ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

LABORATORY OF THEORETICAL PHYSICS

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E- 1534

# ENERGIES OF THE QUADRUPOLE STATES OF STRONGLY DEFORMED EVEN-EVEN NUCLEI

ЖЭТФ, 1964, т.47, в 1, с 252-261.

Дубна 1964

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Submitted to JETP

Дубна 1964

## Abstract

The energies of the beta and gamma vibrational states of even-even strongly deformed nuclei are calculated in the region $152 \leq A \leq 186$ and $228 \leq A \leq 254$. A satisfactory agreement is obtained between the calculated and the corresponding experimental data in case $\kappa_n = \kappa_p = \kappa_{np} = \kappa$ with $\kappa = \frac{10}{A^{4/3}} \hbar\omega_0^0$ in the first region and $\kappa = \frac{12}{A^{4/3}} \hbar\omega_0^0$ in the second one. It is shown that for the isotopes of $Dy$ and $Er$ the energies of the gamma vibrational states lie below the energies of the beta vibrational states in correspondence with the experimental data. It is noted that in the majority of cases the lowest states with $K\pi = 2+$ and $K\pi = 0+$ possess the pronounced collective properties and their energies are considerably less than the energies of the nearest poles in the secular equations. In some cases the energies of the quadrupole states are near the poles and their wave functions are very close to the two-quasi-particle ones. The calculated probabilities of the $E2$ transitions do not contradict the experimental data.



## Аннотация

Рассчитаны энергии $\beta$ и $\gamma$ вибрационных состояний четно-четных сильнодеформированных ядер в областях $152 \leq A \leq 186$ и $228 \leq A \leq 254$. Получено удовлетворительное согласие между расчетами и соответствующими экспериментальными данными в случае $\kappa_n = \kappa_p \equiv \kappa_{np} \equiv \kappa$ с $\kappa = \frac{10}{A^{4/3}} \hbar\omega_0^0$ в первой области и $\kappa = \frac{12}{A^{4/3}} \hbar\omega_0^0$ – во второй. Показано, что для изотопов $Dy$ и $Er$ энергии $\gamma$-вибрационных состояний лежат ниже энергий $\beta$-вибрационных состояний в соответствии с опытными данными. Отмечено, что в большинстве случаев наинизшие состояния с $K\pi = 2+$ и $K\pi = 0+$ обладают ярко выраженными коллективными свойствами и энергии их значительно меньше энергий ближайших полюсов в секулярных уравнениях. В ряде случаев энергии квадрупольных состояний лежат вблизи полюсов, и волновые функции их весьма близки к двухквазичастичным. Вычисленные вероятности $E2$-переходов не противоречат экспериментальным данным.

1. Many theoretical and experimental works are devoted to the investigation of the collective excited states of even-even nuclei. Rather rich experimental information on the energies of beta and gamma vibrational states in even-even strongly deformed nuclei and on the reduced probabilities of the electromagnetic transition has been accumulated. The method of approximate second quantization suggested by N.N. Bogolubov in 1947 [1] and the mathematical methods developed in constructing the theory of superconductivity [2] are the basis of the theoretical investigations of the microscopic nature of the collective states. The most completely studied are the collective states of spherical nuclei [3-5]. In the region of strongly deformed nuclei the investigations [6] carried out before 1963 are restricted to obtaining the basic equations and to investigation of the problem of elimination of a spurious state. In refs. [7,8] the energies of the octupole states of strongly deformed even-even nuclei have been calculated and a rather close agreement with the corresponding experimental data has been obtained. In ref. [9] the energies of gamma vibrational states and the probabilities of the electromagnetic $E2$ -transitions have been calculated. However in this paper the energies of beta vibrations are not calculated and the energies of gamma vibrations calculated using the asymptotic wave functions of the Nilsson potential are about 1,5 time as high as the experimental ones.

It is interesting to calculate the energies of the beta and gamma vibrational states for the same value of the constants of a quadrupole-quadrupole interaction using the exact wave functions of the Nilsson potential and to compare theory with experiment. This is just the aim of the present paper.

2. The collective state energies are calculated on the basis of the superfluid nuclear model. In this model the Hamiltonian of interaction between nucleons in a nucleus is written in the form of three terms

$$H = H_{av} + H_{pair} + H_{coll} , \qquad (1)$$

describing the average nuclear field, interactions leading to pairing correlations of the superconducting type and interactions responsible for the collective effects. In calculating the energies of the quadrupole collective states $H_{coll}$ is taken in the form

$$H_{coll} = -\sum_{\mu=0,2}\Big\{\frac{\kappa_n^{(2)}}{2}\,Q_{2\mu}^{+}(n)Q_{2\mu}(n)+\frac{\kappa_p^{(2)}}{2}\,Q_{2\mu}^{+}(p)Q_{2\mu}(p)+ \qquad (2)$$

$$+\frac{\kappa_{np}^{(2)}}{2}\,(Q_{2\mu}^{+}(n)Q_{2\mu}(p)+Q_{2\mu}^{+}(p)Q_{2\mu}(n))\Big\},$$

where

$$Q_{2\mu}(n)=\sum_{\substack{\sigma\sigma'\\ ss'}} f_{\sigma\sigma'}^{2\mu}(ss')\,a_{s\sigma}^{+}\,a_{s'\sigma'},$$

$$f^{20}=r^2Y_{20},\qquad f^{22}=\frac{1}{\sqrt{2}}\,r^2(Y_{22}+Y_{2\,-2}),$$

$\kappa_n^{(2)}, \kappa_p^{(2)}, \kappa_{np}^{(2)}$ are the constants of the quadrupole-quadrupole interaction.

