ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

LABORATORY OF THEORETICAL PHYSICS

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E - 1520

ASYMPTOTIC RELATIONS BETWEEN SCATTERING AMPLITUDES IN LOCAL FIELD THEORY

Ann. of Phys., 1965, v 31, n 1, p 203-234

Дубна 1964

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Submitted to "Annals of Physics" and to "Uspekhi fiz nauk".

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Introduction

Most of the papers concerned with the investigation of the asymptotic properties of the matrix elements at high energies start either from some semiphenomenological assumptions, or rest on some particular hypotheses. So, for instance, in papers of Pomeranchuk et al [1,2] it is assumed (from the analysis of the experimental data) that the differential cross sections for the charge exchange scattering processes vanish at high energies. In some papers the hypothesis about the diffractional character of elastic scattering at high energies is suggested (see e.g.,[3]). In other papers[4] an assumption is made about the existence of new properties of symmetry in strong interactions which must be demonstrated only at energies much higher than particle masses. Finally, one should mention in this connection the papers in which it is postulated that the asymptotic behaviour of the scattering amplitude at high energies is determined by one Regge pole[5-7]. The validity of each of these papers depends on the results of its experimental verification. If experiment confirmed the predictions contained in such papers, this would mean a discovery of some new laws in strong interactions and therefore would be of great importance. On the other hand, the experimental refutation of the assumptions of any of these papers would not be of great interest, since this would mean the rejection of some particular hypotheses or theoretical speculations, but would not break the basic principles of quantum theory. Recently the second tendency–the tendency to a refutation is rather to be observed, (particularly, this concerns the hypothesis of only one Regge pole). It is natural that there appears an increasing interest to such assertions which are deduced only from the general principles of local field theory[8,9].

The purpose of this paper is to obtain some rigorous asymptotic relations between the observed quantities at high energies.

It should be remembered that among the basic postulates of relativistic quantum field theory such as: invariance under the inhomogeneous Lorentz group, existence of a complete system of physical states with positive energy and microcausality, there is an assumption of a mathematical nature: it is required that the elements of the scattering matrix should be tempered distributions (see, e.g.[10,11]). The importance of this requirement which seems purely formal at the first glance is understood in studying the growth of the analytical continuation of the matrix elements in the momentum space. It turns out to ensure, e.g., the polynomial boundedness of the Fourier transform of the retarded amplitude throughout the region of analiticity of this function (see[11], theorem 1). The weakening of this requirement leads to the fact that the analytical continuation of the matrix elements in the momentum space may have any growth (for instance, exponential) what would correspond to a non-renormalizable theory in the Lagrangian formalism[x]. At the same time it is not at all obligatory that such an increase would be observed for the cross sections along the real axis as it might seem in studying the n-th term of the perturbation theory series which is growing polynomially along any direction in the complex energy plane. Since the infinity in this case is an essential singularity the amplitude may grow exponentially along some directions, remaining bounded along other ones.

The assumption that the matrix elements are tempered distributions is included into the basic postulates of local field theory because one cannot obtain, without using this assumption, the dispersion relations - practically the only consequence from the general principles of relativistic quantum theory which can be experimentally checked. Besides, as is pointed out in[8], in order to compensate the exponential growth of the amplitude in the upper half-plane of the energy E it is necessary to introduce a factor of the type e^{iaE}, where the positive constant a has dimension of length and may be interpreted as a certain measure of the non-locality of the theory ("elementary length").

If the general principles of the local theory are supplemented by the physical assumption that the scattering amplitude does not oscillate, but has a definite growth (e.g., polynomial or logarithmic), when the energy tends to infinity at fixed momentum transfer, then it is possible to get a number of asymptotic relations between the matrix elements for different processes which can be verified experimentally. The first relation of such a kind - the equality of the total cross sections for particle and antiparticle interaction at high energies - was obtained by Pomeranchuk[12]. Various generalizations and justification of Pomeranchuk's statement may be found in[13-16]. A simple and rigorous proof of this statement under more general assumptions than in[12] was given by Meiman[17] on the basis of Phragmén-Lindelöf's theorem in the theory of analytical functions[18,19].

In the present paper[xx], Phragmén-Lindelöf's theorem is used to establish a number of asymptotic relations not only between the total but also between the

x/ See discussion of this problem in[9].

xx/ See also[20-22], where a part of the results discussed here was obtained. We review here some earlier results in order to make the present paper independent of the previous publications of the authors.

differential cross sections for different processes, as well as between different polarization effects. In Sec. 1, the scalar particle scattering is taken as an example to illustrate in detal the method of proof. In particular, the equality of the differential cross sections for particles and antiparticles is established.[x] In the following sections the method described in Sec.1 is applied to the study of more interesting cases from the physical point of view: scattering of particles with spin and form-factors. The main physical results are summarized in Sec. 7.

1. Asymptotic Properties of the Scattering Amplitude of Scalar Particles

1. Phragmén-Lindelöf's Theorem and Asymptotic Equality of Differential Cross Section

Consider the related processes of scalar particle scattering

$$a_1 + b_1 \to a_2 + b_2 \qquad \text{(I)}$$

and

$$\bar{a}_2 + b_1 \to \bar{a}_1 + b_2 , \qquad \text{(II)}$$

where the bar stands for the transition to the antiparticle. Let q_1 and p_1 be the momenta of particles a_1 and b_1 at the beginning of each process while q_2 and p_2 be their momenta at the end of the reaction; the masses of the particles a_1 and b_1 are denoted by m_1 and M_1 ($q_1^2 = m_1^2$, $p_1^2 = M_1^2$ in the process (I), $q_1^2 = m_2^2$, $p_1^2 = M_1^2$ in the process (II). The differential cross section for process (I) is expressed in terms of the invariant amplitude of this process as

$$\frac{d\sigma^I(s,t)}{dt} = \frac{\pi}{k_1 k_2}\frac{d\sigma^I}{d\Omega} = \frac{1}{64\pi s k_1^2} | T^I(s,t) |^2 . \qquad (1.1)$$

Here $s = (p_1 + q_1)^2$, $t = (p_1 - p_2)^2$; k_1 are the magnitudes of the three-dimensional momenta in the c.m.s.

$$k_1^2 = \frac{1}{4s}[s^2 - 2s(M_1^2 + m_1^2) + (M_1^2 - m_1^2)^2] . \qquad (1.2)$$

A similar formula holds for the cross section of process (II) either.

The amplitude of process (II) is connected with the amplitude $T^I(s,t)$ for real s and t by the crossing symmetry relation

x/ A part of the results of this section was obtained in another way (namely by a generalization of Pomeranchuk's method[12]) in a recent paper by Van Hove[23]. In[23], only scalar particle scattering is considered and it is assumed that the scattering amplitude behaves as s at $s \to \infty$ for all fixed momentum transfers t.

$$T^{II}(s,t;\ M_1^2, m_2^2;\ M_2^2, m_1^2) = T^I(u,t;\ M_1^2, m_1^2;\ M_2^2, m_2^2)^* \qquad (1.3)$$

(the star designates complex conjugation). Relation (1.3) is readily obtained from the equality

$$T^{II}(s',t;\ M_1^2, m_2^2; M_2^2, m_1^2) = T^I(u',t;\ M_1^2, m_1^2; M_2^2, m_2^2) \qquad (1.4)$$

valid for complex s' and u'.

Let us assume the masses m_i and M_i and the interactions of particles be such that one would be able to deduce, from the principles of the local theory, the analyticity of the amplitude $T^I(s,t)$ for fixed t in the complex s plane with cuts along the real axis. Besides these cuts, the amplitude has, as a rule, a finite number of poles along the real axis s. When one investigates the asymptotic behaviour at $s \to \infty$ it is convenient beforehand to subtract the pole terms from the amplitude; these terms have the well-known asymptotic behaviour $1/s$. In what follows we shall denote by $T^J(s,t)$ the amplitude with subtracted pole terms.

In order to cover a sufficiently large class of amplitudes which have a "regular" behaviour, we introduce an auxilliary notion. We call the function $\phi(s,t)$ admissible if at fixed t (from a certain interval) the function $1/\phi(s,t)$ 1) is analytical and less than any exponent $e^{\epsilon|s|}$, $\epsilon>0$, at $s\to\infty$ in the upper half-plane, 2) is continuous along the real axis and, 3) if

$$\lim_{s\to\infty}\frac{\phi(s,t)}{\phi(-s,t)} = e^{-i\pi a(t)}, \qquad (1.5)$$

where $a(t)$ is an arbitrary real function. An example of admissible function may be given by

$$\phi(s,t) = (s+i)^{a(t)}\,[\ln(s+i)]^{\beta(t)}\,[\ln\ \ln(s+i)]^{\gamma(t)} \ldots,$$

where $a(t)$, $\beta(t)$ and $\gamma(t)$ are real.

The following theorem holds:

T h e o r e m 1. Let for fixed t and for some choice of the admissible function $\phi(s,t)$ there exist the finite limits

$$V^I(t) = \lim_{s\to\infty}\frac{T^I(s,t)}{\phi(s,t)}, \qquad V^{II}(t) = \lim_{s\to\infty}\frac{T^I(s,t)^*}{\phi(-s,t)}. \qquad (1.6)$$

Then in local field theory, these limits coincide

$$V^I(t) = V^{II}(t). \qquad (1.7)$$

Hence the differential cross sections for processes (I) and (II) at high energy are equal

$$\lim_{s\to\infty}\frac{d\sigma^I(s,t)}{dt}\left[\frac{d\sigma^{II}(s,t)}{dt}\right]^{-1} = 1 \quad \text{or} \quad \frac{d\sigma^I(s,t)}{dt} \sim \frac{d\sigma^{II}(s,t)}{dt}. \qquad (1.8)$$

Proof. In virtue of the assumptions concerning the amplitude $T^I(s,t)$, the function

$$V(s,t) = \frac{T^I(s,t)}{\phi(s,t)} \qquad (1.9)$$

is analytical and does not exceed any exponent $e^{\epsilon|s|}$ in the upper half-plane s and is bounded along the real axis. Besides, it follows from (1.3), (1.6) that

$$\lim_{s\to\infty} V(s,t) = V^I(t), \qquad \lim_{s\to-\infty} V(s,t) = V^{II}(t). \qquad (1.10)$$

Therefore, one may apply Phragmén-Lindelöf's theorem /17-19/, which may be stated as follows:

T h e o r e m 2. I. Let $f(z)$ be an analytical function of $z = re^{i\theta}$, which is regular in the domain D confined between the two rays Γ_1 and Γ_2 which form the angle π/r with a vertex at the origin. Let further $f(z)$ be bounded on these rays ($|f(z)| \le C$ along Γ_1 and Γ_2). Then the following alternative takes place: either $|f(z)| \le C$ at all the points of the domain D, or there exists a sequence r_n tending to infinity so that

$$M(r) = \max_{\substack{|z|=r_n\\ z\subset D}} |f(z)| \ge \exp(\nu r_n^r), \quad \nu>0, \qquad (1.11)$$

If, on the other hand, the magnitude $|f(z)|$ is less than any exponent in the angle D, then the first possibility must be realized, i.e. $f(z)$ is bounded throughout the domain D by a constant.

