ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

LABORATORY OF THEORETICAL PHYSICS

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E - 1550

HIGHER SYMMETRIES OF STRONG INTERACTIONS AND ASYMPTOTIC RELATIONS BETWEEN MESON-BARYON SCATTERING CROSS SECTIONS

Phys. Lett, 1964, v 10, n 1, p 130-131.
Nuovo Cim., 1964, v 33, n 5, p 1312-28

Дубна 1964

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Submitted to Physics Letters

Дубна 1964

1. Introduction

After the works on the classification schemes of elementary particles by Gell-Mann and Nishijima, where the isospin invariance and hypercharge conservation hold, there were some attempts in building up the schemes of strong interaction with higher symmetries: the "global symmetry" of Gell-Mann[1] and Schwinger[2], the dublet symmetry of Pais[3], the cosmic symmetry of Sakurai[4], the universal vector theory of strong interactions of Sakurai[5] and Kobzarev and Okun[6], the unitary symmetry (the $SU(3)$ group) in the triplet model of Sakata[7-9] and in the octet model of Gell-Mann[10] and Ne'eman[11], the symmetry group G_2 in seven-dimensional charge space of Behrends and Sirlin[12], the theory with degeneration in states whose hypercharge equals zero of Baldin and Komar[13] and other symmetries. The possible experimental tests of the unitary symmetry and the symmetry group G_2 have already been discussed in a series of papers. In particular, some relations between cross sections of the meson-baryon and the baryon-baryon scattering processes in these two models have been obtained in[12,14-18].

In the symmetry schemes of strong interactions under discussion all the mesons and their antiparticles belong to the same multiplet. If the total cross sections of meson-baryon and antimeson-baryon interactions tend to constant limits at $s \to \infty$, then according to the Pomeranchuk theorem these limits equal each other. This circumstance reduces the number of independent scattering amplitudes in the models with higher symmetries[20,21].

As it was shown in[22,23] the differential cross sections of crossing processes equals each other asymptotically at high energies and fixed momentum transfer, and there are some asymptotic relations between the amplitudes of these crossing reactions. Since in the models with higher symmetries mesons and antimesons belong to the same multiplet and as there are crossing processes among the meson-baryon scattering processes, then the asymptotic relations between amplitudes of crossing processes and also the symmetry properties of strong interaction lead to subsidiary asymptotic equalities between cross sections of processes under considerations. For instance, it was shown in[23] that from the asymptotic relations between scattering amplitudes of π^+- and π^--mesons on proton and from isospin invariance follows the asymptotic equality between the total interaction cross sections of charged and neutral π -mesons with proton.

In the present paper we shall deduce some asymptotic relations between cross sections of meson-baryon scattering processes in the triplet and octet models of unitary symmetry and in the model with symmetry group G_2 with the help of the methods given in /23/. Comparison of the relations obtained in this paper with experimental data might throw light on the symmetry properties at high energies[1/].

2. Kinematical Considerations

We consider the following meson-baryon scattering processes:

$$\pi^+ + p \to \pi^+ + p \quad (1a), \qquad \pi^- + p \to \pi^- + p \quad (1b),$$
$$\pi^0 + p \to \pi^+ + n \quad (2a), \qquad \pi^- + p \to \pi^0 + n \quad (2b),$$
$$K^+ + p \to K^+ + p \quad (3a), \qquad K^- + p \to K^- + p \quad (3b),$$
$$K^0 + p \to K^0 + p \quad (4a), \qquad \bar{K}^0 + p \to \bar{K}^0 + p \quad (4b),$$
$$K^0 + p \to K^+ + n \quad (5a), \qquad K^- + p \to \bar{K}^0 + n \quad (5b),$$
$$\pi^+ + p \to K^+ + \Sigma^+ \quad (6a), \qquad K^- + p \to \pi^- + \Sigma^+ \quad (6b),$$
$$\pi^- + p \to K^+ + \Sigma^- \quad (7a), \qquad K^- + p \to \pi^+ + \Sigma^- \quad (7b),$$
$$\pi^- + p \to K^0 + \Sigma^0 \quad (8a), \qquad \bar{K}^0 + p \to \pi^+ + \Sigma^0 \quad (8b),$$
$$\pi^- + p \to K^0 + \Lambda \quad (9a), \qquad \bar{K}^0 + p \to \pi^+ + \Lambda \quad (9b),$$
$$\pi^0 + p \to K^+ + \Sigma^0 \quad (10a), \qquad K^- + p \to \pi^0 + \Sigma^0 \quad (10b)$$
$$\pi^0 + p \to K^+ + \Lambda \quad (11a), \qquad K^- + p \to \pi^0 + \Lambda \quad (11b)$$
$$\pi^0 + p \to K^0 + \Sigma^+ \quad (12a), \qquad \bar{K}^0 + p \to \pi^0 + \Sigma^+ \quad (12b),$$
$$\bar{K}^0 + p \to K^+ + \Sigma^0 \quad (13a), \qquad K^- + p \to K^0 + \Sigma^0 \quad (13b),$$
$$K^- + p \to K^+ + \Sigma^- \quad (14).$$

Under crossing transformation processes (ja) go over to processes (jb), while process (14) goes over to itself.