In the microscopic treatment of the superfluid nuclear model the wave functions of the collective states are the superposition of the wave functions of the two-quasi-particle states. The collective states are considered side by side with the two-quasi-particles ones, any restrictions to the collective state energies being absent. The terms beta-vibrational and gamma vibrational states should be considered as conventional. The beta vibrational states are the lowest excited states with $K\pi = 0+$ i.e. with the projection of the momentum of the axis of the nuclear symmetry being zero and the positive parity (different from the rotational states), and the gamma vibrational states are the lowest states with $K\pi = 2+$.

3. Consider the gamma vibrational states. In investigating these states we should bear in mind that in addition to the matrix elements $f_{\sigma\sigma}^{22}(s_1,s_2)\equiv f(s_1,s_2)$, where $K_1 \pm 2 = K_2$ ( $K_1$ and $K_2$ are the projections of the momenta of two states) we must take into account the matrix elements $f_{\sigma,-\sigma}^{22}(s_1,s_2)\equiv \bar{f}(s_1,s_2)$ with $K_1 + K_2 = \pm 2$ disregarded in some papers.

On the basis of the variational principle, in the framework of the method of approximate second quantization we get a secular equation defining the energies $\omega_i$ of the excited states with $K\pi = 2+$ in the form

$$1 = 2\kappa_n^{(2)} \sum_{ss'} \frac{(f(ss')^2 + \bar{f}(ss')^2)\, u^2_{ss'}}{\epsilon(s)+\epsilon(s') - \frac{\omega_i^2}{\epsilon(s)+\epsilon(s')}} + 2\kappa_p^{(2)} \sum_{\nu\nu'} \frac{(f(\nu\nu')^2 + \bar{f}(\nu\nu')^2)\, u^2_{\nu\nu'}}{\epsilon(\nu)+\epsilon(\nu') - \frac{\omega_i^2}{\epsilon(\nu)+\epsilon(\nu')}} + \quad (3)$$

$$+ 4((\kappa_{np}^{(2)})^2 - \kappa_n^{(2)} \kappa_p^{(2)}) \sum_{ss'} \frac{(f(ss')^2 + \bar{f}(ss')^2)\, u^2_{ss'}}{\epsilon(s)+\epsilon(s') - \frac{\omega_i^2}{\epsilon(s)+\epsilon(s')}} \sum_{\nu\nu'} \frac{(f(\nu\nu')^2 + \bar{f}(\nu\nu')^2)\, u^2_{\nu\nu'}}{\epsilon(\nu)+\epsilon(\nu') - \frac{\omega_i^2}{\epsilon(\nu)+\epsilon(\nu')}},$$

the summation $ss'(\nu\nu')$ being made over one-particle levels of the average field of the neutron (proton) system,

$$\epsilon(s) = \sqrt{C_n^2 + |E(s) - \lambda_n|^2}, \quad \epsilon(\nu) = \sqrt{C_p^2 + |E(\nu) - \lambda_p|^2},$$

$$u_{ss'} = u_s v_{s'} + u_{s'} v_s ;$$

The index $i$ in $\omega_i$ denotes the first, second and so on roots of the secular equation.

In case $\kappa_n^{(2)} = \kappa_p^{(2)} = \kappa_{np}^{(2)} \equiv \kappa$ to which we shall restrict ourselves in what follows, the secular equation will be of simpler form, namely:

$$\frac{1}{2\kappa} = \sum_{ss'} \frac{(f(ss')^2 + \bar{f}(ss')^2)\, u^2_{ss'}}{\epsilon(s)+\epsilon(s') - \frac{\omega_i^2}{\epsilon(s)+\epsilon(s')}} + \sum_{\nu\nu'} \frac{(f(\nu\nu')^2 + \bar{f}(\nu\nu')^2)\, u^2_{\nu\nu'}}{\epsilon(\nu)+\epsilon(\nu') - \frac{\omega_i^2}{\epsilon(\nu)+\epsilon(\nu')}} . \quad (4)$$