II. Let the function $w = f(z)$ be regular and bounded in the angle D. We denote by E_i, $i=1,2$ the set of the limit points of w when $z\to\infty$ along the ray Γ_i. Then, either the sets E_1 and E_2 have a common point, or one of them is around the other, separating it thereby from the circumference $|w| = C$.

In particular, if there exist finite limits a_1 and a_2 when $z \to \infty$ along Γ_1 and Γ_2, then $a_1 = a_2 = a$, and $f(z) \to a$ uniformly in D when $z \to \infty$.

The function $V(s, t)$ (1.9) satisfies all the conditions of theorem 2 (in our case D is the upper half-plane s, $r = 1$). As far as this function is bounded by some power of s, the limits (1.10) have to coincide. Thus, equality (1.7) is proved.

If the rejected pole terms decrease at $s \to \infty$ quicker than the function $T^J(s,t)$ itself, then at $s \to \infty$ the formula (1.1) remains valid for this part of the amplitude as well. So, we obtain the asymptotic equality (1.8) between the differential cross sections. If the amplitude behaves at infinity as $1/s$ then a direct account of the pole terms shows that equality (1.8) holds true also for this case. Theorem 1 is proved.

2. The Case of Elastic Scattering

In the particular case of elastic scattering (in this case $m_1 = m_2 = m$, $M_1 = M_2 = M$) the corollary of theorem I about the asymptotic equality of the differential cross sections may be obtained if weaker requirements are imposed. Let us assume that there exist only the limits

$$\lim_{s \to \infty} \frac{64 \pi s k^2}{|\phi(s,t)|^2} \frac{d\sigma^J(s,t)}{dt} = [a^J(t)]^2, \quad J = I, II, \qquad (1.12)$$

and that the imaginary part of the amplitude is non-negative for small t and $s \to \infty$. This second assumption seems quite natural in view of the following heuristic argument. Owing to the unitary condition and the positive definiteness of the metric in the Hilbert space, all the coefficients $Im\ f_\ell^J(s)$ in the expansion

$$\frac{1}{8\pi\sqrt{s}} Im\ T^J(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, Im\ f_\ell^J(s) P_\ell\left(1 + \frac{t}{2k^2}\right) \qquad (1.13)$$

are non-negative. On the other hand $\frac{t}{2k^2} \to 0$ at $s \to \infty$ and since $P_\ell(1) = 1$ each term of the series (1.13) becomes non-negative at s large enough. In the case of physical interest the series (1.13) converge non uniformly with respect to s x/, so that we cannot derive from here that the imaginary part

x/ If the series (1.13) were uniformly convergent, then the asymptotic behaviour of the absorptive part at $s \to \infty$ would not depend on t (and the imaginary part would coincide with that at $t = 0$).

$Im\ T^J(s,t) \geq 0$ for large s. But if we assume that the main contribution in the series (1.13) is given by the terms with $\ell \sim k$, then for large k (and ℓ)

$$P_\ell\left(1 + \frac{t}{2k^2}\right) \approx J_0\left(\frac{\ell}{k}\sqrt{-t}\right).$$

The Bessel function is non-negative for $\frac{\ell}{k}\sqrt{-t} < j_1 \approx 2{,}4048$ Hence, under our assumptions $Im\ T^J(s,t) \geq 0$ for sufficiently small t and large s. We also mention that the imaginary part of the amplitude is non-negative in the part of the non-physical region $t \geq 0$, in which the series (1.13) converge (because $P_\ell(x) \geq 1$ for $x \geq 1$).

We will show that under the assumptions made the limits $a^I(t)$ and $a^{II}(t)$ coincide. Indeed, the assumption about the existence of the limits (1.12) means that the absolute value of the function (1.9) tends to definite limits at $s \to \pm\infty$. The limit sets E_1 and E_2 for the function $V(s,t)$ itself lie on the two concentric circumferences: $|V(s,t)| = a^I(t)$ and $|V(s,t)| = a^{II}(t)$. In virtue of the second part of theorem 2 either the manifolds E_1 and E_2 intersect, what is possible only if $a^I(t) = a^{II}(t)$, or one of them surrounds the other, i.e. consists of all the points of the circumference $|V(s,t)| = \max[a^I(t), a^{II}(t)]$. However, in the case under consideration the second possibility is not realized, since $Im\ T^I(s,t)$ is non-negative at $s \to \infty$, each of the sets E_i may occupy not more than half a circumference, and, therefore, cannot surround the second one. It follows that $a^I(t) = a^{II}(t)$. Therefore the asymptotic equality of the differential cross sections (1.8) is valid.

Other conditions for which the asymptotic equalities between the differential cross sections hold are given in a recent paper by Meiman/24/. The results of this paper may be stated in the following way. Let the elastic scattering amplitudes $T^J(s,t)$ have no real zeros and let the following integrals be convergent

$$\int_{-\infty}^{\infty} |\ln|T^J(s,t)|| \frac{ds}{1+s^2} < \infty, \qquad J = I, II.$$

It follows from here that the products $\pi(s,t) = \prod \frac{1 - s/s_k^J}{1 - s/s_k^{J*}}$ are convergent, where s_k^J are zeros of the amplitude $T^J(s,t)$ in the upper half-plane s. Further, let there exists the limit of the ratio of the absolute values of the amplitudes for processes (I) and (II) at $s \to \infty$

$$\lim_{s\to\infty} \left| \frac{T^{I}(s,t)}{T^{II}(s,t)} \right| = \gamma .$$

It is assumed that this ratio is limited from above and from below along the whole real axis s by some positive function of t. Then $\gamma = 1$, if the argument (the phase) of the ratio

$$\frac{T^{I}(s,t)}{T^{II}(s,t)^{*}} \cdot \frac{\pi^{II}(s,t)^{*}}{\pi^{I}(s,t)}$$

increases (or decreases) slower than $\ln s$ (respectively, $1/\ln s$) at $s\to\infty$; γ has a finite positive value not equal to unity, if the phase of this ratio grows (decreases) as $\ln s(1/\ln s)$; $\gamma = 0$ or 1, if the phase of the above ratio increases (decreases) quicker than $\ln s(1/\ln s)$.

If it is additionally assumed 1) that the function in (1.5) satisfies the condition

$$a(0) = 1 \tag{1.14}$$

(this is so, if the forward elastic scattering amplitude behaves as $s(\ln s)^{\beta(0)}$ at $s\to\infty$) and 2) that the real part of the amplitude increases not faster than its imaginary part, then from theorem I follows Pomeranchuk's theorem about the asymptotic equality of the total cross sections for particle and antiparticle interaction. It suffices to note that in virtue of the optical theorem, the total cross sections $\sigma^{J}_{tot}(s)$ corresponding to processes (I) and (II) are expressed in terms of the imaginary parts of the amplitudes of these processes by the following formula

$$\sigma^{J}_{tot}(s) = \frac{1}{2k\sqrt{s}} \operatorname{Im} T^{J}(s,0), \quad J = I, II. \tag{1.15}$$

If a is a neutral, scalar (or pseudoscalar) particle coinciding with its antiparticle, then the amplitudes of processes (I) and (II) coincide

$$T^{I}(s,t) = T^{II}(s,t) = T(s,t), \tag{1.16}$$

while in the case of forward scattering (with $a(0) = 1$) theorem I leads to

$$\lim_{s\to\infty} \frac{T(s,0)}{T(s,0)^{*}} = -1. \tag{1.17}$$

Using (1.17) we conclude that the amplitude is purely imaginary at $s\to\infty$

$$\lim_{s\to\infty} \frac{\operatorname{Re} T(s,0)}{\operatorname{Im} T(s,0)} = 0 . \tag{1.18}$$

The following asymptotic relation between the differential and the total cross section of the process under consideration may be drawn from (1.1), (1.15) and (1.18):

$$\left.\frac{d\sigma(s,t)}{dt}\right|_{t=0} \approx \frac{1}{16\pi} [\sigma_{tot}(s)]^{2}. \tag{1.19}$$

3. The Upper Bounds for Cross Sections at High Energies

All the results obtained so far are correct for arbitrary high power (and logarithmic) growth of the amplitude. However, our assumptions together with the unitarity condition for the partial amplitudes in (1.13)

$$|f^{J}_{\ell}(s)|^{2} \le \operatorname{Im} f^{J}_{\ell}(s) \le 1 \tag{1.20}$$

enable us to prove that the growth of the elastic scattering amplitudes is bounded.

Using the analiticity of the absorptive part in the Lehmann ellipse with semi-major axis $\cos\theta_0 = 1 + \frac{c}{k^2 s}$, $c>0$ and foci at ± 1, Greenberg and Low /25/ have found the following upper bounds for the scattering amplitude x)

$$|T(s,0)| < A\, s^{2}(\ln s)^{2},$$
$$|T(s,t)| < B\, \frac{s^{7/4}(\ln s)^{3/2}}{|t|^{1/4}}. \tag{1.21}$$

These inequalities give the following limitations on the growth of the cross sections

$$\sigma_{tot}(s) < A_1\, s(\ln s)^{2},$$
$$\frac{d\sigma(s,t)}{dt} < B_1 \frac{s^{3/2}(\ln s)^{3}}{|t|^{1/2}}. \tag{1.22}$$

In proving Pomeranchuk's theorem about the equality of the total scattering cross sections for particles and antiparticles we supposed that the forward elastic scattering amplitude behaves as s (multiplied by a certain power of $\ln s$). Such a behaviour corresponds to the upper bound for the scattering amplitude ob-

x/ In fact, in /25,27/ an estimate of the amplitude growth is given for fixed angle θ. Formulae (1.21) and (1.23) valid for fixed $t \neq 0$ are obtained in a similar way.

tained by Froissart on the basis of Mandelstam representation. Froissart's results were obtained under a weaker assumption by Martin[27] who showed that if the amplitude is analytical with respect to $\cos\theta$ in a certain ellipse with semi-major axis $\cos\theta_0 = 1 + \frac{t_0}{2k^2}$, $t_0 > 0$ then

$$|T(s,0)| < A\, s(\ln s)^2 ,$$
$$|T(s,t)| < B\, \frac{s(\ln s)^{3/2}}{|t|^{1/4}} , \tag{1.23}$$

and therefore

$$\sigma_{tot}(s) < A_1 (\ln s)^2 ,$$
$$\frac{d\sigma(s,t)}{dt} < B_1 \frac{(\ln s)^3}{|t|^{1/2}} . \tag{1.24}$$

Note that it is not possible to improve the upper bound (1.23) for the forward scattering amplitude even if the validity of the Mandelstam representation is assumed. On the contrary, the upper bound on the amplitude at fixed non-zero angle may be improved if the analiticity of the amplitude in a wider domain[27] is assumed.

