$ which is written in the form:

$$\frac{1}{Y_n^i + \bar{Y}_n^i + Y_p^i + \bar{Y}_p^i} \{ \sum_{ss'} y_i(ss') + \sum_{\nu\nu'} y_i(\nu\nu') \} = 1 \qquad (10)$$

$$y_i(ss') = \frac{(f(ss')^2 + \bar{f}(ss')^2)\, U^2_{ss'}\, \omega_i\, (\epsilon(s) + \epsilon(s'))}{[(\epsilon(s) + \epsilon(s'))^2 - \omega_i^2]^2} .$$

As an example we shall consider the structure of the octupole states of U^{234}. In Tables 5-8 are summarized data relative to the first and second roots of the secular equation with $K\pi$ = 0-, 1-, 2-, 3- . One gives the most important two-quasi-particle states[x] in the neutron and proton systems, the values of the matrix elements $f(\rho\rho')$ (in dimensionless units[6]), the values of the first and second poles $\epsilon(\rho) + \epsilon(\rho')$ (in MeV). Further one gives the contribution of certain terms of secular equation (4) for the first $X_1(\rho\rho')$ and the second $X_2(\rho\rho')$ roots. For the sake of convenience these terms are multiplied by 100. And, finally, one gives the contribution (in percent) of certain two-quasi-particle states to the wave function $y_1(\rho\rho')$ of the first octupole state and $y_2(\rho\rho')$ of the second state with a given $K\pi$. From these tables it is seen that in the secular equations the matrix elements corresponding to the states particle-hole with large $\epsilon(\rho) + \epsilon(\rho')$ are very important, so e.g., the term corresponding to states 651↓ - 761↓ in Table 5, while in the wave functions such states are less important. The two-quasi-particle states corresponding to nearest poles give much larger contribution

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x/ $Nn_z\Lambda\uparrow$ denotes the state $K\pi[Nn_z\Lambda]$ of the Nilsson potential with $K = \Sigma + \Lambda$ and $Nn_z\Lambda\downarrow$ is the state $K\pi[Nn_z\Lambda]$ with $K = \Lambda - \Sigma$.

to the wave functions as compared with the secular equations.

From Table 5 it is seen that the first excited state with $K\pi$ = 0- in U^{234} is a collective one since three two quasi-particle states give to the wave function a contribution more than 10% each two states - more than 5% and four states - more than 3% each and so on. The energy of this state is by 0.8 MeV lower than that of the first pole. The properties of the first state with $K\pi$ = 0- in U^{234} is not an exception. The overwhelming majority of the first states with $K\pi$ = 0- are collective, the exception appears to be the isotopes of Fm . Especially clearly the collective properties are displayed about the isotopes of Th , U , Pu , what is seen from Table 9. From Figs. 2 and 3 it is seen that the energies of most first states with $K\pi$ = 0- are lowered by more than 0.5 MeV as compared to the first poles. In the wave functions of the second octupole states with given $K\pi$ the summary contribution from the first and second poles amounts usually to more than 90%, and in some case it reaches (95-98%). This particularity of the wave functions structure corresponding to the values of the second roots is well manifested by the example for the state with $K\pi$ = 0- in Table 5 and with $K\pi$ = 2- in Table 7.

In most cases the lowest states with $K\pi$ = 1- are rather close in their properties to the two-quasi-particle states. For example, in U^{234} the contribution of the neutron state 633↓ - 743↑ , as is seen from Table 6, is 83,8%, the contribution of the neutron state 622↑ - 743↑ in Pu^{240} is 87% and so on. However, in Th^{230} the lowest state with $K\pi$ = 1- is collective what is seen from Table 10. The energy of this state is lowered by 0,23 MeV as compared to the first pole. Table 11 gives the contributions of various two-quasi-particle states to the lowest states of Er^{166} with $K\pi$ = 0- and $K\pi$ = 1- While the state with $K\pi$ = 0- is collective, the state with $K\pi$ = 1- is fairly close to the two-quasi-particle one, since the contribution of the neutron state 633↑ - 523↓ is 97.6%. This roughly proves the correctness of the interpretation of state with $K\pi$ = 1- in Er^{166} given in[11].

We notice that in some cases the second states with a given $K\pi$ possess the collective properties while the first states have the structure close to that of the two-quasi-particle states. An example, may be the states of U^{234} with $K\pi$ = 1- what it seen from Table 6.

The lowest states with $K\pi$ = 2-, on the average, appear to be more collectivized as compared to the first states with $K\pi$ = 1- . However, both states with $K\pi = 1-$ and 2- are considerably less collectivized as compared with the states with $K\pi = 0-$ what is clearly seen from Fig. 1.

From Table 7 it is seen that the first state of U^{234}, with $K\pi = 2-$ contains the contribution from the two-quasi-particle states equal to 77.7%, 15%, 1.4%, 1.1% and so on, i.e. the collective properties of this state are clearly seen. It should be noted that the region of the isotopes of Th, and U is the most favourable for the existence of low-lying collective octupole states. This is related to states with $K\pi = 0-$ as well and to a somewhat smaller degree to states with $K\pi = 1-, 2-$. Table 12 gives the structure of the most low-lying states with $K\pi = 2-$ in Dy^{160}, Yb^{174}, W^{182}. In these nuclei the contribution from the most important two-quasi-particle states is 87.6%, 90.8% and 94.8% respectively. The interpretation of states with $K\pi = 2-$ in Dy^{160}, Yb^{174}, and W^{182} given in /11/ as two-quasi-particle led to a certain overestimation of their energies in Dy^{160} and Yb^{174} whereas in W^{182} the effect of the octupole-octupole interaction on the energy of state with $K\pi = 2-$ is rather small.

All the states with $K\pi = 3-$ in their structure are close to the two-quasi-particle ones. This is seen from Tables 8, 13 and Figs. 8 and 9. We notice that the energies of the first and second states with $K\pi = 3-$ are the same as those corresponding to the first and second poles. The fact that the octupole-octupole interactions for states with $K\pi = 3-$ are of little importance is a consequence of the increase of the number of terms in (4) as well as, in the main, the increase of the number of terms with large matrix elements $f(\rho\rho')$ and $\bar{f}(\rho\rho')$ what was mentioned earlier. The nearness of the structure of states with $K\pi = 3-$ to the two-quasi-particle one is demonstrated in Fig. 1. From Fig. 1 it is seen that a comparatively small increase of $\kappa^{(3)}$ leads to significant increase of the collectivization of states with $K\pi = 1-$, 2- while for states with $K\pi = 3-$ the increase of must be larger. We notice that in case $K\pi = 3-$ the interval of values of $\kappa^{(3)}$ where these states are collective and the first roots of (4) exist is extremely small. We note that the apparent collectivization of the second state with $K\pi = 3-$ in U^{234}, as is seen from Table 4, is due to the energies of the second and third poles being practically equal to each other. From the investigations made it follows that the interpretation of both states with $K\pi = 3-$ in Er^{168} given in /11/ is true since the admixtures of other states, as is seen from Table 13, do not exceed 0.5%.