The wave function of the $i$-th state with $K\pi = 2+$ is $Q_i^+ \Psi$ where the operator $Q_i$ is equal to

$$Q_i = \tfrac{1}{2} \{ \sum_{ss'} (\psi^i_{ss'} A(ss') - \phi^i_{ss'} A(ss')^+ + \bar{\psi}^i_{ss'} \bar{A}(ss') - \bar{\phi}^i_{ss'} \bar{A}(ss')^+) + \quad (5)$$

$$+ \sum_{\nu\nu'} (\psi^i_{\nu\nu'} A(\nu\nu') - \phi^*_{\nu\nu'} A(\nu\nu')^+ + \bar{\psi}^i_{\nu\nu'} \bar{A}(\nu\nu') - \bar{\phi}^i_{\nu\nu'} \bar{A}(\nu\nu')^+) \},$$

in this case

$$A(ss') = \frac{1}{\sqrt{2}} \sum_\sigma \sigma a_{s'\sigma} a_{s-\sigma}, \qquad \bar{A}(ss') = \frac{1}{\sqrt{2}} \sum_\sigma a_{s\sigma} a_{s'\sigma},$$

Here $a_{s\sigma}$ , $a_{\nu\tau}$ are the quasi-particle operators. The functions $\psi^{t}_{ss'}$, $\phi^{t}_{ss'}$ , $\bar{\psi}^{t}_{ss'}$ , $\bar{\phi}^{t}_{ss'}$ , $\psi^{t}_{\nu\nu'}$ , $\phi^{t}_{\nu\nu'}$ , $\bar{\psi}^{t}_{\nu\nu'}$ , $\varphi^{t}_{\nu\nu'}$ are written in the form:

$$\psi^{t}_{ss'} = \tfrac{1}{2}(g^{t}_{ss'} + w^{t}_{ss'}), \quad \bar{\psi}^{t}_{ss'} = \tfrac{1}{2}(\bar{g}^{t}_{ss'} + \bar{w}^{t}_{ss'}),$$

$$\phi^{t}_{ss'} = \tfrac{1}{2}(g^{t}_{ss'} - w^{t}_{ss'}), \quad \bar{\phi}^{t}_{ss'} = \tfrac{1}{2}(\bar{g}^{t}_{ss'} - \bar{w}^{t}_{ss'}),$$

where (6)

$$w^{t}_{ss'} = \frac{\omega_t}{\epsilon(s) + \epsilon(s')}\, g^{t}_{ss'},$$

$$\bar{w}^{t}_{ss'} = \frac{\omega_t}{\epsilon(s) + \epsilon(s')}\, \bar{g}^{t}_{ss'},$$

$$g^{t}_{ss'} = \frac{\sqrt{2}}{\sqrt{Y^{t}_{n} + \bar{Y}^{t}_{n} + Y^{t}_{p} + \bar{Y}^{t}_{p}}} \frac{f(ss')u_{ss'}}{\epsilon(s) + \epsilon(s') - \dfrac{\omega_t^2}{\epsilon(s) + \epsilon(s')}},$$

$$\bar{g}^{t}_{ss'} = \frac{\sqrt{2}}{\sqrt{Y^{t}_{n} + \bar{Y}^{t}_{n} + Y^{t}_{p} + \bar{Y}^{t}_{p}}} \frac{\bar{f}(ss')u_{ss'}}{\epsilon(s) + \epsilon(s') - \dfrac{\omega_t^2}{\epsilon(s) + \epsilon(s')}} \qquad (7)$$

in this case

$$Y^{t}_{n} + \bar{Y}^{t}_{n} = \sum_{ss'} \frac{(f(ss')^2 + \bar{f}(ss')^2)\, u^2_{ss'}\, \omega_t\, (\epsilon(s) + \epsilon(s'))}{[(\epsilon(s) + \epsilon(s'))^2 - \omega_t^2]^2}.$$

Thus, after solving the secular equations (3) or (4) we find the energies of the states with $K^{\pi} = 2^{+}$ and the corresponding wave functions.

4. We consider beta vibrations, i.e. the excited states with $I^{\pi}K = 0^{+}0$. In this case there are diagonal and non-diagonal matrix elements from the operator of the quadrupole momentum and $\bar{f}(ss') = 0$ Thus, for a system consisting of particles of the same kind the secular equation can be written in the form:

$$\frac{1}{2\kappa} = \sum_{s \neq s'} \frac{f(ss')^2 u^2_{ss'}}{\epsilon(s) + \epsilon(s') - \dfrac{\omega_t^2}{\epsilon(s) + \epsilon(s')}} + \sum_{s} \frac{f(ss)^2 c^2}{(4\epsilon(s)^2 - \omega_t^2)\epsilon(s)}, \qquad (8)$$

the contribution of the diagonal terms being as large as (50-70) %.