II. Asymptotic Properties of the Meson-Baryon Scattering Amplitudes

1. Symmetry Properties of the Amplitude

Consider processes (I) and (II) for the case when the particles a_1 have spin 0, whereas the particles b_1 -spin 1/2. Then the amplitudes of these processes may be put as[x]

$$T^J = \bar u(p_2)\,[\,A^J(s,t) + \frac{\hat q_1 + \hat q_2}{2} B^J(s,t)\,]\, u(p_1), \quad J = I, II, \tag{2.1}$$

if the relative parity of particles in the initial state I_i coincides with the relative parity of particles in the final state I_f, or in the form

$$T^J = \bar u(p_2)\,[\,A^J(s,t) + \frac{\hat q_1 + \hat q_2}{2} B^J(s,t)\,]\,\gamma_5\, u(p_1), \quad J = I, II, \tag{2.2}$$

if $I_i = -I_f$. In (2.1) and (2.2) p_1 and q_1 are the 4-momenta of the fermion and boson in the initial state (b_1 and a_1 for process (1) and b_1 and $\bar a_2$ for process (II)), whereas p_2 and q_2 are the 4-momenta in the final state.

[x] We use the representation of the γ-matrices, in which γ_0 is Hermitian, and γ_i, $i = 1,2,3$, are antithermitian, $\gamma_0^2 = 1$, $\gamma_i^2 = -1$. As usual $\hat q = \gamma_0 q_0 - \sum_i \gamma_i q_i$.

The invariant amplitudes $A^J(s,t)$ and $B^J(s,t)$ of processes (I) and (II) are connected by the crossing symmetry relations. Now we proceed to the derivation of these relations[x].

The amplitudes T^J are expressed in terms of the variational derivatives of the S-matrix by

$$T^I(p_1,q_1;p_2,q_2) = -2^4 i \int d^4x\, e^{-i(q_1+q_2)x} \langle b_2(p_2)|\frac{\delta^2 S}{\delta\phi^*_{a_2}(x)\,\delta\phi_{a_1}(-x)}|b_1(p_1)\rangle, \tag{2.3}$$

$$q_1^2 = m_1^2, \quad q_2^2 = m_2^2,$$

$$T^{II}(p_1,q_1;p_2,q_2) = -2^4 i \int d^4x\, e^{i(q_1+q_2)x} \langle b_2(p_2)|\frac{\delta^2 S}{\delta\phi^*_{a_2}(x)\,\delta\phi_{a_1}(x)}|b_1(p_1)\rangle, \tag{2.4}$$

$$q_1^2 = m_2^2, \quad q_2^2 = m_1^2.$$

In the physical region each of the integrals (2.3) and (2.4) should be understood as a limit when the vector $q_1 + q_2$ has a small imaginary part which lies on the future light cone. Therefore, when the vectors p_1 and q_1 are in the physical region of any of processes (I) and (II) expressions (2.3) and (2.4) are Hermitian conjugate i.e.,

$$T^{II}(p_1,q_1;p_2,q_2) = T^{I'}(p_2,-q_1;p_1,-q_2)^*, \tag{2.5}$$

where (I') is the process inverse to (I)

$$a_2 + b_2 \rightarrow a_1 + b_1 . \tag{I'}$$

Processes (I) and (I') are related by the space-time inversion: if

$$T^I(p_1,q_1;p_2,q_2) = \bar u(p_2)\, M^I(p_1,q_1;p_2,q_2)\, u(p_1), \tag{2.6}$$

[x] It is customary to write the crossing symmetry relations for the case of elastic π-meson scattering on the nucleon (see e.g.,[28]). The general case when all the four particles a_i and b_i may be different is treated here by the method used in[29] in studying the symmetry properties of the nucleon-nucleon scattering amplitudes.

then

$$T^{I'}(p_2,q_2;p_1,q_1)=\eta\, u(p_2)[BM^{I}(p_1,q_1;p_2,q_2)B^{-1}]^{T}u(p_2), \qquad (2.7)$$

where the matrix B has the properties

$$B\gamma_\mu B^{-1}=\gamma_\mu^T, \qquad B^T=-B, \qquad (2.8)$$

the upper index T denotes transposition of the matrices, and η is a phase factor, $|\eta|=1$. In the case of elastic scattering $\eta=1$. From (2.5) and (2.7), we get

$$M^{I}(p_1,q_1;p_2,q_2)=\eta\gamma_4[BM^{I}(p_1,-q_2;p_2,-q_1)B^{-1}]^{*}\gamma_4 \qquad (2.9)$$

Relation (2.9) may be written down in a more customary form if we express B through the charge conjugation matrix C

$$B=C^{-1}\gamma_5, \quad C^{-1}\gamma_\mu C=-\gamma_\mu^T, \quad C^T=-C. \qquad (2.10)$$

According to the CPT-theorem /30/

$$\gamma_5 M^{I}(p_1,-q_2;p_2,-q_1)\gamma_5=M^{I}(-p_1,q_2;-p_2,q_1). \qquad (2.11)$$

From (2.9)-(2.11) follows the crossing symmetry relation.

$$M^{II}(p_1,q_1;p_2,q_2)=\eta\gamma_4[C^{-1}M^{I}(-p_1,q_2;-p_2,q_1)C]^{*}\gamma_4. \qquad (2.12)$$

One can easily get the crossing relations between the invariant amplitudes $A^J(s,t)$ and $B^J(s,t)$ processes (I) and (II). Whether the relative parities of particles in the initial and final states coincide or are opposite in sign, we find

$$A^{II}(s,t)=\eta^* A^{I}(u,t)^*, \quad B^{II}(s,t)=-\eta^* B^{I}(u,t)^*, \qquad I_i=I_f,$$
$$A^{II}(s,t)=-\eta^* A^{I}(u,t)^*, \quad B^{II}(s,t)=\eta^* B^{I}(u,t)^*, \qquad I_i=-I_f. \qquad (2.13)$$

In the following we shall put $\eta=1$. This will not lead to a change in the final results since only the product $\eta\eta^*=1$ enters into the expressions for the differential cross sections and polarizations.

Together with processes (I) and (II) in the case when b_i are particles with spin we shall consider also the process

$$a_1+\bar{b}_2\to a_2+\bar{b}_1. \qquad (III)$$

The amplitude of this process can be written down as

$$T^{III}(p_1,q_1;p_2,q_2)=\bar{u}(p_2)M^{III}(p_1,q_1;p_2,q_2)u(p_1). \qquad (2.14)$$

Similarly to what has been said it is possible to obtain the following crossing symmetry relation between the amplitudes M^{III} and M^{I}

$$M^{III}(p_1,q_1;p_2,q_2)=\gamma_4 M^{I}(p_2,-q_1;p_1,-q_2)^{+}\gamma_4, \qquad (2.15)$$

or in terms of the invariant amplitudes

$$A^{III}(s,t)=A^{I}(u,t)^*, \quad B^{III}(s,t)=-B^{I}(u,t)^* \qquad I_i=I_f,$$
$$A^{III}(s,t)=-A^{I}(u,t)^*, \quad B^{III}(s,t)=-B^{I}(u,t)^* \qquad I_i=-I_f. \qquad (2.16)$$

2. Asymptotic Equalities between the Differential Cross Sections

The differential cross section for process (I) is equal to

$$\frac{d\sigma^I(s,t)}{dt}=\frac{1}{64\pi s k_1^2}F^I(s,t) \qquad (2.17)$$

where

$$F^I(s,t)=[(M_2\pm M_1)^2-t]\,|A^I(s,t)|^2+\frac{(u-s)^2-(m_1^2-m_2^2)-[t-2(m_1^2+m_2^2)][t-(M_2\mp M_1)^2]}{4}$$
$$|B^I(s,t)|^2+[(M_2\pm M_1)(s-u)+(M_2\mp M_1)(m_2^2-m_1^2)]\,\mathrm{Re}\,A^I(s,t)B^I(s,t)^*, \qquad (2.18)$$

the upper sign corresponds to the case of identical relative parities $I_i=I_f$, while the lower one - to the case of opposite parities, $I_i=-I_f$.

For the fixed t and $s\to\infty$ (2.18) assumes the form

$$F^I(s,t)\approx|(M_2\pm M_1)A^I(s,t)+sB^I(s,t)|^2-t\,|A^I(s,t)|^2. \qquad (2.19)$$

The differential cross section for process (II) is readily obtained from (2.17) and (2.18) by the substitution $A^{II}(s,t)$ and $B^{II}(s,t)$ for $A^I(s,t)$ and $B^I(s,t)$ and $m_1\leftrightarrow m_2$, while the differential cross section for process (III) - by the substitution $A^{III}(s,t)$ and $B^{III}(s,t)$ for $A^I(s,t)$ and $B^I(s,t)$ and $M_1\leftrightarrow M_2$.

Let us prove the asymptotic equality between the cross sections for processes (I), (II) and (III) for fixed $t\le 0$ and $s\to\infty$. If only one of the two amplitudes $A^J(s,t)$ and $B^J(s,t)$ gives the main contribution to the cross sections for large s then it suffices to consider this amplitude, and the asymptotic equality between the cross sections follows immediately from theorem 1. Therefore, we have to treat the general case when both amplitudes $A^J(s,t)$ and $B^J(s,t)$ give contributions of the same order to the asymptotic behaviour of the cross sections In this case, for some choice of the admissible function $\phi(s,t)$ there exist (by assumption) the finite limits

$$U_{\pm}(t) = \lim_{s\to\pm\infty} \frac{A^I(s,t)}{\phi(s,t)}, \qquad V_{\pm}(t) = \lim_{s\to\pm\infty} \frac{s\,B^I(s,t)}{\phi(s,t)}. \tag{2.20}$$

In virtue of theorem 2 the limiting values (2.20) are equal

$$U_+(t) = U_-(t), \qquad V_+(t) = V_-(t) \tag{2.21}$$

and, therefore, if we take into account the crossing symmetry relations (2.13) and (2.16), we get

$$\lim_{s\to\infty} \frac{A^I(s,t)}{A^{II}(s,t)^*} = \lim_{s\to\infty} \frac{B^I(s,t)}{B^{II}(s,t)^*} = \pm e^{-i\pi\alpha(t)},$$
$$\lim_{s\to\infty} \frac{A^I(s,t)}{A^{III}(s,t)} = \pm \lim_{s\to\infty} \frac{B^I(s,t)}{B^{III}(s,t)} = \pm e^{-i\pi\alpha(t)}, \tag{2.22}$$

where again the upper sign corresponds to the case $l_i = l_f$; whereas the lower one-to the case $l_i = -l_f$.