Thus, the lowest states with $K\pi = 0-$ in most nuclei possess the clear-cut collective properties. The lowest states with $K\pi = 1-, 2-$ in some nuclei

are collective, however, in most cases these states are rather close in their properties to the two-quasi-particle states. So, for them the admixture of the remaining states to the two-quasi-particle one corresponding to the first pole is (2-20)%. The states with $K\pi = 3-$ are practically two-quasi-particle ones since the admixture of other states do not usually exceed 1%.

As was already mentioned, the structure of the octupole states with a given $K\pi$ but $K\pi = 3-$, is different, i.e. some states are collective and others are two-quasi-particle. The average nuclear field defines if the structure of the states will be collective or two-quasi-particle one.

From the investigation made it follows that if the contribution of single two-quasi-particle state to the octupole state exceeds 95% then the energy of such a state should be calculated on the basis of the superfluid nuclear model taking into account the blocking effect, but disregarding the octupole-octupole interaction. In this case the blocking effect is more important as compared with the octupole-octupole interaction.

We have calculated the reduced probabilities of electromagnetic transitions, which will be analysed in other paper. However, it should be noted that the obtained values of the reduced probabilities do not contradict the available experimental data. So, the ratio $B(E3)/B(E3)_{s.p.}$ with $e_{eff} = 0,5$ is 2 - 4 for the collective states and is significantly smaller than the unity for states close to the two-quasi-particle ones.

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TABLE 1

Single-particle levels of the average field, proton system

Z	I	π	[$Nn_z\Lambda$]	E (v)	C_p	λ_p
	5/2	+	422	0,674		
	I/2	+	43I	0,8I6		
	I/2	-	550	0,978		
	7/2	+	4I3	0,987		
	I/2	+	420	I,06		
	9/2	+	404	I,08		
	3/2	-	54I	I,I0		
59	3/2	+	422	I,20	0,I24	I,252
6I	5/2	-	532	I,3I	0,I27	I,325
63	5/2	+	4I3	I,36	0,I29	I,39I
65	3/2	+	4II	I,42	0,I27	I,458
67	7/2	-	523	I,48	0,I23	I,528
69	I/2	+	4II	I,56	0,I2I	I,60I
7I	9/2	-	5I4	I,66	0,I23	I,67I
73	7/2	+	404	I,69	0,I2I	I,737
75	5/2	+	402	I,76	0,II8	I,808
	3/2	+	402	I,86		
	I/2	+	400	I,90		
	I/2	-	54I	I,97		
	II/2	-	505	2,04		
	3/2	-	532	2,I8		
	5/2	-	523	2,46		
	7/2	-	5I4	2,70		

TABLE 2

Single-particle levels of the average field, neutron system

N	$I\ \pi[Nn_z\Lambda]$	E(s)	C_n	λ_n
	I/2 - 54I	0,405		
	7/2 + 404	0,580		
	I/2 + 400	0,597		
	3/2 + 402	0,655		
	I/2 - 530	0,825		
	II/2 - 505	0,850		
	3/2 - 532	0,9I0		
89	I/2 + 660	0,950	0,I37	0,968
9I	3/2 + 65I	I,00	0,I36	I,0I8
93	3/2 - 52I	I,04	0,I3I	I,068
95	5/2 + 642	I,08	0,I20	I,I23
97	5/2 - 523	I,II	0,I04	I,I95
99	7/2 + 633	I,26	0,I06	I,273
I0I	I/2 - 52I	I,30	0,I04	I,34I
I03	5/2 - 5I2	I,36	0,99	I,4I9
I05	7/2 - 5I4	I,48	0,III	I,497
I07	9/2 + 624	I,55	0,I24	I,559
I09	I/2 - 5I0	I,62	0,I35	I,6I3
III	3/2 - 5I2	I,66	0,I42	I,660
II3	7/2 - 503	I,7I	0,I46	I,703
	9/2 - 505	I,74		
	3/2 - 50I	I,75		
	I/2 + 65I	I,78		
	I/2 + 640	I,79		
	I/2 - 770	I,83		
	II/2 + 6I5	I,97		
	5/2 - 503	2,I5		

TABLE 3

Single-particle levels of the average field, proton system

Z	$I\ \pi[Nn_z\Lambda]$	E(v)	C_p	λ_p
	7/2 - 523	0,230		
	9/2 - 5I4	0,475		
	3/2 + 402	0,490		
	I/2 - 54I	0,500		
	I/2 + 660	0,550		
	II/2 - 505	0,600		
	I/2 + 400	0,620		
	3/2 - 532	0,650		
89	3/2 + 65I	0,680	0,I4I	0,753
9I	I/2 - 530	0,750	0,I30	0,803
93	5/2 + 642	0,830	0,I20	0,859
95	5/2 - 523	0,855	0,II0	0,92I
97	3/2 - 52I	0,985	0,I09	0,987
99	7/2 + 633	0,990	0,I04	I,045
	7/2 - 5I4	I,07		
	I/2 - 52I	I,I6		
	9/2 + 624	I,I7		
	5/2 - 5I2	I,22		
	I/2 - 5I0	I,33		
	9/2 - 505	I,36		
	II/2 + 6I5	I,48		
	3/2 - 5I2	I,50		
	7/2 - 503	I,54		
	I/2 + 65I	I,59		
	I3/2 + 606	I,6I		

TABLE 4

Single-particle levels of the average field, neutron system

N	I π [$Nn_z\Lambda$]	E(s)	C_n	λ_n
	9/2 + 624	0,270		
	I/2 + 65I	0,4I0		
	3/2 - 50I	0,435		
	5/2 - 503	0,450		
	II/2 + 6I5	0,475		
	I/2 - 770	0,520		
	I/2 + 640	0,550		
	I/2 - 50I	0,560		
	3/2 + 642	0,590		
	I3/2 + 606	0,625		
	3/2 - 76I	0,660		
I37	3/2 + 63I	0,7I5	0,II9	0,734
I39	5/2 - 752	0,725	0,II2	0,778
I4I	5/2 + 633	0,780	0,I04	0,826
I43	7/2 - 743	0,850	0,099	0,880
I45	I/2 + 63I	0,900	0,097	0,936
I47	5/2 + 622	0,970	0,099	0,994
I49	7/2 + 624	I,03	0,I07	I,048
I5I	9/2 - 734	I,I0	0,II7	I,094
I53	I/2 + 620	I,I7	0,I26	I,I33
	7/2 + 6I3	I,I9		
	3/2 + 622	I,20		
	II/2 - 725	I,22		
	9/2 + 6I5	I,23		
	I/2 - 76I	I,33		
	I/2 - 750	I,35		
	9/2 + 604	I,4I		
	3/2 - 752	I,47		