Note that besides processes (1a,b) - (14), there exist processes obtained from them by the substitutions

$$n \leftrightarrow p, \quad \pi^+ \leftrightarrow \pi^-, \quad K^+ \leftrightarrow K^-, \quad \Sigma^+ \leftrightarrow \Sigma^-, \quad \Xi^0 \leftrightarrow \Xi^-,$$

1/ Note that the higher symmetries break down at low energies.

whose amplitudes coincide with those corresponding to processes (1a,b)-(14) up to a sign. The relations obtained in the following for processes (1a,b)-(14) hold also for the corresponding processes obtained from them by the above substitution. The exceptional cases are processes involving neutral K -mesons in the initial states, for instance, processes (4a) and (4b). Instead of these two processes we shall consider the corresponding mirror processes:

$$K^+ + n \to K^+ + n \quad (4a'), \text{ and } \qquad K^- + n \to K^- + n \quad (4b'),$$

whose amplitudes are equal to amplitudes of (4a) and (4b) respectively, and the physical processes

$$K_2^0 + p \to K_2^0 + p \qquad (4c)$$

and

$$K_2^0 + p \to K_1^0 + p \qquad (4d).$$

We shall denote the amplitudes of process (4a)-(4b) by T_{4a} - T_{4d} respectively. Neglecting the weak interactions, we have

$$T_{4c} = \frac{T_{4a} + T_{4b}}{2}, \quad T_{4d} = \frac{T_{4a} - T_{4b}}{2i}. \qquad (15)$$

Instead of processes (5a), (8b), (9b), (12b) and (13b), the corresponding physical processes

$$K_2^0 + p \to K^+ + n \qquad (5c),$$
$$K_2^0 + p \to \pi^+ + \Sigma^0 \qquad (8c),$$
$$K_2^0 + p \to \pi^+ + \Lambda \qquad (9c),$$
$$K_2^0 + p \to \pi^0 + \Sigma^+ \qquad (12c),$$
$$K_2^0 + p \to K^+ + \Sigma^0 \qquad (13c)$$

are the observable ones in experiment, the cross sections of which equal to those of the corresponding processes with neutral K^0 or $\bar{K}^0$ mesons in the initial state, multiplied by ½ .

In all the models under consideration, all the mesons (π , K and $\bar{K}$) belong to the same multiplet. In the octet model all baryons also belong to the same unitary multiplet. Therefore, the relative parities (πN),($K\Lambda$), ($K\Sigma$) and ($K\Xi$) are the same in this model, and the matrix elements of the processes in question have the form:

$$T_{ja,b} = \bar{u}(p_2)\left[A_{ja,b}(s,t) + i\,\frac{\hat{q}_1 + \hat{q}_2}{2}\,B_{ja,b}(s,t)\right]u(p_1) \qquad (16)$$

where q_1 and p_1 (q_2 and p_2) are the four-momenta of the meson and the baryon in the initial (final) states,

$$s = -(p_1 + q_1)^2, \quad t = -(q_1 - q_2)^2.$$

In the triplet model of Sakata the nucleons and Λ -hyperon belong to the same multiplet. Therefore, in this model the matrix elements of process (ja, b), $j = 1, 2, 3, 4, 5, 9, 11$, have also the form (16). In case of the triplet model we shall consider only these processes. Analogously, in the model with symmetry group G_2, the matrix element of all processes except (9a,b) and (11 a,b) have also the form (16). As for the last two processes with Λ -hyperon (which is not included in the multiplet of other baryons) in the final state, the matrix elements have also the form (16) if the relative parities (πN) and $(K\Lambda)$ are the same. In case of different relative parities, we have:

$$T_{ja,b} = \bar{u}(p_2)[A_{ja,b}(s,t) + i\frac{\hat{q}_1 + \hat{q}_2}{2} B_{ja,b}(s,t)]\gamma_5 u(p_1). \qquad (17)$$

In the following we shall consider the case of equal relative parities in detail. In the other case all calculations can be made analogously and give the same results.

The differential cross section has the form

$$\frac{d\sigma_{ja,b}(s,t)}{dt} = \frac{1}{64\pi s k_1} F_{ja,b}(s,t) \qquad (18)$$

where k_1 is the modulus of the three-momenta of the initial particles in the CMS, while

$$F_{ja,b}(s,t) = [(M_1 + M_2)^2 - t]|A_{ja,b}|^2 + \frac{1}{4}\{(s-u)^2 - (m_1^2 - m_2^2)^2 -$$

$$- [t - 2(m_1^2 + m_2^2)][t - (M_1 - M_2)^2]\}|B_{ja,b}|^2 +$$

$$+ [(M_1 + M_2)(u-s) + (M_1 - M_2)(m_2^2 - m_1^2)] \operatorname{Re} A_{ja,b} B^*_{ja,b} \qquad (19)$$

$$m_1^2 = -q_1^2, \quad M_1^2 = -p_1^2, \quad s + t + u = m_1^2 + m_2^2 + M_1^2 + M_2^2$$