Let us show that the diagonal elements should not be so important. Indeed,

we consider the case when all the diagonal terms are equal to one another, i.e. $f(ss) = f_0$ then the corresponding part of $H_{coll}$ reads

$$-\frac{\kappa}{2} f_0^2 N^2 ,$$

where $N$ is the operator of the number of particles. As far as the number of particles in the ground and excited states is the same, hence, it follows that this term does not lead to the change in the energy difference between the beta vibrational and ground states. If $f(ss) \neq f_0$ then the diagonal terms can lead to the decrease as well as to the increase of the energies of the excited states with $K\pi = 0^+$. Thus, the coherent effect leading to the decrease of the energies of the states with $K\pi = 0^+$ with respect to the nearest pole of the secular equation is, in the main, due to the non-diagonal matrix elements of the quadrupole momentum operator. From these considerations it is seen that the calculations of $\omega_i$ from (8) are very inaccurate.

In [4] one has suggested the method of elimination of spurious states, which allows one to improve essentially the approximation considered above. The secular equation is in this case of more complicated form, namely: (for $\kappa_n^{(2)} = \kappa_p^{(2)} = \kappa_{np}^{(2)} \equiv \kappa$ )

$$\begin{vmatrix} X_n^i + X_p^i - \frac{1}{2\kappa} & V_n^i & V_p^i & W_n^i & W_p^i \\ V_n^i & \sum_s \frac{1}{(2\epsilon(s))(4\epsilon(s)^2-\omega_i^2)} & 0 & \sum_s \frac{u_s^2 - v_s^2}{4\epsilon(s)^2-\omega_i^2} & 0 \\ V_p^i & 0 & \sum_\nu \frac{1}{(2\epsilon(\nu))(4\epsilon(\nu)^2-\omega_i^2)} & 0 & \sum_\nu \frac{u_\nu^2 - v_\nu^2}{4\epsilon(\nu)^2-\omega_i^2} \\ W_n^i & \sum_s \frac{u_s^2 - v_s^2}{4\epsilon(s)^2-\omega_i^2} & 0 & \sum_s \frac{\omega_i^2 - 4C_n^2}{2\epsilon(s)(4\epsilon(s)^2-\omega_i^2)} & 0 \\ W_p^i & 0 & \sum_\nu \frac{u_\nu^2 - v_\nu^2}{4\epsilon(\nu)^2-\omega_i^2} & 0 & \sum_\nu \frac{\omega_i^2 - 4C_p^2}{2\epsilon(\nu)(4\epsilon(\nu)^2-\omega_i^2)} \end{vmatrix} = 0 \qquad (9)$$

where

$$X_n^i = \sum_{ss'} \frac{f(ss')^2 u_{ss'}^2}{\epsilon(s) + \epsilon(s') - \frac{\omega_i^2}{\epsilon(s) + \epsilon(s')}} ,$$

$$V_n^I = \sum_s \frac{f(ss)\,C_n}{\epsilon(s)\,(4\epsilon(s)^2 - \omega_I^2)} ,$$

$$W_n^I = \sum_s \frac{f(ss)\,2C_n(E(s) - \lambda_n)}{\epsilon(s)(4\epsilon(s)^2 - \omega_I^2)} .$$

We notice that in case $f(ss') = f_0$ the diagonal terms fall out from eq. (9). Thus in this case the energy of the excited state with $K\pi = 0+$ does not depend on the diagonal terms what agrees with the above considerations. The diagonal matrix elements in (9) became less important as compared with (8).

The wave functions for the states with $K\pi = 0+$ can be easily obtained using the following expressions for $g^I_{ss'}$ and $w^I_{ss'}$

$$g^I_{ss'} = \frac{\sqrt{2}}{\sqrt{Z_n^I + Z_p^I}} \left\{ \frac{f(ss')\,u_{ss'}}{\epsilon(s) + \epsilon(s') - \frac{\omega_I^2}{\epsilon(s)+\epsilon(s')}} - \delta_{ss'} \frac{2C_n}{4\epsilon(s)^2 - \omega_I^2} \frac{\Gamma_n^I(s)}{\gamma_n^I} \right\}, \quad (10)$$

$$w^I_{ss'} = \frac{\sqrt{2}}{\sqrt{Z_n^I + Z_p^I}} \left\{ \frac{f(ss')\,u_{ss'}\,\omega_I}{(\epsilon(s) + \epsilon(s'))^2 - \omega_I^2} - \delta_{ss'} \frac{\omega_I C_n}{\epsilon(s)(4\epsilon(s)^2 - \omega_I^2)} \frac{\Gamma_n^I(s)}{\gamma_n^I} - \right. \quad (11)$$

$$\left. - \delta_{ss'} \frac{C_n}{\epsilon(s)\,\omega_I} \frac{\xi_n^I}{\gamma_n^I} \right\}$$

and analogous expressions for the proton system where

$$Z_n^I = Y_n^I + 2\frac{\omega_I C_n^2}{\gamma_n^I} \sum_s \frac{(\Gamma_n^I(s))^2}{\epsilon(s)(4\epsilon(s)^2 - \omega_I^2)} - 4\frac{\omega_I C_n^2}{\gamma_n^I} \sum_s \frac{f(ss)\Gamma_n^I(s)}{\epsilon(s)(4\epsilon(s)^2 - \omega_I^2)}$$