TABLE 5

Structure of the states U^{234} with $K\pi$ = 0-

ω_1 = 0,9 MeV, ω_2 = 1,8 MeV

Neutron states	$f(qq')$	$\varepsilon(q)+\varepsilon(q')$	$X_1(qq')$	$y_1(qq')$	$X_2(qq')$	$y_2(qq')$
65I↓- 76I↓	7,2	6,4	I2,8	3,7	I3,6	0,08
642↓- 76I↑	3,2	3,0	I,2	I,6	I,7	0,06
642↓- 752↓	4,0	6,I	4,0	I,2	4,2	0,03
63I↑- 76I↑	-2,5	2,3	I,7	4,2	3,6	0,3
633↓- 752↑	2,3	I,7	4,5	22,9	-48,2	49,4
622↑- 752↑	-2,I	2,2	3,7	I0,9	9,9	I,4
624↓- 743↑	I,6	2,3	I,3	3,4	2,9	0,3
6I5↑- 725↑	-4,I	5,2	5,I	2,2	5,7	0,05
606↑- 7I6↑	-4,6	5,7	5,9	2,2	6,4	0,05
640↑- 750↑	-5,7	5,6	9,I	3,5	9,9	0,08
Proton states						
660↑- 530↑	-4,I	2,9	5,5	8,6	8,2	0,4
400↑- 630↑	-I,5	2,5	I,0	2,3	I,9	0,I
400↑- 5I0↑	3,I	5,2	2,9	I,3	3,2	0,03
65I↑- 532↓	-I,5	2,6	0,7	I,4	I,2	0,08
65I↑- 52I↑	-3,I	2,7	6,0	I0,6	9,6	0,5
642↑- 523↓	-I,3	I,8	I,6	6,9	29,7	46,7
624↑- 5I4↑	-3,0	5,0	2,9	I,4	3,3	0,03

TABLE 6

Structure of the states of U^{234} with $K\pi = 1-$

$\omega_1 = 1{,}4$ MeV , $\omega_2 = 1{,}7$ MeV

Neutron states	$f(qq')$	$\varepsilon(q)+\varepsilon(q')$	$X_1(qq')$	$y_1(qq')$	$X_2(qq')$	$y_2(qq')$
63I↓– 76I↑	0,9	2,2	0,8	0,3	I,4	I,I
642↓– 752↑	-2,2	2,7	I,2	0,2	I,5	0,5
63I↑– 752↑	2,6	2,0	4,9	2,4	I0,5	I4,9
633↓– 743↑	-I,8	I,5	24,0	83,8	-8,7	I4,9
622↑– 743↑	2,5	I,9	7,6	4,5	20,4	43,4
6I5↑– 734↑	-2,7	4,4	2,8	0,2	3,0	0,2
633↓– 76I↑	I,2	2,I	I,I	0,5	2,0	2,2

Proton states						
400↑– 52I↑	4,7	3,0	2,2	0,3	2,6	0,6
402↓– 523↓	-I,6	3,2	I,2	0,I	I,4	0,3
633↑– 523↓	0,8	2,5	0,3	0,08	0,4	0,2
642↑– 52I↑	3,0	2,4	5,9	I,6	8,2	4,2
65I↑– 530↑	3,I	2,2	8,2	3,I	I3,8	I2,0
660↑+ 530↑	-2,3	2,8	I,9	0,3	2,3	0,0I
400↑+ 52I↓	I,8	4,I	I,3	0,09	I,4	0,I

TABLE 7

Structure of the states of U^{234} with $K\pi = 2-$

$\omega_1 = 1{,}55$ MeV , $\omega_2 = 1{,}80$ MeV

Neutron states	$f(qq')$	$\varepsilon(q)+\varepsilon(q')$	$X_1(qq')$	$y_1(qq')$	$X_2(qq')$	$y_2(qq')$
63I↓– 752↑	-0,5	I,8	0,7	0,9	8,5	6,2
642↓– 743↑	I,9	2,4	3,0	I,I	3,9	0,08
63I↑– 743↑	-3,0	I,75	37,I	77,7	-87,I	I8,2
6I5↓– 503↓	-2,8	5,4	2,5	0,I	2,6	0,005
624↑– 7I6↑	3,4	8,0	2,3	0,05	2,4	0,002
6I3↑– 50I↑	-3,3	5,2	3,6	0,2	3,7	0,008
6II↑+ 500↑	-3,5	6,2	3,I	0,I	3,2	0,005

Proton states						
400↑– 5I2↑	2,8	4,4	3,0	0,2	3,2	0,0I
402↓– 5I4↓	2,5	4,3	2,5	0,2	2,7	0,0I
633↑– 52I↑	-3,2	3,0	2,4	0,5	2,8	0,03
402↑– 5I0↑	2,4	6,3	I,5	0,05	I,5	0,002
642↑– 530↑	2,0	I,8	II,3	I5,0	I22,5	75,2
400↑+ 5I2↓	-5,6	6,2	I,9	0,07	2,0	0,003
65I↑+ 530↑	I,5	2,I	2,3	I,4	3,9	0,2

TABLE 8

Structure of the states of U^{234} with $K\pi = 3^-$

$\omega_1 = 1,6$ MeV , $\omega_2 = 1,8$ MeV

Neutron states	$f(qq')$	$\varepsilon(q)+\varepsilon(q')$	$X_1(qq')$	$Y_1(qq')$	$X_2(qq')$	$Y_2(qq')$
640↑– 753↑	I,55	2,7	3,8	0,0I	2,0	0,02
63I↓– 743↑	0,I	I,6	I9,2	99,8	-0,02	0,00I
642↓– 734↑	-I,3	3,7	2,I	0,002	I,0	0,004
63I↑– 734↑	2,5	3,0	9,8	0,02	4,9	0,04
622↑– 725↑	3,5	4,0	2,4	0,002	I,I	0,004
604↑– 50I↑	5,4	6,7	I6,7	0,005	7,4	0,008
63I↓+ 752↑	-0,4	I,8	0,8	0,0I	I5,4	34,6
Proton states						
400↑– 503↑	4,2	6,5	9,7	0,03	4,3	0,005
402↓– 505↓	3,8	6,I	9,2	0,003	4,I	0,005
633↑– 530↑	I,8	2,5	7,4	0,03	4,2	0,07
400↑+ 523↓	0,5	2,4	0,5	0,002	0,3	0,005
642↑+ 530↓	0,8	I,8	4,4	0,06	48,9	65,2