If the initial baryon is unpolarized, then the polarization vector of the final baryon is equal to

$$\xi_\mu^{ja,b} = P_{ja,b}\, n_\mu \qquad (20)$$

where n_μ is the unit space-like four-vector, proportional to the vector $i\epsilon_{\mu\alpha\beta\gamma} q_{1\alpha} p_{1\beta} p_{2\gamma}$ and the polarization $P_{ja,b}$ equals

$$P_{ja,b} = \frac{2s \operatorname{Im} A^*_{ja,b}(s,t) B^*_{ja,b}(s,t)}{F_{ja,b}(s,t)} c(s,t) \qquad (21)$$

where function $c(s,t) \to \sqrt{-t}$ as $s \to \infty$. In case of equal masses $m_1 = m_2 = m$, $M_1 = M_2 = M$, function $c(s,t)$ has a simple form /24/

$$c(s,t) = [t\, \frac{su - (M^2 - m^2)^2}{s^2}]^{1/2}.$$

According to the optical theorem, the total cross section of two particle interaction is proportional to the imaginary part of the elastic scattering amplitude of these particles averaged over spin. We shall denote the total interaction cross section corresponding to the elastic processes (ja, b) $j = 1, 3, 4$, by $\sigma^{tot}_{ja,b}(s)$. The total cross section can be expressed in terms of the imaginary parts of the invariant amplitudes $A_{ja,b}(s,t)$ and $B_{ja,b}(s,t)$ in the following way:

$$\sigma^{tot}_{ja,b}(s) = \frac{1}{2k\sqrt{s}} \operatorname{Im}[2M A_{ja,b}(s,0) + (M^2 + m^2 - s)B_{ja,b}(s,0)] \qquad (22).$$

$$m_1 = m_2 = m, \quad M_1 = M_2 = M$$

From (18), (19) and (22) it is not difficult to see that the total cross section and the differential cross section of forward elastic scattering are expressed by the same function:

$$H_{ja,b}(s) = 2MA_{ja,b}(s,0) + (M^2 + m^2 - s)B_{ja,b}(s,0) \qquad (23)$$

in the following way:

$$\frac{d\sigma_{ja,b}(s,t)}{dt}\Big|_{t=0} = \frac{1}{64\pi s k_1^2} |H_{ja,b}(s)|^2 \qquad (24)$$

$$\sigma^{tot}_{ja,b}(s) = \frac{1}{2k_1\sqrt{s}} \operatorname{Im} H_{ja,b}(s) \qquad (25)$$

3. Relations between Amplitudes

If isospin invariance and other higher symmetries are satisfied, then the amplitudes of the processes under consideration are connected by several equalities. For example, the following well known equalities can be deduced from isospin invariance:

$$T_{1a} - T_{1b} = -\sqrt{2}\, T_{2a} = \sqrt{2}\, T_{2b} \qquad (26)$$

$$T_{4a,b} + T_{5a,b} = T_{3a,b} \qquad (27)$$

$$T_{6a,b} - T_{7a,b} = \sqrt{2}\, T_{8a,b} = -\sqrt{2}\, T_{12a,b} \qquad (28),$$

$$T_{6a,b} + T_{7a,b} = 2T_{10a,b} \qquad (29)$$

$$T_{9a,b} = \sqrt{2}\; T_{11a,b} \qquad (30)$$

$$T_{13a} + T_{13b} = -\, T_{14} \qquad (31)$$

Now let us consider the unitary symmetry. The group of this symmetry is the unitary, unimodular group of rank 2 in three dimensional complex space ($SU(3)$ group)[8-11,25-28]. Every irreducible representation $D(\lambda_1, \lambda_2)$ of this group is characterized by two integral numbers λ_1 and λ_2 and the dimension n of which is equal to

$$n(\lambda_1, \lambda_2) = \tfrac{1}{2}\,(1+\lambda_1)(1+\lambda_2)(2+\lambda_1+\lambda_2)\,.$$

For instance, $n(0,0) = 1$, $n(1,0) = 3$, $n(0,1) = 3^*$, $n(1,1) = 8$, $n(3,0) = 10$, $n(0,3) = 10^*$ etc. The star here is used to denote the inequivalent contragradient representations. There are altogether eight generators of the group, two of which may have diagonal representation simultaneously. One of these operators is connected with hypercharge, and the other is the operator of the third component of isospin. It is convenient to choose the eigenvectors of operators Y, I_z and I^2 as the basic vectors of each irreducible representation. The direct product of two irreducible representations can be decomposed into direct sums of irreducible representations with the help of the Clebsch-Gordan coefficient of the $SU(3)$ group:

$$D(\mu_1, \mu_2) \otimes D(\nu_1, \nu_2) = \sum_{\lambda_1\lambda_2} \oplus\, D(\lambda_1, \lambda_2)$$

The octet model of Gell-Mann[10] and Ne'eman[11] and the triplet model of Sakata[7] are based on the $SU(3)$ group[2]. The difference between these two models is that in the first one the eight baryons as well as the eight mesons (including the π^{∞} -meson) belong to the same multiplets (differing in the baryon number, parity, spin etc. respectively) corresponding to the $D(1,1)$ representation, while in the second one the nucleons and the Λ - -hyperons form a triplet corresponding to the $D(1,0)$ representation. In case of the triplet model we shall consider only process in which the nucleons and the Λ -hyperon participate.