$$\Gamma_n^I(s) = \sum_{s_2 s_2'} \frac{f(s_2 s_2')}{\epsilon(s_2)(4\epsilon(s_2)^2 - \omega_I^2)} .$$

$$\frac{4(E(s'_2)-\lambda_n)(E(s_2)-\lambda_n)-4(E(s)-\lambda_n)(E(s_2)-\lambda_n)+4(E(s)-\lambda_n)(E(s'_2)-\lambda_n)+4C_n^2-\omega_i^2}{\epsilon(s'_2)(4\epsilon(s'_2)^2-\omega_i^2)}$$

$$\gamma_n^i = \sum_{ss'} \frac{4(E(s)-\lambda_n)(E(s')-\lambda_n)+4C_n^2-\omega_i^2}{\epsilon(s)(4\epsilon(s)^2-\omega_i^2)\epsilon(s')(4\epsilon(s')^2-\omega_i^2)},$$

$$\xi_n^i = \sum_{ss'} \frac{f(ss)}{\epsilon(s)(4\epsilon(s)^2-\omega_i^2)} \cdot \frac{4C_n^2-\omega_i^2+4(E(s)-\lambda_n)(E(s')-\lambda_n)}{\epsilon(s')(4\epsilon(s')^2-\omega_i^2)}.$$

The roots of the secular equation (9) as well as (4) were found with the aid of the electronic computer. The determinant of the 6-th order was expanded (the program was compiled for the general case of any values of $\kappa_n$, $\kappa_p$ and $\kappa_{np}$) and transformed to such a form in which the dependence on $\kappa_n$, $\kappa_p$ and $\kappa_{np}$ is clearly seen. Then, the first root was searched in the interval from $\omega = 0$ to the nearest pole $X_n(\omega)$ and $X_p(\omega)$, the second one between the first and the second poles and so on. The roots were found by the bisection, about eleven iterations were used to find the root with the accuracy $10^{-4}$.

5. The calculations of the energies of the beta and gamma vibrational states and the reduced probabilities of electromagnetic transitions have been performed in both regions of strongly deformed nuclei: $152 \leq A \leq 186$ and $228 \leq A \leq 254$. The wave functions and the schemes of the Nilsson potential one-particle levels have been used[10]. All the calculations in the region $152 \leq A \leq 186$ were made using the wave functions with the deformation $\delta = 0.3$ and in the region $228 \leq A \leq 254$ with $\delta = 0.2$ and for the same scheme of the one-particle levels in the neutron (proton) system for each nucleus in each region. In order that the calculations be most unambiguous, the change in the nuclear deformation is not taken into account. Therefore near the boundaries of the regions of strongly deformed nuclei the accuracy of the calculations becames worse because the equilibrium deformation changed but the behaviour of the one-particle levels of the average field was unaffected. The schemes of the one-particle levels of the average field, the values of the correlation functions and the chemical potentials which were used in the calculations are given in ref.[8] Notice that the correlation functions and the chemical potentials were obtained earlier in refs.[11,12].

The constant $\kappa$ of the quadrupole-quadrupole interaction is chosen so as to obtain the closest agreement of the calculated energies of the states with $K\pi = 2+$ with the corresponding experimental data. The values of $\kappa$ depend on the number of transitions in eqs. (4). We have taken into account all the transitions between one-particle states given in [8], about 100-120 transitions. If following [3] we assume that

$$\kappa = \frac{k}{A^{4/3}} \hbar\omega_0^0 = \frac{k^1}{A^{5/3}}$$

then we do not succeed in choosing the same value of $k$ for both regions of strongly deformed nuclei. So, in the region $152 \leq A \leq 186$ $k = 10$ and in the region $228 \leq A \leq 254$ $k = 12$. It is obvious that by increasing the number of transitions in the region $228 \leq A \leq 254$ we may obtain the same value of $k$ for both regions of strongly deformed nuclei. Note that for $\kappa = \frac{1,8}{A} \hbar\omega_0^0$ a rather good agreement it obtained between the calculated energies of the collective states and the experimental data in both regions.

If the energies of the beta vibrational states are calculated by (8) then to obtain even rough agreement with experiment it is necessary to introduce two values of $\kappa$, one for the description of the beta vibration energies and another for the description of the gamma vibrational energies.

6. Now we discuss the results of calculations of the energies of the lowest states with $I^{\pi}K = 0^+0$ and $I^{\pi}K = 2^+2$. The results of calculations and the corresponding experimental data are given in Fig. 1 and 2. The experimental values of the energies are taken from [11,13,14] and others.