TABLE 9

Contribution of two-quasi-particle states to the collective states ω_1

with $K\pi = 0^-$ (in percent)

Neutron system	Th^{230}	U^{232}	Pu^{240}
65I↓– 76I↓	3,4	3,4	4,0
642↓– 76I↑	4,0	4,I	0,2
642↓– 752↓	I,I	I,I	I,4
63I↑– 76I↑	9,7	I0,7	0,4
633↓– 752↑	27,4	30,3	I,4
622↑– 752↑	6,0	6,I	9,3
6I5↑– 725↑	2,0	2,0	2,4
606↑– 7I6↑	I,9	I,8	2,4
624↓– 743↑	I,0	I,0	I7,6
640↑– 750↑	3,I	3,0	3,8
Proton system			
660↑– 530↑	I3,6	8,I	3,3
400↑– 630↑	3,3	I,8	0,8
400↑– 5I0↑	I,0	I,2	I,5
65I↑–532↓	2,6	I,2	0,5
65I↑– 52I↑	6,6	8,8	I3,3
642↑– 523↓	I,8	4,9	I6,2
624↑– 5I4↑	I,2	I,3	I,5

TABLE 10

Contribution of two-quasi-particle states to the collective states ω_1 with $K\pi = 1-$ (in percent)

Neutron states	Th^{230}	Pu^{240}
63I↓ - 76I↑	0,6	0,06
642↓ - 752↑	I,8	0,02
63I↑ - 752↑	39,0	0,I
633↓ - 743↑	30,4	0,8
622↑ - 743↑	I,7	87,0
6I5↑ - 734↑	0,3	0,2
633↓ - 76I↑	3,8	0,02
Proton states		
400↑ - 52I↑	0,6	0,4
402↓ - 523↓	0,4	0,I
633↑ - 523↓	$5{,}I0^{-2}$	0,4
642↑ - 52I↑	I,0	6,9
65I↑ - 530↑	I3,6	0,9
660↑ + 530↑	I,4	0,I
400↑ + 52I↓	0,2	0,I

TABLE 11

Contribution of two-quasi-particle states to the collective states of Er^{166} with $K\pi = 0-$ and with $K\pi = 1-$ (in percent).

Neutron system	$K\pi = 0-$	Neutron system	$K\pi = 1-$
400↑ - 5I0↑	I,4	640↑ - 52I↑	0,03
660↑ - 770↑	I3,7	65I↑ - 522↑	0,05
65I↓ - 54I↓	2,7	624↑ - 505↑	0,02
640↑ - 530↑	I,4	633↑ - 523↓	97,6
402↓ - 5I2↓	I,5	633↑ - 5I2↑	0,9
65I↑ - 52I↑	4,0	642↑ - 52I↑	0,6
642↑ - 523↓	34,5	65I↑ - 52I↓	0,04
642↑ - 5I2↑	I5,9	400↑ + 52I↓	0,02
404↓ - 5I4↓	I,9	660↑ + 52I↓	0,03
6I5↑ - 505↑	I,5	640↑ + 54I↓	0,0I
Proton system		**Proton system**	
43I↓ - 54I↓	I,9	4II↑ - 532↑	0,I
4II↓ - 54I↓	I,4	4I2↑ - 523↑	0,I
4II↑ - 54I↑	0,5	402↑ - 523↑	0,I
402↑ - 532↑	0,8	404↑ - 505↑	0,0I
404↓ - 523↑	2,7	4I3↓ - 532↓	0,02
404↑ - 5I4↑	6,2	402↓ - 550↑	0,02

TABLE 12

Contribution of two-quasi-particle states to the collective states ω_1 with $K\pi = 2-$ (in percent)

Neutron system	Dy^{160}	Yb^{174}	W^{182}
400↑- 5I2↑	0,3	0,04	4.10^{-3}
402↓- 5I4↓	0,2	0,I	0,0I
624↑- 5I2↑	0,04	90,8	4,3
633↑- 52I↑	6,3	0,2	0,0I
642↑- 530↑	2,5	0,06	10^{-3}
660↑+ 52I↑	0,2	2.10^{-4}	3.10^{-5}
Proton system			
420↑- 532↑	0,I	0,0I	2.10^{-3}
4I0↑- 523↓	0,02	0,08	0,07
422↓- 523↑	0,I	8.10^{-3}	5.10^{-4}
4II↑- 523↑	87,6	I,9	0,05
4I3↓- 5I4↑	0,I	0,2	8.10^{-3}
402↑- 5I4↑	0,I	5,3	94,8

TABLE 13

Contribution of two-quasi-particle states to the first and second states of Er^{168} with $K\pi = 3-$ (in percent)

Neutron system	First state	Second state
400↑- 503↑	3.10^{-3}	$4,6.10^{-3}$
402↓- 505↓	3.10^{-3}	$4,2.10^{-3}$
6I5↑- 5I2↑	$I,5.10^{-3}$	$2,3.10^{-3}$
624↑- 52I↑	$I,5.10^{-2}$	$2,7.10^{-2}$
633↑- 52I↓	99,82	$2,0.10^{-2}$
642↑+ 52I↓	$2,5.10^{-3}$	$8,6.10^{-3}$
402↑+ 50I↓	$2,3.10^{-3}$	$3,7.10^{-3}$
Proton system		
420↑- 523↑	$2,3.10^{-3}$	$3,9.10^{-3}$
4II↓- 523↑	$I,I.10^{-3}$	99,53
4II↑- 5I4↑	$0,I3^{3}$	0,38
402↑- 505↑	$I,0.10^{-3}$	$I,6.10^{-3}$