The relations between amplitudes in these models can be obtained by using the standard method. Let us consider, for instance, the octet model. In this model the wave functions of the initial and the final states in the processes (1a,b)-(14)

2/ In some papers the triplet model of Sakata is considered as based on the unitary group $U(3)$,but not on the group $SU(3)$. The difference between these groups does not change the results obtained in this paper.

belong to a reducible representation resulted from the direct product of two irreducible representations $D(1,1)$. This reducible representation can be decomposed into direct sum of irreducible representations in the following way:

$$D(1,1) \otimes D(1,1) = D(0,0) \oplus D(1,1) \oplus D(1,1)_A \oplus D(3,0) \oplus D(0,3) \oplus D(2,2) \qquad (22)$$

In this decomposition the representation $D(1,1)$ appears twice. It is convenient in the following to choose these two representations in such a way that the wave functions of one of which would be symmetric with respect to the Gell-Mann R-reflection[10], while those of the other - antisymmetric. From the invariance with respect to the unitary transformations it follows, that the matrix elements between two states belonging to two inequivalent representations vanish. Since the R -reflection is not included in the $SU(3)$ group, then the matrix elements between two states, one from which belongs to $D(1,1)_S$ and the other belongs to $D(1,1)_A$, are not equal to zero. Thus, the matrix elements of the processes under consideration can be expressed by seven independent amplitudes[3]:

$$T^{(0,0)},\; T^{(1,1)_{SS}},\; T^{(11)_{AA}},\; T^{(1,1)_{AS}},\; T^{(3,0)},\; T^{(0,3)},\; T^{(22)}\,.$$

The coefficients before these independent amplitudes can be calculated with the help of the Clebsch-Gordan coefficients of the $SU(3)$ group[14-17,28,29]. In this way, one can obtain the following relations between the amplitudes of process (1a,b)-(14)[15-18]

$$T_{1a,b} + T_{6a,b} = T_{3a,b} \qquad (32)$$

$$\sqrt{6}\, T_{9a,b} - \sqrt{2}\, T_{8a,b} = 2(T_{7b,a} - T_{5b,a}) \qquad (33)$$

$$\sqrt{3}\, T_{11a,b} + \sqrt{2}\, T_{2a,b} = T_{10a,b} - T_{5a,b} \qquad (34)$$

$$T_{13a,b} = T_{7a,b}\,. \qquad (35)$$

If the subsidary invariance with respect to the R - reflection[4] is assumed, then $T^{(1,1)_{AS}} = 0$, $T^{(3,0)} = T^{(0,3)}$, and we obtain the following relations[14-17]

3/ The equality $T^{(1,1)_{SA}} = T^{(1,1)_{AS}}$ follows from time reflection invariance.

4/ The R -reflection is defined as:

$$p \leftrightarrow -\Xi^-, \quad n \leftrightarrow \Xi^0, \quad \Sigma^+ \leftrightarrow \Sigma^-,$$
$$K^+ \leftrightarrow -K^-, \quad K^0 \leftrightarrow \bar{K}^0, \quad \pi^+ \leftrightarrow \pi^-.$$

$$T_{1a,b} = T_{4a,b} \quad . \tag{36}$$

$$T_{5a,b} = T_{6a,b} \tag{37}$$

more.

From the relations (32)-(37) and the isospin relations (26)-(31) follow equalities:

$$T_{8a} - T_{8b} = T_{2b} \tag{38}$$

$$T_{9a} - T_{9b} = \sqrt{3}\, T_{2b} \tag{39}$$

$$T_{9a} - T_{9b} = \sqrt{3}\,(T_{8a} - T_{8b}), \quad (T_{9a} + T_{9b}) = -\frac{1}{\sqrt{3}}(T_{8a} + T_{8b}) \tag{40}$$

Analogously, in case of the triplet model of Sakata the matrix elements of the processes (ja, b), $j = 1,2,3,4,5,9,11$, in question are expressed by three independent amplitudes $T^{(2,1)}$, $T^{(0,2)}$ and $T^{(1,0)}$, and for which the following equalities can be obtained[14,17,18]:

$$T_{1a,b} = T_{3a,b} \tag{41}$$

$$T_{4a} = T_{4b} \tag{42}$$

$$T_{5a,b} = T_{9a,b} \quad . \tag{43}$$

At last let us consider the model of Behrends and Sirlin in the seven-dimensional charge space[12]. The symmetry group of this model is the G_2 group of rank 2 with 14 parameters[26]. The basic irreducible representation of this group is $D^1(0,0)$, $D^7(1,0)$, $D^{14}(0,1)$, $D^{27}(2,0)$ and $D^{64}(1,1)$. In this model the Λ -hyperon is assigned to a singlet of the one dimensional representation $D^1(0,0)$, while the rest seven baryons are assigned to a multiplet corresponding to the irreducible representation $D^7(1,0)$ this is also the case for the seven well established pseudoscalar mesons. Therefore, the matrix elements of all processes under consideration with participation of the Λ -hyperon are expressed by the same independent amplitude and hence are proportional to one another, namely[5]

$$T_{9a} = \sqrt{2}\, T_{11a} = -T_{9b} = -\sqrt{2}\, T_{11b} \tag{44}$$

while the matrix elements of the rest processes are expressed by four independent amplitudes $T^{(00)}$, $T^{(1,0)}$, $T^{(0,1)}$, $T^{(2,0)}$ and satisfy the following relations[12,16].

5/ We remind the readers that the first and the last equalities in (44) follow from the isospin invariance.

$$T_{1a,b} = T_{4a,b}\,, \tag{45}$$

$$T_{5a,b} = T_{6a,b}\,, \tag{46}$$

$$T_{7a,b} = T_{13a,b}\,, \tag{47}$$

$$T_{8a,b} = T_{2a,b}\,, \tag{48}$$

$$T_{1a,b} + T_{7a,b} = T_{3b,a}\,. \tag{49}$$

We would like to point out that the first three relations here coincide with those in eq. (36), (37) and (35) respectively, and the consequences of the octet model differ very little from those of the G_2 model.