In the region $154 \leq A \leq 182$ the calculated energies of the states with $K\pi = 2+$ are in a rather good agreement with the experimental data. The exception is $W^{182}$ what is due to the change in the deformation. However, if we take $\kappa = \frac{8}{A^{4/3}} \hbar\omega_0^0$ then we obtain the energy of the state with $K\pi = 2^+$ $W^{182}$ equal to1,2 MeV and in $W^{184}$ about 1 MeV. If we choose $\kappa = \frac{11}{A^{4/3}} \hbar\omega_0^0$ then we obtain very close agreement between theory and experiment for $Er^{166}$, $Er^{168}$ and for all the isotopes of $Yb$, however, for the isotopes of $Dy$ the calculated values of the energies of the states $2^+$ will be considerably lower than the experimental ones.

The agreement between the calculated energies of the states $0^+$ with the experimental data is rather satisfactory with the exception of the isotopes of $Hf$ and $W$. In these isotopes the energies of the states are very close to those of the corresponding poles and the blocking effect should therefore be taken into

account which is in this case very important. Taking into account the blocking effect we may obtain a good agreement with experiment in the isotopes of $Hf$ and $W$.

The problem of the relative behaviour of the energies of beta and gamma vibrational states is of interest. According to our calculations the energies of the states with $K\pi = 2+$ are considerably lower than those for the states with $K\pi = 0+$ for the isotopes $Dy$ and $Er$. The lowering of the energies of the gamma vibrational states lower than the energies of the beta vibrational states is defined by the behaviour of the average field levels and the appropriate wave functions. This lowering is in total agreement with experimental data and theoretically is first explained in the present paper.

The next paper will be devoted to the study of the structure of the collective states. However, we note that the majority of low energy states with $K\pi = 2+$ possess the pronounced collective properties. So, in $Er^{166}$ the four two-quasi-particle states give contribution to the collective state with $K\pi = 2+$ from 16% to 32% each. In a number of cases the low energy states with $K\pi = 2+$ are close to the two-quasi-particle ones. So, in $Yb^{172}$ the wave function of the lowest state with $K\pi = 2+$ contains contribution as large as 97,6% from the two-quasi-particle neutron state $5/2+[512] - 1/2+[521]$. Since the admixtures of other states are less than 3% then this state is very close to the two-quasi-particle one, which bears out the results obtained earlier in /11/. However, in $Yb^{172}$ there must be the second state with $K\pi = 2+$ which is by 200-400 KeV higher than the first one and possesses the collective properties.

The results of calculations of the energies of beta and gamma vibrational states in the region $228 \leq A \leq 254$ and the corresponding experimental data are given in Fig. 2. The calculated energies of the states with $K\pi = 2+$ are in rather good agreement with the corresponding experimental data. The disagreement in $Pu^{240}$ is due to the fact that the blocking effect was not taken into account. Whereas according to /12/ where the blocking effect was taken into account the two-quasi-particle state with $K\pi = 2+$ in $Pu^{240}$ has the energy 0.9 MeV. A very large lowering of the states with $K\pi = 2+$ in $Cf^{250}$ and in $Fm^{254}$ is due to a large matrix element of transition between states $7/2+[613]$ and $3/2+[611]$. If the number of transitions increases then the location of the levels $2+$ in $Cf^{250}$, $Fm^{254}$ and the behaviour of the energies of the states with $K\pi = 0+$ must be improved. In Fig. 2 the cross denotes the energies of the states with $K\pi = 2+$ for $Cf^{250}$, $Fm^{252}$ and $Fm^{254}$ calculated with $\kappa = \frac{11}{A^{4/3}} \hbar\omega_0^0$

the experimental data can be improved if we renounce the condition $\kappa_n = \kappa_p = \kappa_{np}$.

7. The reduced probabilities of the $E2$ transitions from states $K\pi = 2+$ to the ground ones are calculated by the formula:

$$B(E2) = 1{,}024A^{2/3} \cdot 10^{-53} \Big( e_{eff} \frac{\sum \frac{(f(ss')^2 + \bar{f}(ss')^2)\, u^2_{ss'}\,(\epsilon(s)+\epsilon(s'))}{(\epsilon(s)+\epsilon(s'))^2 - \omega^2_i}}{\sqrt{Y_n + \bar{Y}_n + Y_p + \bar{Y}_p}} + \Big. \tag{12}$$

$$\Big. + (e + e_{eff}) \frac{\sum \frac{(f(\nu\nu')^2 + \bar{f}(\nu\nu')^2)\, u^2_{\nu\nu'}\,(\epsilon(\nu)+\epsilon(\nu'))}{(\epsilon(\nu)+\epsilon(\nu'))^2 - \omega^2_i}}{\sqrt{Y_n + \bar{Y}_n + Y_p + \bar{Y}_p}} \Big)^2 .$$

The reduced transition probabilities $B(E2)$ are calculated with $e_p = e + e_{eff}$ and $e_n = e_{eff}$ and $e_{eff} = 0{,}8\,e$. In cases for which the experimental data are available [14] the ratio $B(E2)/B(E2)_{e.p.}$ is equal to 1 – 4. So, for example, for $Dy^{160}$ $B(E2)/B(E2)_{e.p.}$ it is 2.8 for $Dy^{162}$ – 3.2 for $Th^{230}$ 1.7 for $U^{238}$ – 1.4 and so on. However, for the state 2 + with an energy 1468 KeV in $Yb^{172}$ $B(E2)/B(E2)_{e.p.} = 0.04$ i.e. 30–50 times less than in the neighbourhood nuclei. This fact stresses once more the two-quasi-particle nature of this state. The experimental determination of $B(E2)$ as well as the finding of the second state with $K\pi = 2+$ in $Yb^{172}$ would be very desirable.