From these asymptotic relations between the amplitudes of processes (I), (II) and (III) follows the asymptotic equality between the cross sections of these processes for fixed t and $s\to\infty$. For instance, in such a way we obtain the asymptotic equalities between the differential cross sections for the processes

$$\begin{array}{lcl} \pi^+ + p \to \pi^+ + p & \text{and} & \pi^- + p \to \pi^- + p, \\ K^+ + p \to K^+ + p & \text{and} & K^- + p \to K^- + p, \\ \pi^+ + p \to K^+ + \Sigma^+ & \text{and} & K^- + p \to \pi^- + \Sigma^+, \\ K^- + p \to K^0 + \Xi^0 & \text{and} & \bar{K}^0 + p \to K^+ + \Xi^0 \end{array} \tag{2.23}$$

(if the Ξ -hyperon has spin 1/2),

$$\Sigma^+ + He \to p + He_\lambda \quad \text{and} \quad \bar{p} + He \to \bar{\Sigma}^+ + He_\lambda.$$

3. Asymptotic Relations between Fermion Polarizations in Final State

Suppose that fermions in the initial state are unpolarized. Denote by n_μ the unit space-like four-vector proportional to $\epsilon_{\mu\alpha\beta\gamma}\, p_{1\alpha}\, q_{1\beta}\, p_{2\gamma}$, where $\epsilon_{\mu\alpha\beta\gamma}$ is the totally antisymmetrical tensor. In the centre-of-mass system $n_0 = 0$, $\vec{n} = [\vec{p}_1 \times \vec{p}_2]/|[\vec{p}_1 \times \vec{p}_2]|$. The polarization state of the fermions in the final state is characterized by the four-dimensional polarization vector ξ^J_μ proportional to the unit vector n_μ.

$$\xi^J_\mu = P^J(s,t)\, n_\mu. \tag{2.24}$$

Calculating $P^J(s,t)$, we get

$$P^J(s,t) = 2s\sqrt{-t}\; C(s,t)\, \frac{Im\, A^J(s,t)^*\, B^J(s,t)}{F^J(s,t)}, \tag{2.25}$$

where the function $C(s,t)$ tends to unity as $s\to\infty$ at fixed t (sec /22/ and the expression (2.35) for the case of equal masses). For process (I) the function $F^I(s,t)$ is determined by (2.18), and at large s - by (2.19), as for processes (II) and (III) the functions $F^J(s,t)$ are obtained from (2.18) with modifications indicated after formula (2.19).

As was shown for fixed t and $s\to\infty$ the functions $F^J(s,t)$are equal for all processes (I), (II) and (III). Therefore, in studying the polarization it is sufficient to consider the quantity $Im\, A^J(s,t)^*\, B^J(s,t)$. It follows from the asymptotic relations (2.22) that

$$\lim_{s\to\infty} \frac{Im\, A^I(s,t)^* B^I(s,t)}{Im\, A^{II}(s,t)\, B^{II}(s,t)} = \pm \lim_{s\to\infty} \frac{Im\, A^I(s,t)^* B^I(s,t)}{Im\, A^{III}(s,t)^* B^{III}(s,t)} = -1, \tag{2.26}$$

where like in (2.22) the sign "+" corresponds to the case $l_i = l_f$, while the sign "-" to the case $l_i = -l_f$.

Thus it was show that for fixed t and $s\to\infty$ the polarizations of fermions in the final states of processes (I), (II) and (III) are connected by

$$\begin{array}{lll} P^I(s,t) = -P^{II}(s,t) = -P^{III}(s,t) & \text{if} & l_i = l_f \\ P^I(s,t) = -P^{II}(s,t) = P^{III}(s,t) & \text{if} & l_i = -l_f. \end{array} \tag{2.27}$$

The results obtained are applicable, in particular, to processes (2.23)

For instance, the polarizations of the recoil protons in the processes

$$\pi^+ + p \to \pi^+ + p \quad \text{and} \quad \pi^- + p \to \pi^- + p$$
$$K^+ + p \to K^+ + p \quad \text{and} \quad K^- + p \to K^- + p$$

for the same values of energy s and momentum transfer t are equal by absolute value and opposite in sign. For the polarizations of Σ^+ - hyperons in the processes

$$\pi^+ + p \to K^+ + \Sigma^+ \quad \text{and} \quad K^- + p \to \pi^- + \Sigma^+$$

or for the polarization of Ξ^o -hyperon in the processes

$$K^- + p \to K^o + \Xi^o \quad \text{and} \quad \bar{K}^o + p \to K^+ + \Xi^o$$

there holds the same asymptotic relations irrespective of the relative parities of particles. However, the asymptotic relation between the nucleon and antihyperon polarizations in the last pair of processes (2.23) depends on the relative parity $I_{\Sigma\Lambda}$: the polarizations of p and $\bar{\Sigma}^+$ are equal by absolute value and opposite in sign, if this parity is $+1$, and they are equal both by absolute value and in sign if the parity $I_{\Sigma\Lambda}$ is -1.

We considered the general case when both the invariant amplitudes contribute to the asymptotic behaviour of the cross sections. As one can easily see from formula (2.31) the polarizations in this case may tend to nonvanishing limits at $s \to \infty$. If only one of these amplitudes gives a contribution to the cross sections at large s, then the polarizations are tending to zero.

Now we will show that there exists a number of processes in which the fermion polarizations in the final states are tending to zero at $s \to \infty$ for fixed (non-vanishing) t irrespective of the relative behaviour of the invariant amplitudes. These are the processes which transform into themselves in the crossing transformation, i.e., the processes for which $\bar{a}_2 = a_1$. In this case, processes (1) and (II) coincide, so that $P^I = P^{II}$. On the other hand, by (2.27), $P^I(s,t) = -P^{II}(s,t)$ irrespective of the relative parities of particles. Therefore, in this case

$$P^I(s,t) = P^{II}(s,t) = 0$$

Thus, for example, the fermion polarizations in the final states of the processes

$$K^o_{1,2} + p \to K^o_{1,2} + p$$

and

$$K^- + p \to K^+ + \Xi^-$$

(if Ξ -hyperon spin is 1/2) are tending to zero at $s \to \infty$ for fixed t irrespective of the relative parities of particles even in the case when both the invariant amplitudes give contributions to the asymptotic behaviour of the cross

sections. If the isotopic invariance is fulfilled then the polarization of the recoil neutron is also tending to zero in the charge exchange process

$$\pi^- + p \to \pi^o + n .$$

Indeed, we saw that the polarizations of the recoil neutrons in this process and in the process

$$\pi^o + p \to \pi^- + n$$

are opposite at $s \to \infty$. However, it follows from the isotopic invariance that the matrix elements of these processes coincide (up to a sign). Therefore, the neutron polarizations in both processes must be equal to each other. It follows from here that at $s \to \infty$ the polarizations of the recoil neutrons of the processes under consideration tend to zero.

To check the obtained asymptotic relations between the polarizations it is probably simpler to use the polarized nucleon target and to measure the asymmetry parameter η^J for different processes which is known to be equal to the polarization (2.25) if the relative parities I_1 and I_t are the same, and have the opposite sign if $I_1 = -I_t$ /31/. Moreover, between the polarizations and the asymmetry parameters in processes (I) and (III) there exist the following relations

$$P^I = -\eta^{III}, \quad P^{III} = -\eta^I$$

irrespective of the parities I_1 and I_t. (2.28)

4. The Complete Set of Measurements for Elastic Meson-Nucleon Scattering

In the case of zero spin meson scattering on the nucleon (with spin 1/2) the complete set of measurements has to yield three real quantities (for each value of the variables s and t)x/. As such quantities one can choose the differential cross section (2.17)-(2.18), the polarization $P^J(s,t)$ of the fermion in the final state by scattering on the unpolarized target (2.25) and the polarization by scattering on the polarization by scattering on the polarized target with the polarization vector $\vec{\zeta}$. The three-dimensional vector of the nucleon polarization in the final state in the process of elastic of meson scattering on a polarized nucleon target in the centre-of-mass system is equal to

$$\vec{\xi} = P^J(s,t)\vec{n} + \vec{\zeta} + Q^J(s,t)[(\vec{\zeta}\vec{n})\vec{n} - \vec{\zeta}], \quad J = I, II, \qquad (2.29)$$

where $P^J(s,t)$ is given by the formula (2.25) and

x/ In /32/, the complete set of measurements is restricted to two quantities since at the energy below the threshold for inelastic processes the remaining quantities can be determined from the elastic unitarity condition.

$$Q^J(s,t) = -32\,t\,C^2(s,t)\left[\frac{s}{(\sqrt{s}+M)^2 - m^2}\right]^2 \frac{|A^J(s,t) - (\sqrt{s}+M)\,B^J(s,t)|^2}{F^J(s,t)}, \quad (2.30)$$

$F^J(s,t)$ is given by (2.18) (and the indications after formula (2.19)),

$$C^2(s,t) = -\frac{su - (M^2 - m^2)^2}{s^2} \to 1, \qquad s \to \infty. \quad (2.31)$$

As a third measurable quantity (along with $\frac{d\sigma^J}{dt}(s,t)$ and $P^J(s,t)$) one can take the quantity $Q^J(s,t)$ in virtue of (2.29). It can be easily seen that under the assumptions we have made the asymptotic values of the quantities $Q^J(s,t)$ for processes I and II must coincide at $s \to \infty$.

In the case of elastic meson-nucleon scattering under consideration the results obtained may be also derived from some weaker assumptions: instead of the existence of two complex (i.e., four real) limits (2.20) it suffices to assume the existence of the limits for the three experimentally measurable quantities $\frac{d\sigma^J}{dt}(s,t)$, $P^J(s,t)$ and $Q^J(s,t)$ or, what is the same, the existence of the limits (at $s \to \infty$) for the quantities

$$\left|\frac{f^J(s,t)}{\phi(s,t)}\right|, \quad \left|\frac{A^J(s,t)}{\phi(s,t)}\right| \quad \text{and} \quad \arg\left[\frac{A^J(s,t)}{f^J(s,t)}\right], \quad (2.32)$$

where

$$f^J(s,t) = 2M\,A^J(s,t) + (s - M^2 - m^2)\,B^J(s,t), \quad (2.33)$$

provided that $Im\, f^J(s,t) \geq 0$ at $s \to \infty$. This latter assumption is reasonable since

$$f^J(s,t) = \sum_{\ell=0}^{\infty} [(\ell+1) f_\ell^{(+)} + \ell f_\ell^{(-)}]\, P_\ell\left(1 + \frac{t}{2k^2}\right) - \frac{t}{2k^2} \sum_{\ell=1}^{\infty} [f_\ell^{(+)} - f_\ell^{(-)}]\, P'_\ell\left(1 + \frac{t}{2k^2}\right), \quad (2.34)$$

and in virtue of the unitarity condition

$$Im\, f_\ell^{(\pm)} \geq 0. \quad (2.35)$$

Under these assumptions the conclusions about the asymptotic equality of the differential cross sections, about the relation between polarizations (2.27) and about the equality of the quantities $Q^J(s,t)$ (2.30) for processes (I) and (II) also hold true. The proof of this assertion is carried out similarly to that for the case of scalar particles (see I).