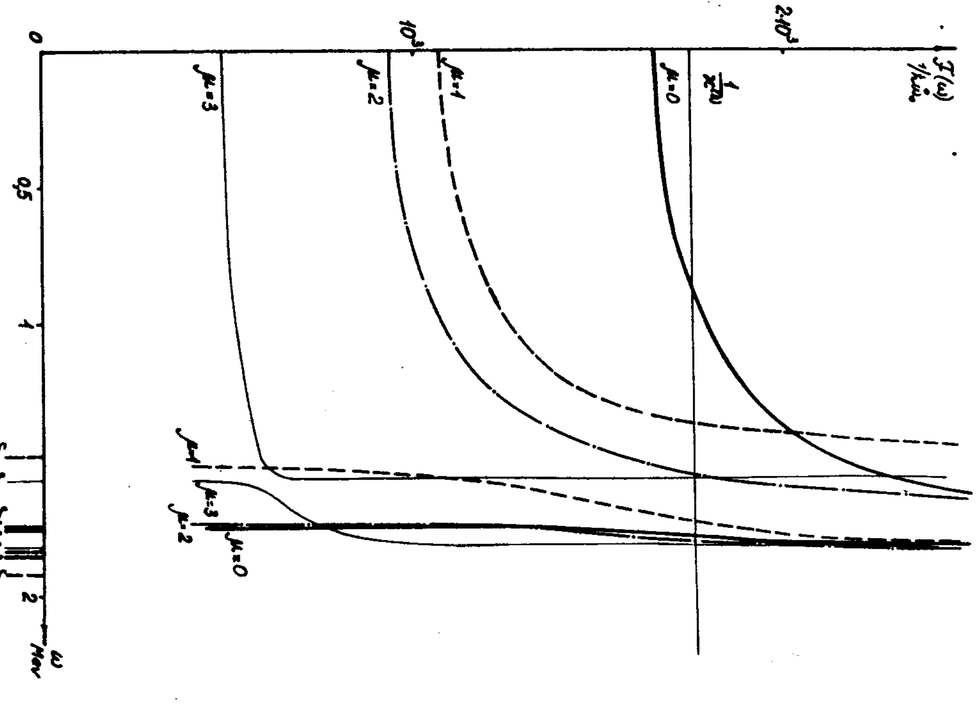


Fig. 1. The behaviour of functions $F(\omega)$ of octupole states of U^{234} with μ = 0,1,2,3.

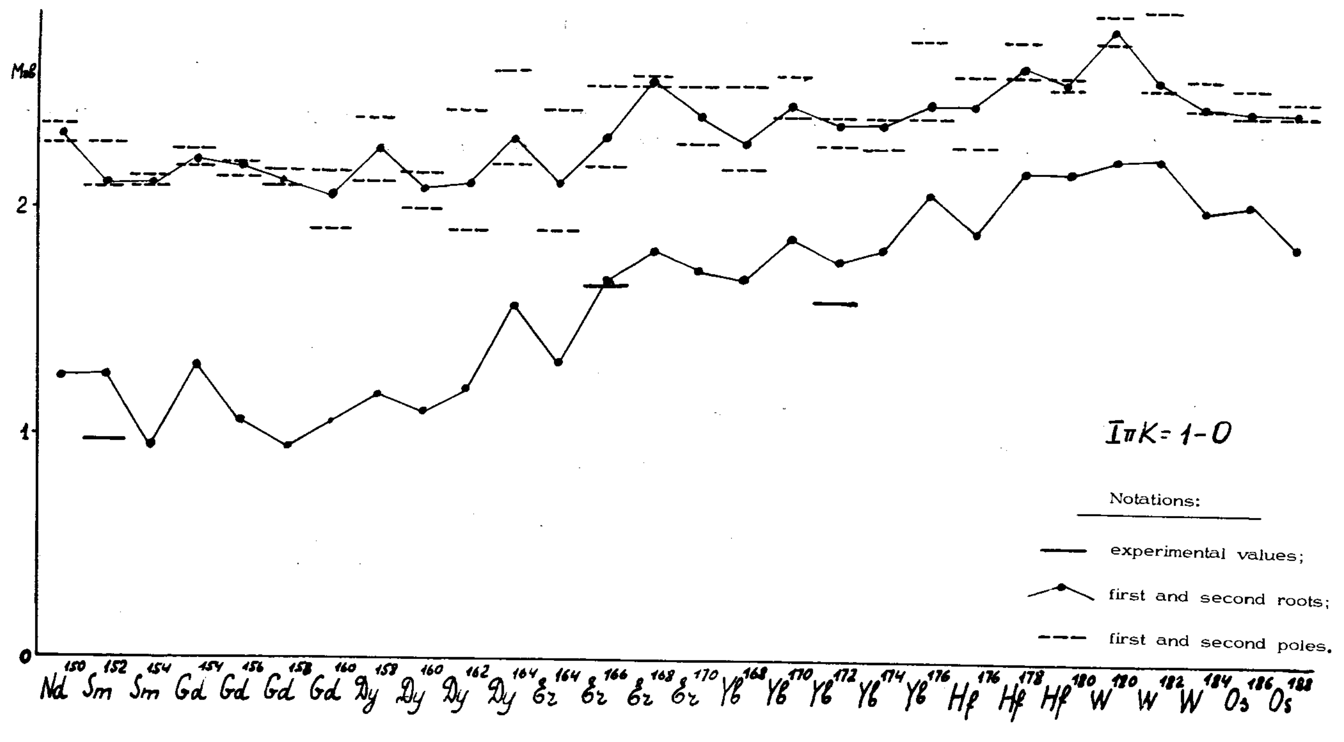
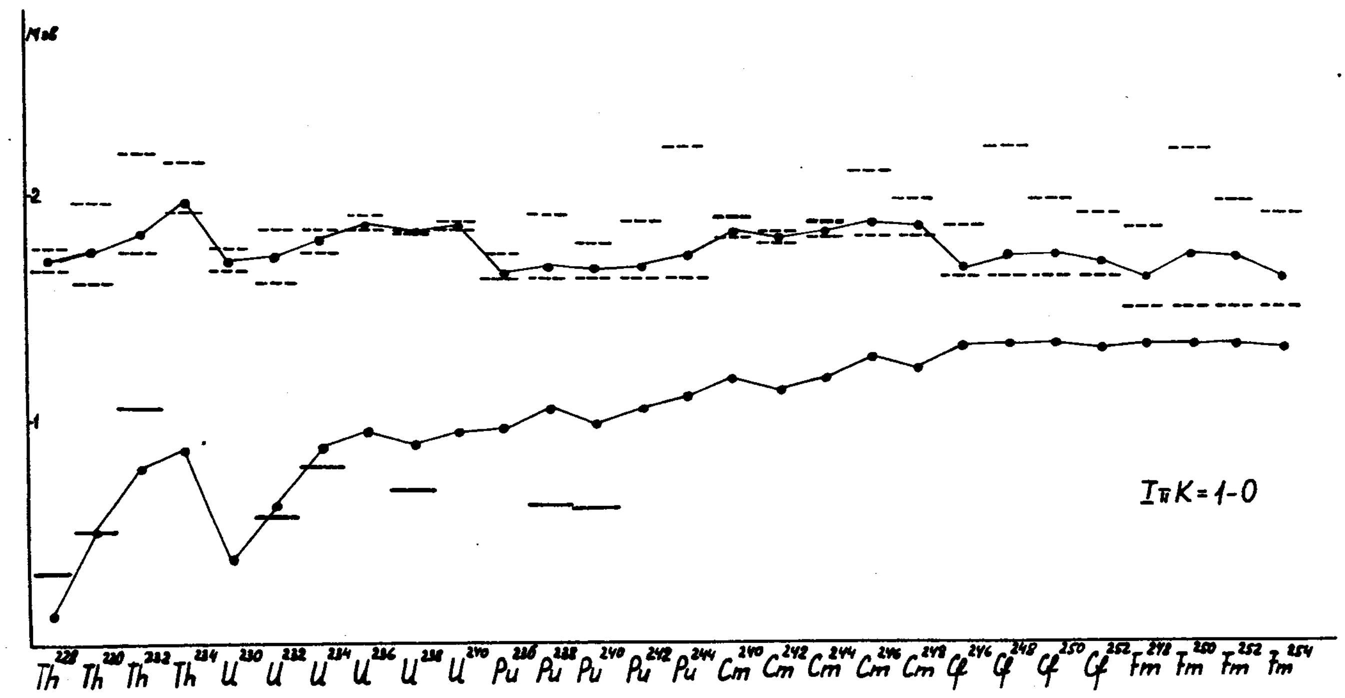
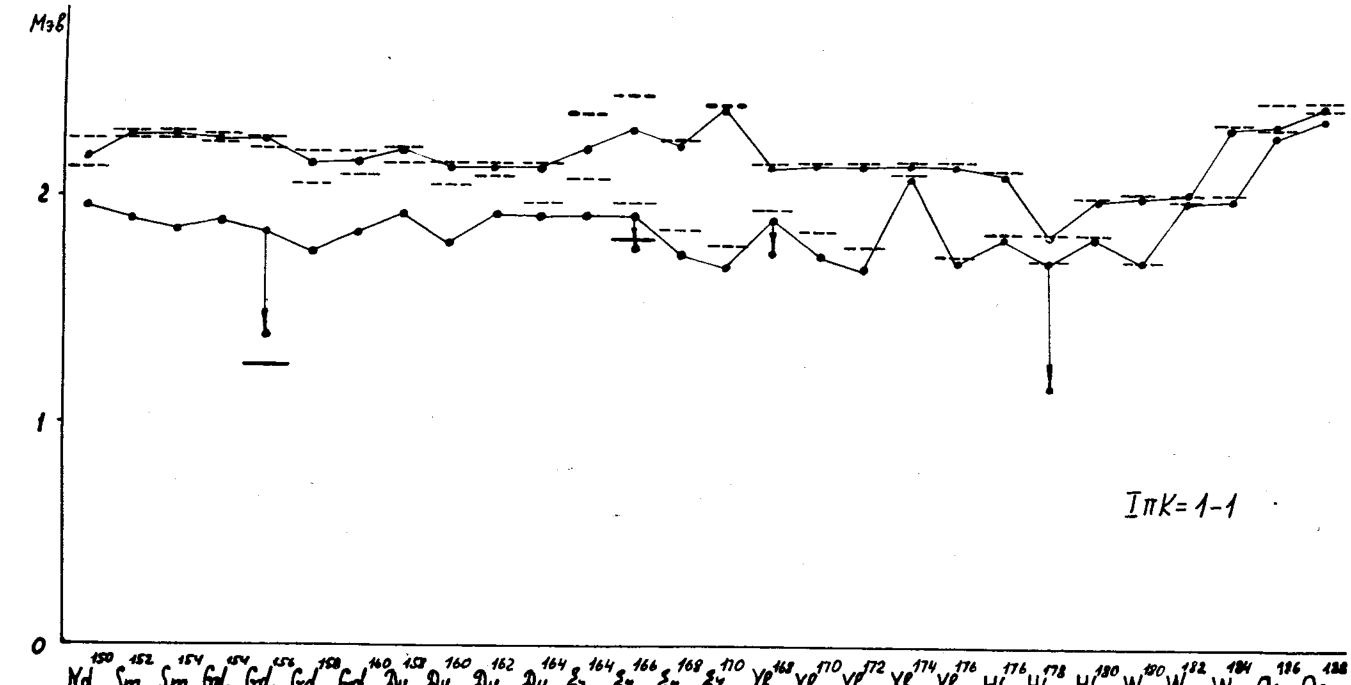