4. Relations between Cross Sections

From the relations (26)-(49) between matrix elements one can obtain a series of relations between cross sections[14-16,18]. We should like to make a remark with respect to these relations. As is well known, all the higher symmetries considered above are destroyed at low energies, and they can hold only at large values of s and t, or at least, when one of them is large. The relations written down above can be understood as those between invariant (scalar) amplitudes $A_{ja,b}(s,t)$ and $B_{ja,b}(s,t)$. In the following (when the elastic scattering at zero angle are to be considered) we shall sometimes assume that these relations are correct at $t = 0$ and large s. We note that even in this case, due to the presence of a kinematic factor, the relations of those processes where the baryons in the initial state and in the final state have different mass, for instance (7a,b) and (13a,b), are not satisfied. Nevertheless the relations between cross sections of elastic scattering type processes, and in particular, relations between total cross sections are satisfied.

Now let us deduce some relations between cross sections of the processes under consideration. We shall consider first the consequences of isospin invariance. The relation (26) can be rewritten in the following form:

$$\begin{aligned} A_{1a}(s,t) - A_{1b}(s,t) &= \sqrt{2}\, A_{2b}(s,t) \\ B_{1a}(s,t) - B_{1b}(s,t) &= \sqrt{2}\, B_{2b}(s,t). \end{aligned} \tag{50}$$

As it was shown in[23], in the general case, when both invariant amplitudes $A_{ja,b}(s,t)$ and $B_{ja,b}(s,t)$ contribute to the asymptote of cross sections[6], if these amplitudes do not oscilate, but have a definite growth as

6/ Other special cases can be considered in a similar manner. All conclusions made in the general case are also correct in these special cases.

$s \to \infty$ then from the crossing symmetry properties and by virtue of the Phragmen-Lindelöf theorem, the following relations between amplitudes of crossing processes (ja) and (jb) can be proved:

$$A_{jb}(s,t) = e^{-i\pi\alpha(t)} A^*_{ja}(s,t) \qquad\qquad s \to \infty \,, \tag{51}$$
$$B_{jb}(s,t) = e^{-i\pi\alpha(t)} B^*_{ja}(s,t)$$

where $\alpha(t)$ is a real function of t.

By use of (51) relations (50) can be rewritten in the form:

$$A_{1a}(s,t) - e^{-i\pi\alpha(t)} A^*_{1a}(s,t) = \sqrt{2}\, A_{2b}(s,t)$$
$$B_{1a}(s,t) - e^{-i\pi\alpha(t)} A^*_{1a}(s,t) = \sqrt{2}\, B_{2b}(s,t)$$

and hence

$$Re\,[e^{i\pi\alpha(t)/2} A_{2b}(s,t)] = Re\,[e^{i\pi\alpha(t)/2} B_{2b}(s,t)] = 0$$

$$Im\,[e^{i\pi\alpha(t)/2} A_{1a}(s,t)] = \sqrt{1/2}\; Im\,[e^{i\pi\alpha(t)/2} A_{2b}(s,t)] \tag{52}$$

$$Im\,[e^{i\pi\alpha(t)/2} B_{1a}(s,t)] = \sqrt{1/2}\; Im\,[e^{i\pi\alpha(t)/2} B_{2b}(s,t)] \,.$$

From these relations and the expressions for the cross section (18) and (19) it is easy to deduce the following inequalities:

$$\frac{d\sigma_{1a,b}(s,t)}{dt} - \frac{1}{2}\frac{d\sigma_{2a}(s,t)}{dt} \geq 0 \,. \tag{53}$$

If $\alpha(0)=1$ then from relations (52) and the expressions for cross sections (23)-(25) follow the equality:

$$\frac{d\sigma_{1a,b}(s,t)}{dt}\Big|_{t=0} - \frac{1}{2}\frac{d\sigma_{2a}(s,t)}{dt}\Big|_{t=0} = \frac{1}{16\pi}\,[\sigma^{tot}_{1a,b}(s)]^2 \,. \tag{54}$$

Analogously, the equality

$$\frac{d\sigma_{13a,b}(s,t)}{dt} - \frac{1}{4}\frac{d\sigma_{14}(s,t)}{dt} > 0 \tag{55}$$

can be deduced from relation (31).

Now, let us consider the consequence of relation (24) under the assumption that $\alpha(0)=1$. From this relation and eq. (15), (23) and (51) it follows

$$Im\,H_{3a,b}(s) - Im\,H_{4c}(s) = Im\,H_{5a,b}(s)$$

and hence according to (24) and (25)

$$|\,\sigma^{tot}_{3a,b}(s) - \sigma^{tot}_{4c}(s)\,| \leq \sqrt{16\pi \frac{d\sigma_{5a,b}(s,t)}{dt}\Big|_{t=0}} \,. \tag{56}$$

We have shown several asymptotic relations between cross sections in case of isospin invariance. It is worth- while to point out that besides these relations there are some other well known equalities and inequalities between cross sections, which can be obtained from eq. (26)-(31) and are correct at all energies.