III. Asymptotic Properties of the Scattering Amplitudes of Spinor Particles

1. Asymptotic Equalities for Differential Cross Sections

Now we proceed to the study of processes (I), (II), (III) in the case when all the particles a_i and b_i have spin 1/2. In this case processes (II) and (III) differ only in notations. Therefore, it suffices to consider processes (I) and (II). The amplitudes of these processes may be put as

$$T^J = \sum_{i=1}^{\infty} \bar{u}_a(q_2)\, \Gamma_i^{(a)}(p)\, u_a(q_1)\, \bar{u}_b(p_2)\, \Gamma_i^{(b)}(q)\, u_b(p_1)\, F_i^J(s,t), \quad J = I, II. \quad (3.1)$$

where

$$q = \frac{q_1 + q_2}{2}, \qquad p = \frac{p_1 + p_2}{2}, \quad (3.2)$$

$u_a(q_1)$ and $u_b(p_1)$ are the spinors with positive energy. In the first process $u_a(q_1)$ and $u_b(p_1)$ characterize the states of the particles a_1 and b_1, $u_a(q_2)$ and $u_b(p_2)$ – the particles a_2 and b_2, and in the second process $u_a(q_1)$ and $u_b(p_1)$ characterize the states of the particles $\bar{a}_2$ and b_1, $u_a(q_2)$ and $u_b(p_2)$ – the particles $\bar{a}_1$ and b_2. The covariant matrices $\Gamma_i^{(a,b)}$ in (3.1) may be chosen (depending on the relative parities of the particles) as follows

$$\Gamma_i^{(a)}(p) = \{1, 1, \hat{p}, \hat{p}, \gamma_5, \gamma_5, \hat{p}\gamma_5, \hat{p}\gamma_5\}, \quad (3.3)$$

(irrespective of the relative parities),

$$\Gamma_i^{(b)}(q) = \{1, \hat{q}, \hat{q}, 1, \gamma_5, \hat{q}\gamma_5, \gamma_5, \hat{q}\gamma_5\} \quad (3.4)$$

if $I_i = I_i$,

$$\Gamma_i^{(b)}(q) = \{\gamma_5, \hat{q}\gamma_5, \hat{q}\gamma_5, \gamma_5, 1, \hat{q}, 1, \hat{q}\} \quad (3.5)$$

if $I_i = -I_i$.

The amplitudes T^{II} and T^I are connected by a relation of type (2.12):

$$\sum_{i=1}^{8} \Gamma_i^{(b)}(q)\, \bar{u}_{a_1}(q_2)\, \Gamma_i^{(a)}(p)\, u_{a_2}(q_1)\, F_i^{II}(s,t) =$$
$$\sum_{i=1}^{8} \gamma_4 [C^{-1} \Gamma_i^{(b)}(q)\, C]^* \gamma_4 [\bar{u}_{a_2}(q_1)\, \Gamma_i^{(a)}(-p)\, u_{a_1}(q_2)]^* F_i^{-I}(u,t)^*. \quad (3.6)$$

It follows from this relation that the invariant amplitudes $F_i^J(s,t)$ of proces ses (I) and (II) are connected by the following crossing symmetry relations

$$F_i^J(s,t) = \pm(-1)^{i+1} F_i^J(u,t)^*, \tag{3.7}$$

where the $+$ sign corresponds to the case $I_i = I_f$, while the $-$ sign - to the case $I_i = -I_f$.

Note, that in the elastic scattering processes the relative parities of particles in the initial and final states are the same: $I_i = I_f$. It follows from the time reversal invariance that in this case the two amplitudes $F_6^J(s,t)$ and $F_7^J(s,t)$ are equal to zero and there exist only six independent amplitudes. Besides, if all the particles are the components of the same isotopic multiplet (for example, in case of nucleon-nucleon scattering), then the amplitudes $F_2^J(s,t)$ and $F_4^J(s,t)$ coincide, and there exist only five independent amplitudes.

The differential cross section for process (I) reads

$$\frac{d\sigma^I(s,t)}{dt} = \frac{1}{64\pi s k_I^2} F^I(s,t), \tag{3.8}$$

$$F^I(s,t) = \sum_{i=1}^{8} A_i B_i |F_i^I(s,t)|^2 + \mathrm{Re} \sum_{i<j} A_{ij} B_{ij} F_i^I(s,t) F_j^I(s,t)^*. \tag{3.9}$$

The coefficients A_i, B_i, A_{ij}, B_{ij} satisfy the equalities

$$\begin{aligned} &A_1 = A_2 = A_{12}, \quad A_3 = A_4 = A_{34}, \quad A_{13} = A_{14} = A_{23} = A_{24}, \\ &A_5 = A_6 = A_{56}, \quad A_7 = A_8 = A_{78}, \quad A_{57} = A_{67} = A_{58} = A_{68}, \\ &B_1 = B_4 = B_{14}, \quad B_2 = B_3 = B_{23}, \quad B_{12} = B_{13} = B_{24} = B_{34}, \\ &B_5 = B_7 = B_{57}, \quad B_6 = B_8 = B_{68}, \quad B_{56} = B_{58} = B_{67} = B_{78}, \\ &A_{ij} = B_{ij} = 0 \qquad |j-i| \geq 4 . \end{aligned} \tag{3.10}$$

The coefficients A_1, A_3 and A_{13} are equal

$$\begin{aligned} &A_1 = (m_1 + m_2)^2 - t \\ &A_3 = \tfrac{1}{4} \{ (s-u)^2 - (M_1^2 - M_2^2)^2 - [t - 2(M_1^2 + M_2^2)][t - (m_1 - m_2)^2] \}, \\ &A_{13} = (m_1 + m_2)(s-u) + (m_2 - m_1)(M_2^2 - M_1^2), \end{aligned} \tag{3.11}$$

and A_5, A_7 and A_{57} are readily obtained from A_1, A_3 and A_{13}

respectively, by the substitution $m_1 \to -m_1$. As for the coefficients B_i and B_{ij} they are determined depending on the relative parities of particles: in the case $I_i = I_f$, B_1, B_2, B_{12}, B_5, B_6 and B_{56} are obtained from A_1, A_3, A_{13}, A_5, A_7 and A_{57}, respectively, by the substitution $m_1 \leftrightarrow M_1$; and in the case $I_i = -I_f$ - by the same substitution from A_5, A_7, A_{57}, A_1, A_3 and A_{13}, respectively.

The differential cross section for process (II) is determined in a similar manner. It is obtained from the differential cross section of process (I) by the substitution $F_i^{II}(s,t)$ for $F_i^I(s,t)$ and $m_1 \leftrightarrow m_2$.

We will consider the general case when all eight amplitudes $F_i^J(s,t)$ contribute to the asymptotic behaviour of the cross sections. It follows from (3.10) and (3.11) and the indications after these formulas that in this case the functions

$$F_1^J(s,t), \; sF_2^J(s,t), \; s^2F_3^J(s,t), \; sF_4^J(s,t), \; F_5^J(s,t), \; sF_6^J(s,t), \; sF_7^J(s,t), \; s^2F_8^J(s,t) \tag{3.12}$$

have the same asymptotic behaviour at $s \to \infty$ and for fixed t. As well as in the case of scalar particle scattering on spinor particles, it follows from Phragmen-Lindelöf's theorem x/ and from the crossing symmetry relations that

$$\lim_{s\to\infty} \frac{F_i^I(s,t)}{F_i^{II}(s,t)^*} = \begin{cases} \pm e^{-i\pi\alpha(t)}, & i = 1,2,\dots,6, \\ \mp e^{-i\pi\alpha(t)}, & i = 7,8. \end{cases} \tag{3.13}$$

From relations (3.13) and the expressions for the differential cross section for processes (I) and (II) one can easily see that these cross sections are asymptotically equal. In particular, at $s \to \infty$ and for fixed t there occurs the asymptotic equality between the differential cross sections for the processes xx/

x/ Note that, while the dispersion relations for pion-nucleon scattering are proved on the basis of general principles of local field theory/ 8/, the analytical properties of nucleon-nucleon scattering amplitude necessary for proving theorem 2 are proved only in any order of perturbation theory/ 33,34/. As for the hyperon scattering on the nucleon, then, as is shown in/ 35/, the usual dispersion relations do not hold on the lowest orders of perturbation thery even for the case of forward scattering. The analytical properties from which the equalities of the cross sections for processes (3.14) follow are the hypothetical ones.

xx/ As was pointed out in the previous footnote, the hypothesis about the meromorphy of the hyperon-nucleon scattering amplitude in the upper half-plane s (from which the asymptotic equalities between the cross sections follow) is not so far proved. This also holds for the equalities between the polarizations for the processes involving hyperons.

$$
\begin{aligned}
&p + p \to p + p &&\text{and} && \bar{p} + p \to \bar{p} + \bar{p}\,, \\
&\Sigma^+ + p \to \Sigma^+ + p &&\text{and} && \bar{\Sigma}^+ + p \to \bar{\Sigma}^+ + p\,, \\
&\Sigma^- + p \to \lambda + n &&\text{and} && \bar{\lambda} + p \to \bar{\Sigma}^- + n\,, \\
&\Sigma^+ + p \to p + \Sigma^+ &&\text{and} && \bar{p} + p \to \bar{\Sigma}^+ + \Sigma^+\,, \\
&\Sigma^- + p \to n + \lambda &&\text{and} && \bar{n} + p \to \bar{\Sigma}^- + \lambda\,.
\end{aligned}
\tag{3.14}
$$

It should be noted that in the two latter reactions of (3.14) the momentum transfer t is measured between the initial proton and the final hyperon (as usual, this transfer is denoted by u). Therefore, in these cases we have the equality between the cross section for backward elastic hyperon-proton scattering and the annihilation cross section of the proton-antiproton pair into the hyperon-antihyperon pair.

2. Asymptotic Relations between Polarizations

We denote by ξ_μ^{Ia} and ξ_μ^{Ib} the four-dimensional polarization vectors of the particles a_2 and b_2, respectively, in the final state of process (I), by ξ_μ^{IIa} and ξ_μ^{IIb} the polarization vectors of the particles a_2 and b_2, respectively, in the final state of process (II), and by n_μ the unit space-like four-vector proportional to $\epsilon_{\mu\alpha\beta\gamma}\; p_{1\alpha}\; q_{1\beta}\; p_{2\gamma}$. If the particles in the initial state are unpolarized then the polarization vectors ξ_μ^{Ja} and ξ_μ^{Jb} are proportional to the unit vector n_μ

$$
\xi_\mu^{Ja} = P^{Ja}(s,t)\, n_\mu\,, \qquad \xi_\mu^{Jb} = P^{Jb}(s,t)\, n_\mu\,. \tag{3.15}
$$

Let us calculate the polarization of the particle b_2 in the final states of processes (I) and (II). For the first process we have

$$
\begin{aligned}
P^{Ib}(s,t) = 2s\sqrt{-t}\,\frac{C(s,t)}{F^I(s,t)}\, Im\, \{ & A_{12}F_1^I(s,t)^* F_2^I(s,t) + A_{13}F_1^I(s,t)^* F_3^I(s,t) \\
& + A_{24}F_2^I(s,t)F_4^I(s,t)^* + A_{34}F_3^I(s,t)F_4^I(s,t)^* + A_{56}F_5^I(s,t)^* F_6^I(s,t) \\
& + A_{78}F_7^I(s,t)^* F_8^I(s,t) + A_{58}F_5^I(s,t)^* F_8^I(s,t) + A_{67}F_6^I(s,t)F_7^I(s,t)^* \}\,,
\end{aligned}
\tag{3.16}
$$

the function $C(s,t)$ tending to unity at $s \to \infty$ for fixed t. The expression for $P^{IIb}(s,t)$ may be obtained from (3.16) by substituting $F_i^{II}(s,t)$ for $F_i^I(s,t)$ and $m_1 \leftrightarrow m_2$.