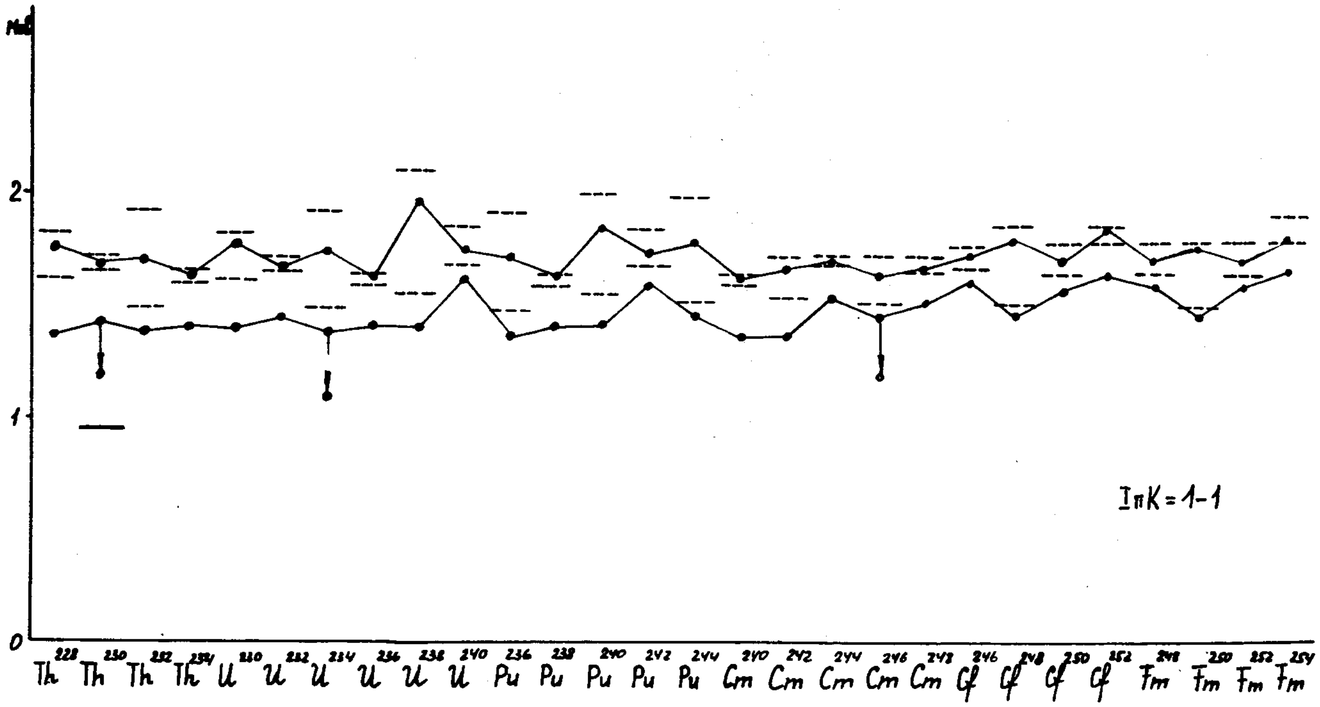
Fig. 2. Energies of states with $K\pi = 0-$