The difference between the energies of the states with $K\pi = 2+$ and the ratios $B(E2)/B(E2)_{e.p.}$ obtained by us and those calculated in [9] are accounted for by the fact that our calculations are carried out successively on the basis of the superfluid nuclear model while in [9] the phenomenological treatment of the unified model is combined with the microscopic approach of the superfluid model. This difference in the approaches was brought out, for example, in choosing the value of the constant of the quadrupole-quadrupole interaction. Then, the difference is due to the fact that in [9] the asymptotic wave functions of the Nilsson potential has been used, while in the present paper the exact wave functions of the Nilsson potential have been used. Here the difference is due not only to the fact that the matrix elements with the asymptotic wave functions are calculated insufficiently accurately but mainly to the fact that a large number of

poles in (4) is lost in working with the asymptotic wave functions. Note that in some cases near poles even with comparatively small values of matrix elements $f(ss')$ and $\bar{f}(ss')$ are very important. The difference in the values of $B(E2)/B(E2)_{s.p.}$ is mainly due to the fact that in [9] the calculations were made by the formula following from the phenomenological theory.

8. Thus, the calculated energies of the beta and gamma vibrational states agree rather well with the corresponding experimental data for the same values of the constants of the quadrupole-quadrupole interaction. The calculations show that in some cases the energies of the gamma vibrational states lie lower than the beta vibrational ones, so, e.g., in the middle of the region $154 \leq A \leq 182$. The behaviour of the energies of the quadrupole states is determined by the average nuclear field. In most cases the lowest states with $K\pi = 2+$ and $K\pi = 0+$ possess clearly pronounced collective properties and their energies become considerably low with respect to the nearest poles in the secular equations. In some cases the energies of the states with $K\pi = 2+$ and $K\pi = 0+$ are near the lowest poles and their wave functions are close to the two-quasi-particle ones. This fact has been noted earlier in [7] in studying the behaviour of the energies of the octupole states with $K\pi = 0-$.

It should be noted that the results of calculations essentially depend on the wave functions of the Nilsson potential. It is quite possible that some errors of calculations are due to the disadvantages characterising the wave functions of the Nilsson potential. However, a rather good general agreement between theory and experiment as to the energies of the states with $K\pi = 2+$ and $K\pi = 0+$ is one more evidence for the fact that the Nilsson potential allows one to describe rather well the average field of strongly deformed nuclei.

In conclusion we express our gratitude to N.N. Bogolubov and P. Vogel for interesting discussions and also K.M. Zhelesnova and L.V. Korneichuk for the aid in making the numerical calculations.

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Received by Publishing Department
on January 6, 1964.