As was shown, at $s \to \infty$ for fixed t the functions $F^I(s,t)$ and $F^{II}(s,t)$ are asymptotically equal. It follows from relations (3.13) that for the six first terms in (3.16)

$$
Im\, F_i^I(s,t)^* F_j^I(s,t) = -\, Im\, F_i^{II}(s,t)^* F_j^{II}(s,t)\,, \tag{3.17a}
$$

while for the two latter ones

$$
Im\, F_i^I(s,t)^* F_j^I(s,t) = Im\, F_i^{II}(s,t)^* F_j^{II}(s,t)\,. \tag{3.17b}
$$

However, the coefficients A_{58} and A_{67} of these two latter terms are proportional for large s to the mass difference $m_1 - m_2$ and in the transition from $P^{Ib}(s,t)$ into $P^{IIb}(s,t)$ one should make the substitution $m_1 \leftrightarrow m_2$, i.e., in this transition the coefficients A_{58} and A_{67} change their signs, and the other coefficients do not change (asymptotically). Thus, for large s the polarizations $P^{Ib}(s,t)$ and $P^{IIb}(s,t)$ are equal by absolute value and have opposite signs, irrespective of the particle parities

$$
P^{Ib}(s,t) = -\, P^{IIb}(s,t)\,. \tag{3.18}
$$

Consider now the polarization properties of the particles a_2 and $\bar{a}_1$ in processes (I) and (II), respectively. For the first process we have

$$
\begin{aligned}
P^{Ia}(s,t) = 2s\sqrt{-t}\,\frac{C(s,t)}{F(s,t)}\, Im\{ & B_{13}F_1^I(s,t)^* F_3^I(s,t) + B_{14}F_1^I(s,t)^* F_4^I(s,t) \\
& + B_{23}F_2^I(s,t)^* F_3^I(s,t) + B_{24}F_2^I(s,t)^* F_4^I(s,t) + B_{58}F_5^I(s,t)^* F_8^I(s,t) \\
& + B_{57}F_5^I(s,t)^* F_7^I(s,t) + B_{67}F_6^I(s,t)^* F_7^I(s,t) + B_{68}F_6^I(s,t)^* F_8^I(s,t) \}\,,
\end{aligned}
\tag{3.19}
$$

$F^I(s,t)$ and B_{ij} being determined as earlier, while the polarization of the particle $\bar{a}_1$ in process (II) is obtained from (3.19) by substituting $F_i^{II}(s,t)$ for $F_i^I(s,t)$ and $m_1 \to m_2$. Note, that for large s B_{ij} are independent of the masses m_1 and m_2. Therefore, they do not change in the transition from $P^{Ia}(s,t)$ to $P^{IIa}(s,t)$. It follows from asymptotic relations (3.13) that for the four first terms in (3.19)

$$
Im\, F_i^I(s,t)^* F_j^I(s,t) = -\, Im\, F_i^{II}(s,t)^* F_j^{II}(s,t) \tag{3.20a}
$$

and for the four remaining ones

$$Im\ F^{I}_{i}(s\,t)^{*}\ F^{I}_{i}(s\,t) = Im\ F^{II}_{i}(s\,t)^{*}\ F^{II}_{i}(s\,t)\,. \qquad (3.20b)$$

Thus, in the general case, when all the amplitudes $F^{J}_{i}(s\,t)$ contribute to the asymptotic behaviour of the cross sections there exists no simple relation between $P^{Ia}(s\,t)$ and $P^{IIa}(s\,t)$.

Consider the case of elastic particle scattering. In this case, as was pointed out, $F^{J}_{6}(s\,t) = F^{J}_{7}(s\,t) = 0$ i.e., the three last terms in (3.19) are equal to zero. Moreover, in this case $M_1 = M_2$, B_{58} does not contain the higher term like in (3.11) and the fifth term in (3.19) does not give a contribution $s \to \infty$. Therefore, $P^{Ia}(s\,t)$ are determined by the four first terms which change their signs in the transition from $P^{Ia}(s\,t)$ to $P^{IIa}(s\,t)$. Thus, in the case of elastic particle scattering there holds the following asymptotic relation between the polarization of the particles a_2 and $\bar{a}_1 = \bar{a}_2$ in processes (I) and (II)

$$P^{Ia}(s\,t) = -P^{IIa}(s\,t). \qquad (3.21)$$

Consider some examples. In virtue of (3.18), at $s \to \infty$ and for fixed t the recoil proton polarizations in the processes

$$p + p \to p + p \qquad \bar{p} + p \to \bar{p} + p$$

have the same magnitude and opposite signs; the recoil neutron polarizations in the processes

$$\Sigma^{-} + p \to \lambda + n \qquad \text{and} \qquad \bar{\lambda} + p \to \bar{\Sigma}^{-} + n$$

and the hyperon polarizations in the processes

$$\Sigma^{+} + p \to p + \Sigma^{+} \qquad \text{and} \qquad \bar{p} + p \to \bar{\Sigma}^{+} + \Sigma^{+}$$

are also opposite in signs and equal by the absolute value. In virtue of (3.21) the nucleon polarization (correspondingly- hyperon) and the antinucleon (correspondingly- antihyperon) in the elastic scattering processes

$$p\,(\Sigma^{+}) + p \to p\,(\Sigma^{+}) + p \qquad \bar{p}\,(\bar{\Sigma}^{+}) + p \to \bar{p}\,(\bar{\Sigma}^{+}) + p$$

are also opposite.

If the initial particles are polarized, then in the angular distributions of final particles there will be the left- right asymmetry. We denote by $\eta^{Ia}(s\,t)$ and $\eta^{Ib}(s\,t)$ the asymmetry parameters in the processes with polarized particles a and b, respectively. For these quantities we have expressions analogous to (3.16) and (3.19) (but not exactly the same). From these expressions we can also prove the asymptotic relation analogous to (3.18)

$$\eta^{Ib}(s\,t) = -\eta^{IIb}(s\,t) \qquad (3.18')$$

In the general case the connection between $\eta^{Ia}(s\,t)$ and $\eta^{IIa}(s\,t)$ is not so simple, but in the case of elastic scattering we have

$$\eta^{Ia}(s\,t) = -\eta^{IIa}(s\,t) \qquad (3.21')$$

by analogy with (3.21). In the general case we have the following asymptotic relations

$$P^{Ia}(s\,t) = -\eta^{IIa}(s\,t), \qquad P^{IIa}(s\,t) = -\eta^{Ia}(s\,t), \qquad (3.22)$$

irrespective of the relative parities I_i and I_f.

Remind that in the case of boson- fermion scattering we have also similar relations (2.28).

IV. Asymptotic Properties of the Amplitudes of Pion- Photoproduction and Compton- Effect on Nucleon

We will show that the isotopic invariance of strong interactions and Phragmén- Lindelöf's theorem lead to the asymptotic equality between the differential cross sections of the processes

$$\gamma + p \to \pi^{+} + n \qquad (4.1)$$

and

$$\gamma + n \to \pi^{-} + p\,. \qquad (4.2)$$

The amplitudes of the processes under consideration may be written as

$$T^{a} = \sum_{i=1}^{4} \bar{u}(p_2)\, T_i\, u(p_1)\, F^{a}_{i}(s\,t) \qquad (4.3)$$

where

$$\begin{aligned} T_1 &= i\gamma_5 \hat{\epsilon}\hat{k}\,, \\ T_2 &= 2i\,\gamma_5\,[(q\epsilon)(pk) - (qk)(p\epsilon)]\,, \\ T_3 &= i\,\gamma_5\,[\hat{\epsilon}(qk) - \hat{k}(q\epsilon)]\,, \\ T_4 &= 2i\,\gamma_5\,[\hat{\epsilon}(pk) - \hat{k}(p\epsilon)]\,, \end{aligned} \qquad (4.4)$$

p_1 and p_2 are the 4- momenta of the nucleons in the initial and final states, respectively, k and q are the 4- momenta of the photon and the π -meson, ϵ_μ is the 4- vector of the photon polarization,

$$p=\frac{p_1+p_2}{2}\,,\quad s=(p_1+k)^2,\quad u=(p_1-q)^2,\quad t=(k-q)^2,$$

and a is the isotopic index of the pion. It follows from the isotopic invariance of strong interactions that the amplitudes $F_i^a(s\,t)$ have the following structure

$$F_i^a(s,t)=\delta_{a3}F_i^{(+)}(s\,t)+\tfrac{1}{2}[\tau_a,\tau_3]F_i^{(-)}(s\,t)+\tau_a F_i^{(0)}(s\,t). \qquad (4.5)$$

The amplitudes $F_i^{(+)}(s\,t)$ and $F_i^{(0)}(s\,t)$ satisfy the crossing symmetry relations

$$F_i^{(+,0)}(s\,t)=F_i^{(+,0)}(u\,t)^*,\quad i=1,2,4 \qquad (4.6)$$

$$F_3^{(+,0)}(s\,t)=-F_3^{(+,0)}(u\,t)^*,$$

and $F_i^{(-)}(s\,t)$ satisfy the relations

$$F_i^{(-)}(s\,t)=-F_i^{(-)}(u\,t)^*,\qquad i=1,2,4 \qquad (4.7)$$

$$F_3^{(-)}(s\,t)=F_3^{(-)}(u\,t)^*.$$

The amplitudes $F_i^{I}(s\,t)$ and $F_i^{II}(s\,t)$ of processes (4.1) and (4.2), respectively, are related to the amplitudes $F_i^{(\pm 0)}(s\,t)$ in the following manner

$$F_i^{I}(s\,t)=\sqrt{2}\,[F_i^{(0)}(s\,t)+F_i^{(-)}(s\,t)], \qquad (4.8)$$

$$F_i^{II}(s\,t)=\sqrt{2}\,[F_i^{(0)}(s\,t)-F_i^{(-)}(s\,t)].$$

Therefore, the amplitudes $F_i^{J}(s\,t)$ of processes (4.1) and (4.2) are connected by the relations

$$F_i^{II}(s\,t)=F_i^{I}(u\,t)^*,\quad i=1,2,4$$

$$F_3^{II}(s\,t)=-F_3^{I}(u\,t)^*. \qquad (4.9)$$

The differential cross sections of the processes under consideration are equal

$$\frac{d\sigma^J(s\,t)}{dt}=\frac{1}{64\pi s k^2}\{4(s-M^2)(M^2-u)|F_1^J(s\,t)|^2+$$

$$\tfrac{1}{2}[t^2(u-s)^2-t(t-4M^2)(t-m^2)^2]|F_2^J(s\,t)|^2+[-t(s-M^2)^2-t(u-M^2)^2+2M^2(t-m^2)^2]|F_3^J(s\,t)|^2$$

$$+[(4M^2-t)(s-M^2)^2+(4M^2-t)(u-M^2)^2+2M^2(s-u)^2]|F_4^J(s\,t)|^2+[t(u-s)^2-(t-4M^2)(t-m^2)^2]\,\mathrm{Re}\,F_1^J(s\,t)$$

$$F_2^J(s\,t)^*+4M(s-u)(m^2-t)\,\mathrm{Re}\,F_1^J(s\,t)F_3^J(s\,t)^*+2M(u-s)^2\mathrm{Re}\,F_1^J(s\,t)F_4^J(s\,t)^*$$

$$+2M[t(u-s)^2-(t-4M^2)(t-m^2)^2]\,\mathrm{Re}\,F_2^J(s\,t)F_4^J(s\,t)^* \qquad (4.10)$$

$$+2(s-u)(t-4M^2)(t-m^2)\,\mathrm{Re}\,F_3^J(s\,t)F_4^J(s\,t)^*\}\,,$$

where M and m are the nucleon and pion masses. For fixed t at $s\to\infty$ this expression reads:

$$\frac{d\sigma^J(s,t)}{dt}\approx\frac{1}{64\pi s k^2}\{|2sF_1^J(s,t)+tsF_2^J(s\,t)+2M s F_4^J(s,t)|^2 \qquad (4.11)$$

$$+|tsF_2^J(s,t)+2MsF_4^J(s,t)|^2+8M^2s^2|F_4^J(s,t)|^2$$

$$-2ts^2|F_3^J(s,t)|^2\}.$$

Using (4.11) and the crossing symmetry relations (4.9) and applying Phragmen-Lindelöf's theorem it is not difficult to prove the asymptotic equality between the differential cross sections

$$\frac{d\sigma^I(s\,t)}{dt}\quad\text{and}\quad\frac{d\sigma^{II}(s\,t)}{dt}$$

for processes (4.1) and (4.2).