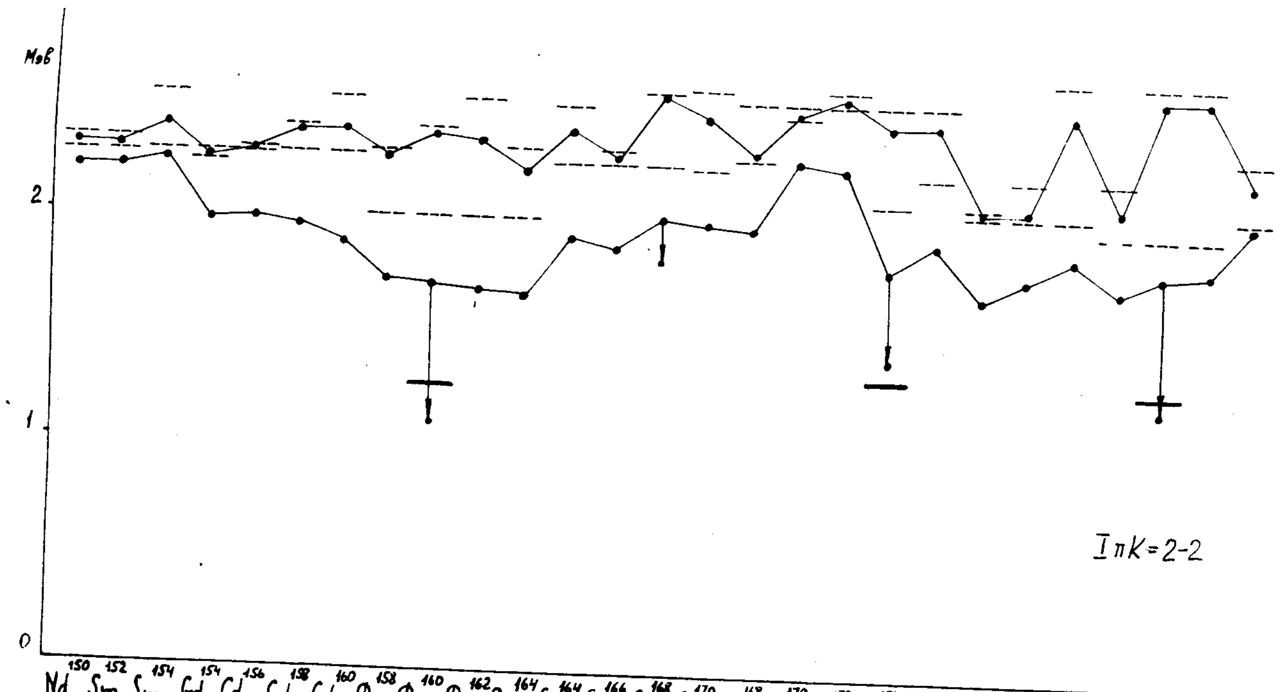
F i g. 3. Energies of states with $K\pi = 0-$. (For notations see Fig. 2).



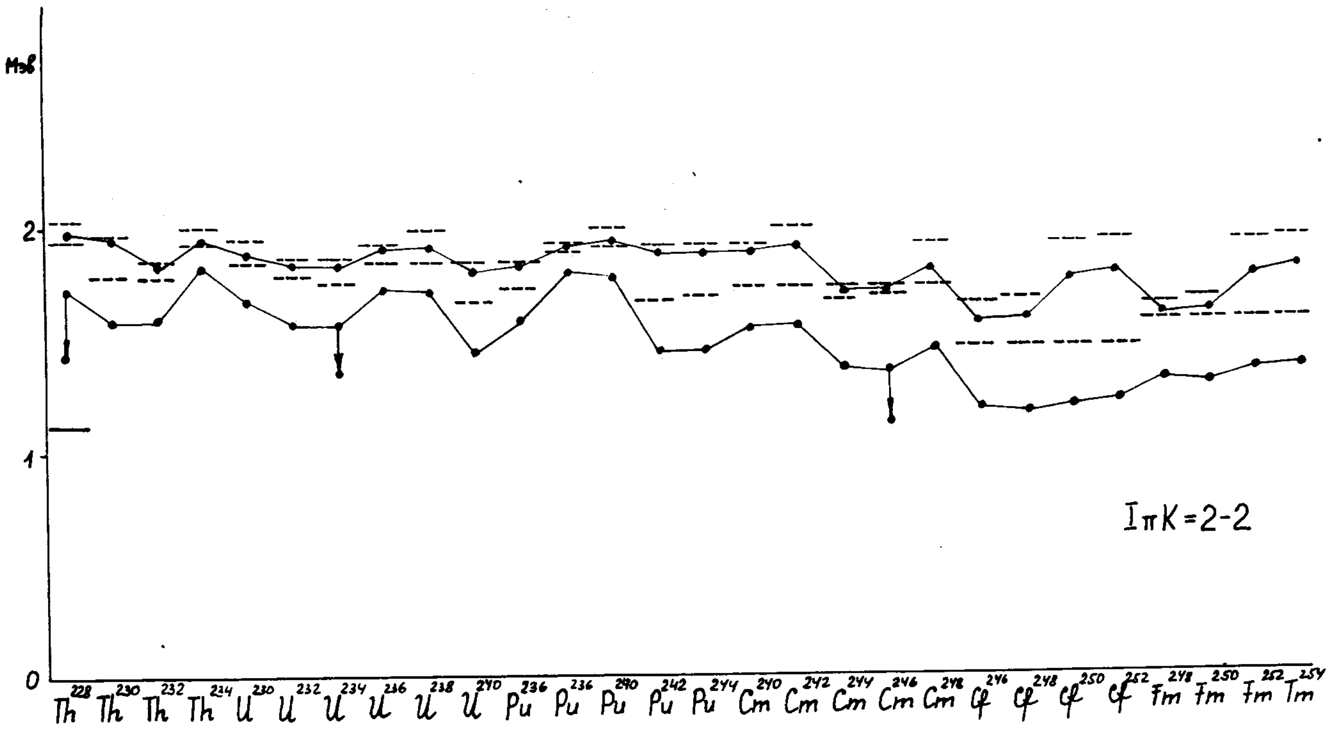
F i g. 4. Energies of states with $K\pi = 1-$. (For notations see Fig. 2).