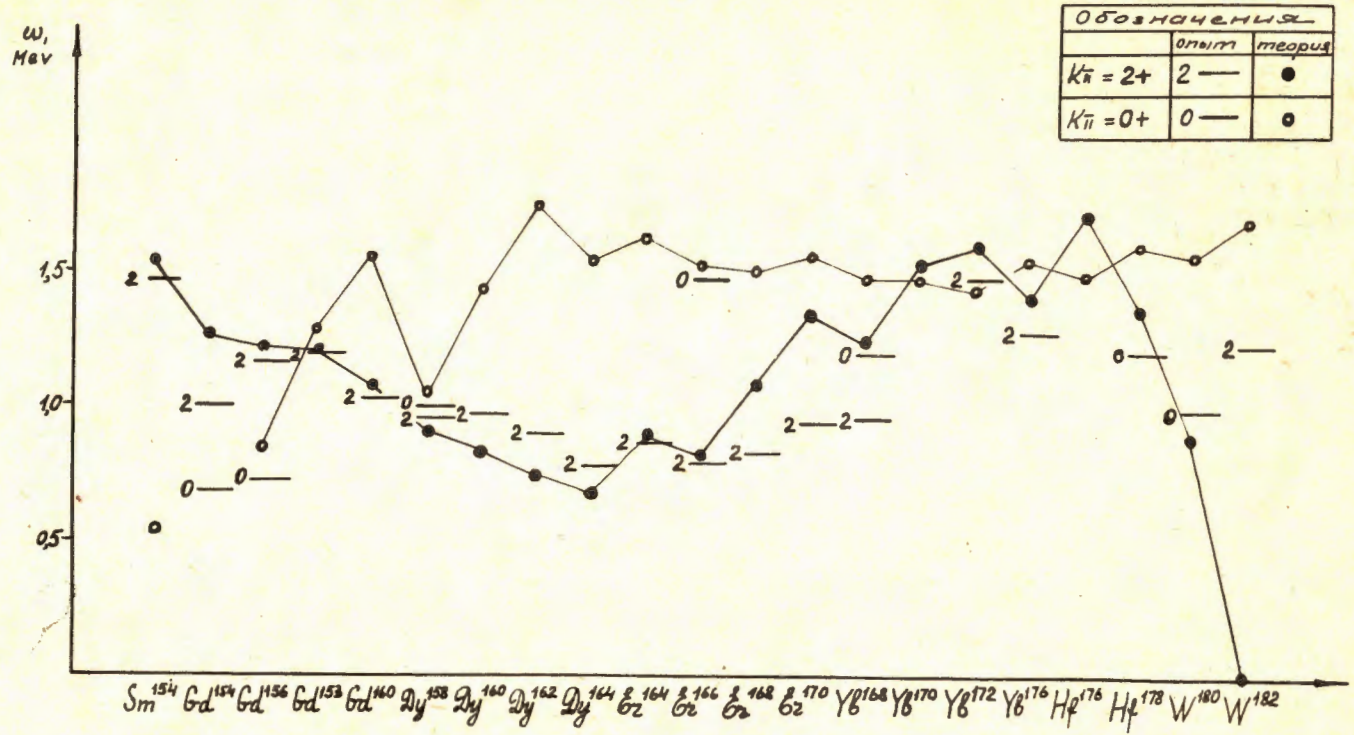


Рис. 1.

Энергии $\beta$ и $\gamma$ вибрационных состояний при $\varkappa^{(2)} = \frac{10}{A^{4/3}}\ \hbar\mathring{\omega}_0 = \frac{410}{A^{5/3}}$ Mev

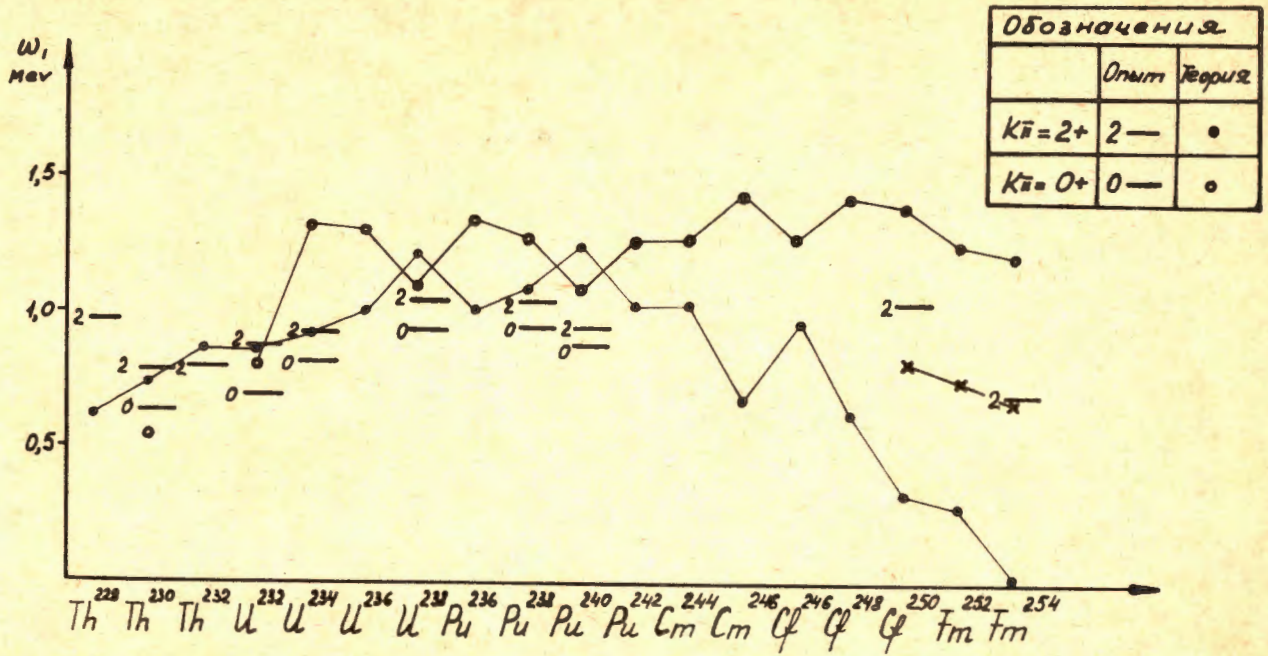


Рис. 2.

Энергии $\beta$ и $\gamma$ вибрационных состояний при $\varkappa^{(2)}=\frac{12}{A^{4/3}}\hbar\omega_0=\frac{480}{A^{5/3}}$ Mev.