Consider now the elastic photon-nucleon scattering. This is a self-crossed process. In studying the processes of scalar particle scattering on spinor particles we showed that in the processes of this type the recoil fermion polarization is tending to zero at $s\to\infty$ for fixed t. We prove that this holds for the given case as well.

The amplitude of the process we are considering may be written as

$$T=\bar{u}(p_2)\Big\{\frac{(\epsilon_2P')(\epsilon_1P')}{P'^2}[F_1(s,t)+\hat{k}F_2(s,t)]+\frac{(\epsilon_2N)(\epsilon_1N)}{N^2}[F_3(s,t)+\hat{k}F_4(s,t)] \qquad (4.12)$$

$$+i\frac{(\epsilon_2P')(\epsilon_1N)-(\epsilon_2N)(\epsilon_1P')}{\sqrt{2P'^2N^2}}\gamma_5F_5(s,t)+i\frac{(\epsilon_2P')(\epsilon_1N)+(\epsilon_2N)(\epsilon_1P')}{\sqrt{2P'^2N^2}}\gamma_5\hat{k}F_6(s,t)\Big\}\,,$$

where p_1 and p_2 are the 4-momenta of nucleons in the initial and final states, respectively, k_1 and ϵ_1 are the 4-momentum and the polarization vector of the photon before scattering, and k_2 and ϵ_2 are the same quantities after scattering,

$$k = \frac{k_1+k_2}{2}, \quad p = \frac{p_1+p_2}{2}, \quad P' = p - \frac{(pk)}{k^2}\cdot k ,$$

$$N_a = \epsilon_{a\rho\gamma\delta} \, P'_\beta \, k_\gamma \, (k_1 - k_2)_\delta .$$

The polarization $P^\gamma(s,t)$ of the recoil nucleon is

$$P^\gamma(s,t) = 2s\sqrt{-t}\,\frac{C(s,t)}{F(s,t)}\, Im\{F_1(s,t)^* F_2(s,t) + F_3(s,t)^* F_4(s,t)\}, \qquad (4.13)$$

$$F(s,t) = (4M^2 - t)\{|F_1(s,t)|^2 + |F_3(s,t)|^2\}$$

$$+ \tfrac{1}{4}[(s-u)^2 - t^2]\{|F_2(s,t)|^2 + |F_4(s,t)|^2\} - t\,|F_5(s,t)|^2 + (M^4 - su)|F_6(s,t)|^2 \qquad (4.14)$$

$$+ 2M(s-u)\, Re\{F_1(s,t) F_2(s,t)^* + F_3(s,t) F_4(s,t)^*\} .$$

The amplitudes $F_i(s,t)$ have the following crossing symmetry properties

$$F_i(s,t) = F_i(u,t)^*, \quad i = 1,3,5,6 \qquad (4.15)$$

$$F_i(s,t) = -F_i(u,t)^*, \quad i = 2,4.$$

Using the expressions (4.13), (4.14), relations (4.15) and applying Phragmén-Lindelöf's theorem it is not difficult to see that the recoil nucleon polarization is tending to zero at $s \to \infty$ for fixed t even in the general case when all the independent invariant amplitudes give a contribution to the asymptotic behaviour of the cross section.

V. Asymptotic Relations between Forward Elastic Scattering Amplitudes

We have seen from the example of scalar particles (see I) that in the case of forward elastic scattering one can obtain the equality of the total cross sections, as well as some other asymptotic relations of type (1.19) if an additional assumption (1.14) $a(0) = 1$ is made, and if the real part of the amplitude increases not faster than the imaginary one. We shall show that for elastic scattering of particles with spin at zero angle it is possible to obtain (assuming

(1.14)) some new relations besides the equalities between the differential cross sections. The number of the relations which can be checked experimentally increases, if the isotopic invariance of strong interactions is taken into account.

Let us start with a consideration of elastic pion-nucleon scattering. The amplitudes $A(s,t)$ and $B(s,t)$ of this process (2.1), (2.2) have the following isotopic structure

$$A^{\beta a} = A^{(+)}\delta_{\beta a} + A^{(-)}\tfrac{1}{2}[\tau_\beta, \tau_a], \quad B^{\beta a} = B^{(+)}\delta_{\beta a} + B^{(-)}\tfrac{1}{2}[\tau_\beta, \tau_a]. \qquad (5.1)$$

The amplitudes $A(s,t)$ of the physical processes

$$a_I)\ \pi^+ + p \to \pi^+ + p, \qquad a_{II})\ \pi^- + p \to \pi^- + p \qquad (5.2)$$

$$b_I)\ \pi^- + p \to \pi^0 + n \qquad b_{II})\ \pi^0 + p \to \pi^+ + n,$$

$$c)\ \pi^0 + p \to \pi^0 + p$$

and of the processes obtained from (5.2) by the substitution $p \to n$, $\pi^+ \to \pi^-$ are connected with amplitudes (5.1) by

$$A^{a_I} = A^{(+)} - A^{(-)}, \qquad A^{a_{II}} = A^{(+)} + A^{(-)}, \qquad (5.3)$$

$$A^{b_I} = -A^{b_{II}} = -\sqrt{2}\,A^{(-)}, \qquad A^c = A^{(+)};$$

The same equalities hold for amplitudes $B(s,t)$ either.

Note, that in case of forward scattering the differential and total cross sections are expressed in terms of the same function

$$\frac{d\sigma^i(s,t)}{dt}\Big|_{t=0} = \frac{1}{64\,\pi\, s^2}\,|f^i(s)|^2, \qquad (5.4)$$

$$i = a_I, a_{II}, b_I, b_{II}, c,$$

$$\sigma^i_{tot}(s) = \frac{1}{2k\sqrt{s}}\, Im\, f^i(s), \quad i = a_I, a_{II}, c,$$

where $f^i(s)$ is given by formula (2.33) at $t = 0$

$$f^i(s) = f^i(s,0) = 2MA^i(s,0) + (s - M^2 - m^2)\,B^i(s,0). \qquad (5.5)$$

One can apply theorem I to the functions $f^i(s)$, what leads, with account of (1.14), to the asymptotic relations

$$f^{a_I}(s) \approx -f^{a_{II}}(s)^*, \qquad f^{b_I}(s) \approx -f^{b_{II}}(s)^*, \quad f^c(s) \approx -f^c(s)^*. \qquad (5.6)$$

It follows from (5.3) and (5.6) that

$$Im\, f^{(-)}(s) \approx 0, \quad Re\, f^{(+)}(s) \approx 0, \tag{5.7}$$

and therefore

$$Im\, f^{a_I}(s) \approx Im\, f^{a_{II}}(s) \approx Im\, f^{c}(s), \quad Im\, f^{b_I}(s) \approx Im\, f^{b_{II}}(s) \approx 0,$$
$$Re\, f^{a_I}(s) \approx -Re\, f^{a_{II}}(s) \approx \frac{1}{\sqrt{2}} f^{b_I}(s) \approx -\frac{1}{\sqrt{2}} f^{b_{II}}(s), \quad Re\, f^{c}(s) \approx 0. \tag{5.8}$$

All the relations (5.8) can be checked experimentally. The equality of the imaginary parts for the processes a_I , a_{II} , c leads to the asymptotic equality of the total cross sections

$$\sigma_{tot}(\pi^+ p) \approx \sigma_{tot}(\pi^- p) \approx \sigma_{tot}(\pi^0 p). \tag{5.9}$$

The first of these equalities is Pomeranchuk's theorem/12/, and the second one was suggested in/1,2/ from the analysis of the experimental data. Further, we note that the charge exchange scattering amplitudes $f^{b_I}(s)$ and $f^{b_{II}}(s)$ for large are real. Then, using the equalities (5.8) we get the following interesting relation between the differential and the total cross sections

$$\left[\frac{d\sigma^{a_I}(s,t)}{dt} - \frac{1}{2}\frac{d\sigma^{b_I}(s,t)}{dt}\right]_{t=0} \approx \frac{1}{16\pi}\left[\sigma_{tot}(\pi^+ p)\right]^2. \tag{5.10}$$

The relation (5.10) is a generalization of equality (1.19) to the case of charged meson scattering. It is seen from (5.8) that equality (1.19) holds, without change, for the cross sections of π^0 - meson scattering on the proton(process 'c'). In such manner one can show that relation (1.19) holds also for the cross sections of K_1^0 and K_2^0 scattering on the nucleon (if the weak interactions are neglected, the amplitudes of both these processes are equal). Point out also that if (1.14) holds, the amplitude for the process $K_2^0 + p \to K_1^0 + p$ is real at $t = 0$ and $s \to \infty$.