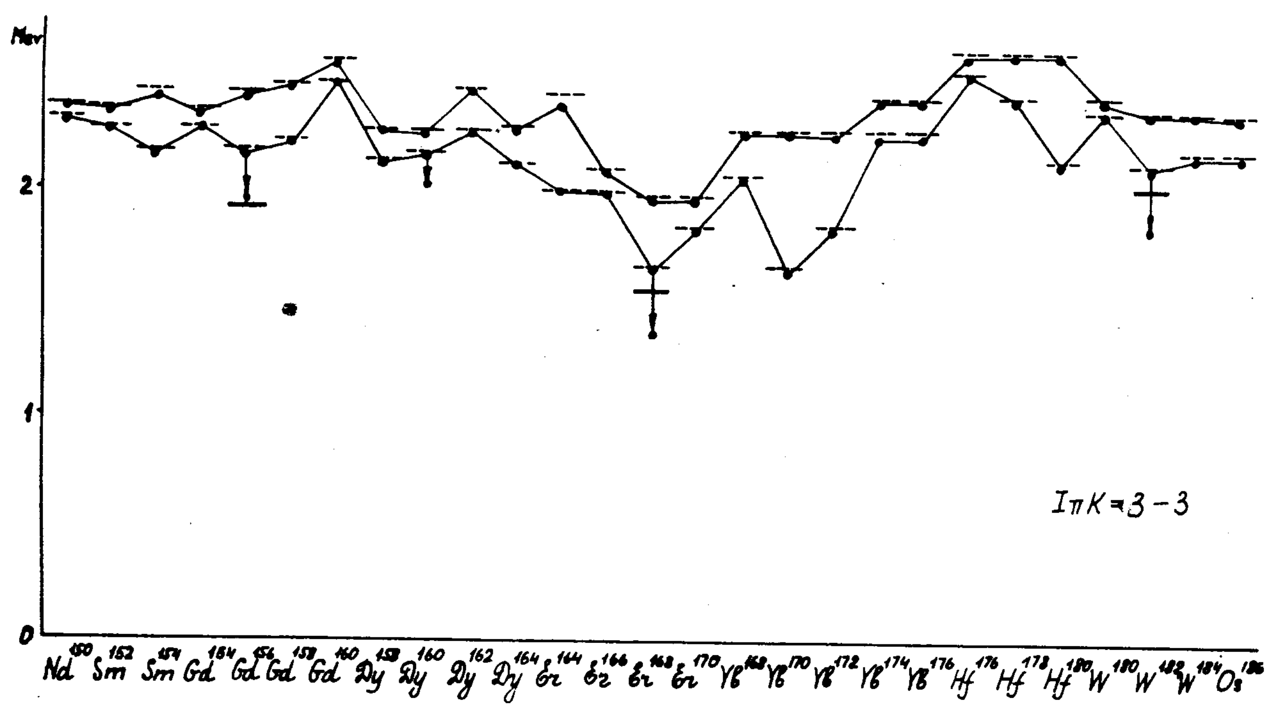
F i g. 5. Energies of states with $K\pi = 1-$. (For notations see Fig. 2).



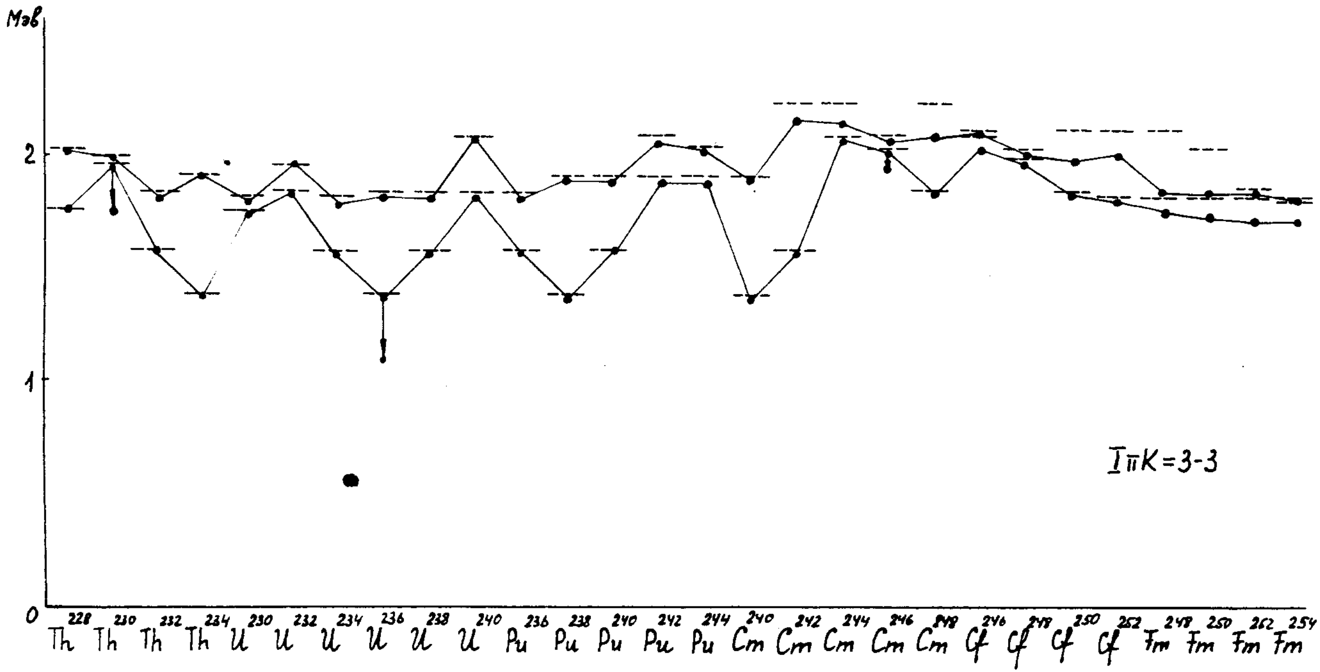
F i g. 6. Energies of states with $K\pi = 2-$. (For notations see Fig. 2) .



F i g. 7. Energies of states with $K\pi = 2-$. (For notations see Fig. 2).



F i g. 8. Energies of states with $K\pi = 3-$. (For notations see Fig. 2).



F i g. 9. Energies of states with $K\pi = 3-$. (For notations see Fig. 2).