Let us go over to study the consequences of the unitary symmetry of the octet model. We shall assume R-reflection invariance. In this case relations (32)-(40) hold between amplitudes of the processes under consideration. Relations (35)-(37) give us three well known equalities between differential cross sections and an equality between total cross sections of $\pi^{\pm}$-proton and $K^{\pm}$-neutron interactions/14,15,17/. By the procedure explained above and using relations (38) and (39) we can deduce the following inequalities:

$$\frac{d\sigma_{8a,b}(s,t)}{dt} - \frac{1}{4}\frac{d\sigma_{2b}(s,t)}{dt} \geq 0 \,, \tag{57}$$

$$\frac{d\sigma_{9a,b}(s,t)}{dt} - \frac{3}{4}\frac{d\sigma_{2b}(s,t)}{dt} > 0 \,. \tag{58}$$

Now let us consider the consequence of equality (36). According to (15) and (26), we have

$$T_{4c} = \frac{1}{2}(T_{1a} + T_{1b}) \,, \tag{59}$$

$$T_{4d} = \frac{1}{2i}(T_{1a} - T_{1b}) = \frac{1}{\sqrt{2}\,i}\, T_{2b} \,, \tag{60}$$

then from eq. (60) the equalities

$$\frac{d\sigma_{4d}(s,t)}{dt} = \frac{1}{2}\frac{d\sigma_{2b}(s,t)}{dt} \tag{61}$$

follows immediately.

By use of eq. (51), (59) and (60) can be rewritten in the following form:

$$A_{4c}(s,t) = \tfrac{1}{2}[A_{1a}(s,t) + e^{-i\pi\alpha(t)} A^*_{1a}(s,t)] \qquad (62)$$

$$B_{4c}(s,t) = \tfrac{1}{2}[B_{1a}(s,t) + e^{-i\pi\alpha(t)} B^*_{1a}(s,t)] ,$$

$$A_{4d}(s,t) = \tfrac{1}{2}[A_{1a}(s,t) + e^{-i\pi\alpha(t)} A^*_{1a}(s,t)] \qquad (63)$$

$$B_{4d}(s,t) = \tfrac{1}{2}[B_{1a}(s,t) - e^{-i\pi\alpha(t)} B^*_{1a}(s,t)] .$$

These relations show that

$$|A_{1a}(s,t)|^2 = |A_{4c}(s,t)|^2 + |A_{4d}(s,t)|^2 ,$$

$$|B_{1a}(s,t)|^2 = |B_{4c}(s,t)|^2 + |B_{4d}(s,t)|^2 ,$$

$$Re\, A_{1a}(s,t) B^*_{1a}(s,t) = Re\, A_{4c}(s,t) B^*_{4c}(s,t) + Re\, A_{4d}(s,t) B^*_{4d}(s,t) ;$$

and hence

$$\frac{d\sigma_{1a,b}(s,t)}{dt} = \frac{d\sigma_{4c}(s,t)}{dt} + \frac{d\sigma_{4d}(s,t)}{dt} . \qquad (64)$$

Now consider relations (40). Using eq. (51), we can rewrite these relations in the following form:

$$A_{9a}(s,t) - e^{-i\pi\alpha(t)} B^*_{9a}(s,t) = \sqrt{3}\,[A_{8a}(s,t) - e^{-i\pi\alpha(t)} A^*_{8a}(s,t)], \qquad (65)$$

$$B_{9a}(s,t) - e^{-i\pi\alpha(t)} B^*_{9a}(s,t) = \sqrt{3}\,[B_{8a}(s,t) - e^{-i\pi\alpha(t)} B^*_{8a}(s,t)].$$

$$A_{9a}(s,t) + e^{-i\pi\alpha(t)} A^*_{9a}(s,t) = -\frac{1}{\sqrt{3}}[A_{8a}(s,t) + e^{-i\pi\alpha(t)} A^*_{8a}(s,t)],$$

$$B_{9a}(s,t) + e^{-i\pi\alpha(t)} B^*_{9a}(s,t) = -\frac{1}{\sqrt{3}}[B_{8a}(s,t) + e^{-i\pi\alpha(t)} B^*_{8a}(s,t)] . \qquad (66)$$

From these relations and the expressions for differential cross section (18) and (19) follows:

$$\frac{d\sigma_{8a,b}(s,t)/dt}{d\sigma_{9a,b}(s,t)/dt} = \frac{3 + \frac{1}{3}\frac{G}{E}}{1 + \frac{G}{E}} , \qquad (67)$$

where G is obtained from $F_{9a,b}(s,t)$ by the substitutions

$$|A_{9a,b}(s,t)|^2 \to [Im\, A_{9a,b}(s,t)\, e^{\frac{i\pi\alpha(t)}{2}}]^2 ,$$

$$|B_{9a,b}(s,t)|^2 \to [Im\, B_{9a,b}(s,t)\, e^{\frac{i\pi\alpha(t)}{2}}]^2 ,$$

$$Re A_{9a,b}(s,t) B^*_{9a,b}(s,t) \to Im\,[A_{9a,b}(s,t)\, e^{\frac{i\pi\alpha(t)}{2}}]\, Im\,[B_{9a,b}(s,t)\, e^{\frac{i\pi\alpha(t)}{2}}] ,$$

while E is obtained from G by the substitution $Im \to Re$. The ratio G/E can very from 0 to ∞. Therefore from (68) follows:

$$\frac{1}{3} \le \frac{d\sigma_{8a,b}(s,t)/dt}{d\sigma_{9a,b}(s,t)/dt} \le 3 . \qquad (68)$$