A somewhat more complicated situation occurs in the case of elastic scattering of particles with spin 1/2, when we are dealing with six independent invariant functions (or with five functions in the case of nucleon-nucleon scattering). Instead of using the formulae (3.1)-(3.4) with $F_6^J(s,t) = F_7^J(s,t) = 0$, it is more convenient in this case to write down the amplitudes of processes (I) and (II) as

$$T^J = \sum_{i=1}^{6} \bar{u}_b(p_2)\, \Gamma_i^{(b)}(q)\, u_b(p_1)\, \bar{u}_a(q_2)\, \Gamma_i^{(a)}(p)\, u_a(q_1)\, F_i^J(s,t) \tag{5.11}$$

where

$$\Gamma_i^{(a)} = \{ 1, \gamma_a, \gamma_5, 1, \hat{p}, \hat{p} \} \tag{5.12}$$

$$\Gamma_i^{(b)} = \{ 1, \gamma_a, \gamma_5, \hat{q}, \hat{q}, 1 \}, \tag{5.13}$$

while p and q are given by the formula (3.2). For nucleon-nucleon scattering $F_4^J(s,t) = F_6^J(s,t)$ The crossing symmetry relation preserves its form (here $l_1 = l_t$). Besides, the hermitian and antihermitian parts of the amplitudes are given by

$$D^J = \sum_{i=1}^{6} \bar{u}_b(p_2)\, \Gamma_i^{(b)}(q)\, u_b(p_1)\, \bar{u}_a(q_2)\, \Gamma_i^{(a)}(p)\, u_a(q_1)\, Re\, F_i^J(s,t),$$
$$A^J = \sum_{i=1}^{6} \bar{u}_b(p_2)\, \Gamma_i^{(b)}(q)\, u_b(p_1)\, \bar{u}_a(q_2)\, \Gamma_i^{(a)}(p)\, u_a(q_1)\, Im\, F_i^J(s,t). \tag{5.14}$$

The choice of the matrices (5.13) is convenient because it allows to distingush the amplitude $F_2^J(s,t)$ with the same matrix structure as the Coulomb one. For forward scattering (under assumption (1.14)) theorem 2 enables us to prove the following asymptotic relations

$$D^I(s,0) \approx -D^{II}(s,0),$$
$$A^I(s,0) \approx A^{II}(s,0) \tag{5.15}$$

The second equality leads to Pomeranchuk's theorem about the asymptotic equality of the total cross sections

$$\sigma_{tot}(\bar{p}p) \approx \sigma_{tot}(pp), \quad \sigma_{tot}(\bar{\Sigma} p) \approx \sigma_{tot}(\Sigma p)$$

The first equality (5.15) may also be verified experimentally.

VI. The Asymptotic Behaviour of the Vertex Part

On the basis of the perturbation theory considerations one may assert/36/ that the vertex part $\Gamma(M^2, M_1^2, t)$ is an analytical function of the variable with a cut along the positive real axis $t = q^2$. It is well-known that in the local theory the form-factor increases not faster than a polynomial in the complex plane t . This permits to draw certain conclusions about the asymptotic behaviour of the processes which are described in the e^2 -approximation in terms of the electromagnetic form-factors. Consider, for instance, the processes of electron-proton scattering and the proton and antiproton annihilation into electron and positron. The cross section for these processes is expressed in e^2 - approximation in terms of the electromagnetic form-factors $F_1(t)$ and $F_2(t)$ or in terms of their linear combinations /37/

$$G_E(t) = F_1(t) + \frac{t}{4M^2}\mu F_2(t), \quad G_\mu(t) = F_1(t) + \mu F_2(t). \qquad (6.1)$$

In the first process t is negative, while in the second one t is positive.

The behaviour of the form-factors for large negative t was studied on the basis of numerous experiments on electron-proton scattering. Under the assumptions we have made it should be expected that the same behaviour must be observed for the form-factors at large positive t (to be more exact, the absolute values of the ratios $G_j(t)/G_j(-t)$, $j = E, \mu$ must tend to 1 at $t \to \infty$).

Let us emphasize also that since for $t < 0$ the functions $F_j(t)$ and $G_j(t)$ $j = E, \mu$ are real, then in virtue of theorem 2 at $t \to +\infty$ these functions must be real and $G_j(t) \to G_j(\infty)$ for $|t| \to \infty$ along any complex way, if this limit exists for $t \to -\infty$ (as this is indicated by experiment). Here, just as in case of the two-particle Green function, the vertex functions satisfy the dispersion relation

$$G_j(t) = G_j(\infty) + \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{Im\, G_j(t')}{t'-t}\, dt', \quad j = E, \mu, \qquad (6.2)$$

$$Im\, G_j(\infty) = 0.$$

Summary of Results

Basing on the principlex of local field theory and on the assumption that the amplitudes of the scattering processes for fixed momentum transfer t do not oscillate, when the energy squared $s \to \infty$, but have a definite (e.g., power or logarithmic) growth, we deduce the following physical consequences.

1. At high energies and for fixed momentum transfer there take place the asymptotic equalities between the differential cross sections $\frac{d\sigma(s,t)}{dt}$ for following pairs of processes

$$\pi^+ + p \to \pi^+ + p \qquad \pi^- + p \to \pi^- + p, \qquad (7.1)$$

$$K^+ + p \to K^+ + p \qquad K^- + p \to K^- + p, \qquad (7.2)$$

$$\pi^+ + p \to K^+ + \Sigma^+ \qquad K^- + p \to \pi^- + \Sigma^+ \qquad (7.3)$$

$$\pi^- + p \to K^\circ + \Lambda \qquad \bar{K}^\circ + p \to \pi^+ + \Lambda, \qquad (7.4)$$

$$K^- + p \to K^\circ + \Xi^\circ \qquad \bar{K}^\circ + p \to K^+ + \Xi^\circ \qquad (7.5)$$

$$\Sigma^+ + He \to p + He_\Lambda \qquad \bar{p} + He \to \bar{\Sigma}^+ + He_\Lambda \qquad (7.6)$$

$$p + p \to p + p \qquad \bar{p} + p \to \bar{p} + p, \qquad (7.7)$$

$$\Sigma^+ + p \to \Sigma^+ + p \qquad \bar{\Sigma}^+ + p \to \bar{\Sigma}^+ + p, \qquad (7.8)$$

$$\Sigma^- + p \to \Lambda + n \qquad \bar{\Lambda} + p \to \bar{\Sigma}^- + n, \qquad (7.9)$$

$$\Sigma^+ + p \to p + \Sigma^+ \qquad \bar{p} + p \to \bar{\Sigma}^+ + \Sigma^+, \qquad (7.10)$$

$$\Sigma^- + p \to n + \Lambda \qquad \bar{n} + p \to \bar{\Sigma}^- + \Lambda. \qquad (7.11)$$

(For proving the equalities between the cross sections for processes (7.6) and (7.8)-(7.11) it is necessary to make certain assumption about the analyticity of the amplitudes of these processes). In all these processes t is determined as the momentum transfer between the first particle at the beginning and the first particle at the end of the reaction. In more customary notations processes (7.10) and (7.11) are treated for fixed u (there occurs the equality between the differential cross section for the backward hyperon-proton scattering and the annihilation cross section of proton and antiproton into hyperon and antihyperon).

2. The recoil fermion polarizations in processes (7.1)-(7.5) are equal in absolute value and opposite in signs at $s \to \infty$ (if the meson is scatterred on an polarized hydrogen target, the asymmetry coefficient in the first five pairs of reactions are also equal by magnitude and opposite in sign).

The measurement of the fermion polarizations in the final state in processes (7.6) allows to determine the relative parity of Σ and λ hyperons: if $I_{\lambda\Sigma} = 1$, then these polarizations must be opposite in sign (and be the same by absolute value) at high energies. If, on the other hand, $I_{\lambda\Sigma} = -1$, then the polarizations must coincide both by absolute value and in sign.

The fermion polarizations in reactions (7.7) and (7.8) should be opposite $P^{Ip} = -P^{IIp}$, $P^{I\lambda} = -P^{II\bar{\lambda}}$ etc. The same holds true for the neutron polarizations in processes (7.9).

The recoil fermion polarizations in the reactions

$$K^{o}_{1,2} + p \to K^{o}_{1,2} + p \,, \qquad (7.12)$$

$$K^{-} + p \to K^{+} + \Xi^{-}, \qquad (7.13)$$

$$\gamma + p \to \gamma + p \qquad (7.14)$$

are vanishing at high energies. If the isotopic invariance of strong interactions is assumed, one can show that the recoil neutron polarization in the process

$$\pi^{-} + p \to \pi^{0} + n \qquad (7.15)$$

must vanish at high energies.

3. If the absorptive part of the elastic scattering amplitude behaves like e.g. $s(\ln s)^{\beta}$, where β is an arbitrary real number, then there occur the asymptotic equalities between the total cross sections for particle and antiparticle interaction at high energies

$$\sigma_{tot}(\pi^{+} p) \approx \sigma_{tot}(\pi^{-} p), \quad \sigma_{tot}(K^{+} p) \approx \sigma_{tot}(K^{-} p), \qquad (7.16)$$

$$\sigma_{tot}(\bar{p} p) \approx \sigma_{tot}(p p), \quad \sigma_{tot}(\bar{\Sigma} p) \approx \sigma_{tot}(\Sigma p). \qquad (7.17)$$

Under the same assumption the differential cross section for forward elastic scattering of neutral meson on proton is proportional to the total cross section squared

$$\left.\frac{d\sigma(K^{o}_{1,2} + p \to K^{o}_{1,2} + p)}{dt}\right|_{t=0} \approx \frac{1}{16\pi}\left[\sigma\,(K^{o}_{1,2}\, p)\right]^{2}. \qquad (7.18)$$

If we take into account the isotopic invarnance of strong interactions, we get

$$\left[\frac{d\sigma'(\pi^{+} + p \to \pi^{+} + p)}{dt} - \frac{1}{2}\frac{d\sigma(\pi^{-} + p \to \pi^{0} + n)}{dt}\right]_{t=0} \approx \frac{1}{16\pi}\left[\sigma_{tot}(\pi^{+} p)\right]^{2}; \qquad (7.19)$$

$$\sigma_{tot}(\pi^{+} p) \approx \sigma(\pi^{0} p). \qquad (7.20)$$

Everywhere the sign $A \approx B$ should be understood as a symbolic writing of the equality $\lim\limits_{s \to \infty} \frac{A}{B} = 1$).

4. If one takes into account the isotopic invariance (together with the principles of the local theory) it is possible to prove the asymptotic equality between the differential cross sections for the processes

$$\gamma + p \to \pi^{+} + n \quad \text{and} \quad \gamma + n \to \pi^{-} + p \qquad (7.21)$$

for fixed momentum transfer.

5. The limits of the form-factors at $t \to \pm\infty$ are equal. It follows from here that in the e^{2} approximation at high energies the differential cross sections for the processes

$$e^{-} + p \to e^{-} + p \quad \text{and} \quad \bar{p} + p \to e^{-} + e^{+} \qquad (7.22)$$

must coincide.

In conclusion the authors express their deep gratitude to S.M. Bilenky, R.M. Ryndin and O.A. Khrustalev, in collaboration with whom some results of the present paper were obtained. They are also grateful to N.N.Bogolubov, D.I.Blokhintsev, R.T. Denchev, M.A. Markov, N.N. Meiman, Ja. Khristov, P. Suranyi and to the participants of the seminars of the Laboratory of Theoretical Physics of the Joint Institute for Nuclear Research, of Steklov's Mathematical Institute and of the Institute of Theoretical and Experimental Physics for interest in the work and useful remarks.

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Received by Publishing Department
on December 28, 1963.

Note Added in Proof

In a recent paper by Logunov A.A., Nguyen van Hieu and Hsien Ting-chang (preprint of JINR E-1550) the method developed here was applied to study the consequences of the higher symmetries of strong interactions. In particular, it was shown that in any model with higher symmetries (unitary symmetry, G_2 - model) all the total cross sections of meson-baryon interactions are asymptotically equal, and the polarizations in the charge exchange processes tend to zero at $s \to \infty$.