In the case of the unitary symmetry of the triplet model the amplitudes of processes under consideration are related by eq. (41)-(43). Several identities between cross sections deduced from these relations were given in /13,17/. We shall deduce some other relations between cross sections and polarizations. It is not difficult to see from eq. (15) and (42) that

$$\frac{d\sigma_{4c}(s,t)}{dt} = \frac{d\sigma_{4a'}(s,t)}{dt} = \frac{d\sigma_{4b'}(s,t)}{dt}$$

$$\sigma^{tot}_{4c}(s) = \sigma^{tot}_{4a'}(s) = \sigma^{tot}_{4b'}(s) \qquad (69)$$

$$\frac{d\sigma_{4d}(s,t)}{dt} = 0$$

Furthermore, relations (42) and (51) show that the amplitudes of processes (4a') and (4b') have the following property:

$$A_{4a',b'}(s,t) = e^{-i\pi\alpha(t)} A^*_{4a',b'}(s,t) \qquad (70)$$

$$B_{4a',b'}(s,t) = e^{-i\pi\alpha(t)} B^*_{4a',b'}(s,t) .$$

From eq. (70) and (21) it follows that the polarization of the recoil neutrons in processes (4a') and (4b') should tend to zero at $s \to \infty$ and fixed t, not depending on the relative behaviour of the invariant amplitudes. Moreover, relation (43) gives an equality between cross sections of processes (5c)and(9c):

$$\frac{d\sigma_{5e}(s,t)}{dt} = \frac{d\sigma_{9e}(s,t)}{dt} . \qquad (73)$$

Now let us consider the model with symmetry group G_2. Relation (44) together with eq. (51) show that the polarization of the Λ -hyperon in processes (9a,c) and (11a,b) should tend to zero at $s \to \infty$ and fixed t, not depending on the relative behaviour of the invariant amplitudes. Such conclusion is also correct for processes (8a,c) because relation (45)-(47) coincide with (36),(37) and (35). Therefore, several relations in case of the octet model, for instance, (64) and (65) hold also in case of the model with the symmetry group G_2.

At last let us deduce some common asymptotic properties of the meson-baryon scattering amplitudes for all the models under consideration. The asymptotic relations (51) can be rewritten in the form

$$e^{i\frac{\pi\alpha(t)}{2}} A_{ja}(s,t) = [e^{i\frac{\pi\alpha(t)}{2}} A_{jb}(s,t)]^*,$$
$$e^{i\frac{\pi\alpha(t)}{2}} B_{ja}(s,t) = [e^{i\frac{\pi\alpha(t)}{2}} B_{jb}(s,t)]^* \qquad (73)$$
$$s \to \infty$$

from which follows:

$$Re[e^{i\frac{\pi\alpha(t)}{2}} A_{ja}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} A_{jb}(s,t)]^*$$
$$Re[e^{i\frac{\pi\alpha(t)}{2}} B_{ja}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} B_{jb}(s,t)]^*. \qquad (74)$$

In all the models with higher symmetries under consideration amplitudes $A_{ja,b}(s,t)$ and $B_{ja,b}(s,t)$ are expressed in terms of those between states corresponding to irreducible representations $A^{(\lambda_1,\lambda_2)}(s,t)$ and $B^{(\lambda_1,\lambda_2)}(s,t)$ where λ_1 and λ_2 characterize the representations. As an example, in the triplet model of Sakata from the expressions of $A_{ja,b}(s,t)$ and $B_{ja,b}(s,t)$ in terms of $A^{(\lambda_1,\lambda_2)}(s,t)$ and $B^{(\lambda_1,\lambda_2)}(s,t)$ and (74), it follows that

$$Re[e^{i\frac{\pi\alpha(t)}{2}} A^{(2,1)}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} A^{(0,2)}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} A^{(1,0)}(s,t)]$$
$$(75)$$
$$Re[e^{i\frac{\pi\alpha(t)}{2}} B^{(2,1)}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} B^{(0,2)}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} B^{(1,0)}(s,t)]$$

and therefore for all charge exchange processes, we have

$$Re[e^{i\frac{\pi\alpha(t)}{2}} A_{ja,b}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} B_{ja,b}(s,t)] = 0, \qquad (76)$$

while for the elastic processes, we have

$$Re[e^{i\frac{\pi\alpha(t)}{2}} A_{1a,b}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} A_{3a,b}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} A_{4a,b}(s,t)] \quad (77)$$
$$Re[e^{i\frac{\pi\alpha(t)}{2}} B_{1a,b}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} B_{3a,b}(s,t)] = Re[e^{i\frac{\pi\alpha(t)}{2}} B_{4a,b}(s,t)].$$

Properties (76) and (77) hold not only in the Sakata model, but also in other models under consideration. From (76) we may deduce that at $s \to \infty$ and fixed t, the polarization of the final baryon equals zero in the charge exchange processes, when the baryon in the initial state is unpolarized, if the strong interactions satisfy one of the above discussed symmetries.

Consider now the elastic forward scattering processes. Suppose $\alpha(0) = 1$ /23/, then

$$Re[e^{i\frac{\pi\alpha(0)}{2}} A_{ja,b}(s,0)] = -Im\, A_{ja,b}(s,0)$$
$$Re[e^{i\frac{\pi\alpha(0)}{2}} B_{ja,b}(s,0)] = -Im\, B_{ja,b}(s,0). \qquad (78)$$

Eqs. (77) together with (78) show that the imaginary part of all the elastic forward scattering amplitudes equal one another[7]. From which follows the asymptotic equality of all the total meson-baryon interaction cross sections in the models under consideration:

$$\sigma^{tot}(\pi^{\pm} p) = \sigma^{tot}(K^{\pm} p) = \dots = \sigma^{tot}(\pi\Xi). \qquad (79)$$

Nevertheless, without subsidiary assumptions concerning the real part of amplitudes, no conclusion with respect to the differential cross sections can be made.

We would like to point out that we have written down only the independent relations between cross sections, from which some other dependent relations could be obtained. Moreover, we have not written down those obvious triangular inequalities, which could be obtained from relations of type (32), (38), (39) etc. between matrix elements.

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[7] This result has been also obtained in a recent preprint of Ryder and Smith /30/.

5. Resume of Results

In conclusion we shall tabulate all relations obtained in [13-15,17] and in the present paper. The well known isotopic relations, triangular inequalities and asymptotic identities between cross sections of crossing processes will not be included in this table. We shall denote, for example, the differential cross section of process $K^- + p \to \pi^- + \Sigma^+$ at given t by $\sigma(K^- p \to \pi^- \Sigma^+)$, the polarization of the Σ^+-hyperon in this process while the proton is unpolarized by $P(K^- p \to \pi^- \Sigma^+)$ and the total cross section of π^+-meson-proton interaction by $\sigma^{tot}(\pi^+ p)$.

The models in which the given relations hold and whether the Phragmen-Lindelöf theorem (or the Pomeranchuk theorem) is used in the deduction of these relations are listed also in the table. We shall use the following notations in the table:

- I - isotopic invariance;
- O - the octet model with R-reflection invariance;
- T - the triplet model;
- G_2 - the model with symmetry group
- $PH-L$- the Phragmen-Lindelöf theorem.

The authors are indebted to N.N.Bogoliubov, M.A. Markov, V.I. Ogievetsky, Ya.A. Smorodinsky, I.T. Todorov and O.A. Khrustalev for interest in this work and valuable discussions.

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TABLE OF RELATIONS

Relations	Models	Reference
$\sigma(\pi^+ p \to \pi^+ p) - \frac{1}{2}\sigma(\pi^- p \to \pi^0 n) \geq 0$	$I, PH-L$	
$\sigma(\pi^+ p \to \pi^+ p) - \frac{1}{2}\sigma(\pi^- p \to \pi^0 n)\Big\vert_{t=0} = \frac{1}{16\pi}[\sigma^{tot}(\pi^+ p)]^2$	$I, PH-L$	
$\sigma(K^- p \to K^0 \Xi^0) - \frac{1}{4}\sigma(K^- p \to K^+ \Xi^-) \geq 0$	$I, PH-L$	
$\sigma^{tot}(K^+ p) \to \sigma^{tot}(K^0_2 p) \vert \leq \sqrt{16\pi\sigma(K^- p \to K^0 n)\vert_{t=0}}$	$I, PH-L$	
$\sigma(K^- p \to K^0 \Xi^0) = \sigma(K^- p \to \pi^+ \Sigma^-)$	$0, G_2$	[15, 16, 18]
$\sigma(K^- p \to \bar{K}^0 n) = \sigma(K^- p \to \pi^- \Sigma^+)$	$0, G_2$	[15, 16, 18]
$\sigma(\pi^- p \to K^0 \Sigma^0) - \frac{1}{4}\sigma(\pi^- p \to \pi^0 n) \geq 0$	$0, PH-L$	
$\sigma(\pi^- p \to K^0 \Lambda) - \frac{3}{4}\sigma(\pi^- p \to \pi^0 n) \geq 0$	$0, PH-L$	
$\sigma(K^0_2 p \to K^0_1 p) = \frac{1}{2}\sigma(\pi^- p \to \pi^0 n)$	$0, G_2, PH-L$	
$\sigma(\pi^+ p \to \pi^+ p) = \sigma(K^0_2 p \to K^0_2 p) + \sigma(K^0_2 p \to K^0_1 p)$	$0, G_2, PH-L$	
$\frac{1}{3} \leq \frac{\sigma(\pi^- p \to K^0 \Sigma^0)}{\sigma(\pi^- p \to K^0 \Lambda)} \leq 3$	$P\ 0, PH-L$	
$\sigma(\pi^+ p \to \pi^+ p) \to \sigma(K^+ p \to K^+ p)$	T	[14, 18]
$\sigma(K^- p \to \bar{K}^0 n) = \sigma(\pi^- p \to K^0 \Lambda)$	T	[14, 18]
$\sigma(K^0_2 p \to K^0_2 p) = \sigma(K^+ n \to K^+ n)$	T	
$\sigma(K^0_2 p \to K^0_1 p) = 0$	T	
$\sigma(K^0_2 p \to K^+ n) = \sigma(K^0_2 p \to \pi^+ \Lambda)$	T	
$\sigma^{tot}(K^0_2 p) = \sigma^{tot}(K^+ n)$	T	
$P(K^+ n \to K^+ n) = 0$	$T, PH-L$	
$P(m_1 B_2 \to m_2 B_2) = 0, \quad m_1 \neq m_2, \ B_1 \neq B_2$	$0, G_2, T, PH-L$	
$\sigma^{tot}(\pi^\pm p) = \sigma^{tot}(K^\pm p) = \ldots = \sigma^{tot}(mB)$	$0, G, T, PH-L$	