ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

V.G.Soloviev, P.Vogel, A.A.Korneichuk

E-1356

ENERGIES OF THE OCTUPOLE COLLECTIVE STATES WITH $I\pi K=1\text{-}0$ OF EVEN-EVEN STRONGLY DEFORMED NUCLEI

ДАН СССР, 1964, т154, н 1, с 72-75.

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Submitted to DAN

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A number of authors[2] have made general investigations of the collective properties of atomic nuclei on the basis of the method of approximate second quantization[1]. The most progress has been achieved in the domain of spherical nuclei where the collective state energies and the electromagnetic transition probabilities were calculated[3-5]. In the region of strongly deformed nuclei the investigations[6] are restricted to the obtaining of the basic equations and to the investigation of the problem of elimination of a spurious state. It is only in ref[7] that the gamma oscillation energies and the $E2$ transition probabilities are calculated. In the present paper the energies of the octupole collective oscillations with $I\pi K = 1-0$ of even-even nuclei in the regions $228 \leq A \leq 254$ and $152 \leq A \leq 186$ are calculated in the framework of the superfluid nuclear model using the method of approximate second quantization.

The secular equation determining the frequencies of the octupole oscillation is of the form

$$1 = 2\varkappa_n^{(3)} \sum_{ss'} - \frac{f^n(ss')^2 (u^n_{ss'})^2}{\varepsilon_n(s) + \varepsilon_n(s') - \frac{\omega^2}{\varepsilon_n(s) + \varepsilon_n(s')}} + 2\varkappa_p^{(3)} \sum_{\nu\nu'} \frac{f^p(\nu\nu')^2 (u^p_{\nu\nu'})^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu') - \frac{\omega^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu')}} +$$

$$+ 4\left(\varkappa_{np}^{(3)2} - \varkappa_n^{(3)} \varkappa_p^{(3)}\right) \sum_{ss'} \frac{f^n(ss')^2 (u^n_{ss'})^2}{\varepsilon_n(s) + \varepsilon_n(s') - \frac{\omega^2}{\varepsilon_n(s) + \varepsilon_n(s')}} \sum_{\nu\nu'} \frac{f^p(\nu\nu')^2 (u^p_{\nu\nu'})^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu') - \frac{\omega^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu')}} \quad (I)$$

where $\varkappa_n^{(3)}$, $\varkappa_p^{(3)}$, $\varkappa_{np}^{(3)}$ are the constants of octupole-octupole interactions, $f(ss')$ is the matrix element of the octupole momentum operator, the index n is related to the neutron system and p to the proton one; the summation $ss'(\nu\nu')$ is made over the single-particle levels of the average field of the neutron (proton) system. The pairing correlations of a superconducting type are treated as in[8] $\varepsilon(s) = \sqrt{C^2 + (E(s) - \lambda)^2}$ $u_{ss'} = u_s v_{s'} + v_s u_{s'}$. The values of the correlation functions C, the chemical potentials λ and the schemes of the average field levels are taken from[9] and[10].

The frequencies of the octupole oscillations are found by numerical solution of the secular equation (I) on the electronic computer. The first root of ω is found in the interval

$$0 < \omega < \min(\varepsilon_s + \varepsilon_{s'},\ \varepsilon_\nu + \varepsilon_{\nu'}) \quad (2)$$

by a successive division of the interval into two parts. In the case $\varkappa_n^{(3)} = \varkappa_p^{(3)} = \varkappa_{np}^{(3)} = \varkappa^{(3)}$ there is no root in the interval (2), provided $\varkappa^{(3)} > \varkappa^{(3)}_{max}$. The second and the subsequent roots (I) are situated between successive poles in the right-hand side of (I), they exist for any values of $\varkappa^{(3)}$.

The values of ω calculated from (I) depend on the wave functions and the eigenvalues of the average field potential and also on what levels of the average field were taken into account in (I). The terms in (I) with s and s' corresponding to the particle and hole states in all the nuclei with $|E(s)-\lambda| \gg C$ and $|E(s')-\lambda| \gg C$ lead only to the renormalization of constants $\varkappa^{(\lambda)}$ as in[3]. The same terms in (I) which in some nuclei correspond to the particle and hole states and in another - only to particle (hole) states, lead not only to the renormalization of $\varkappa^{(3)}$ but also to the change of ω in some nuclei as compared to another.

The correctness of the location of the Nilsson scheme levels in the regions $61 \leq Z \leq 79$; $89 \leq N \leq 115$ and $87 \leq Z \leq 99$, $137 \leq N \leq 155$ is proved by the experimental data on the single-quasi-particle levels of odd-A nuclei. As to the behaviour and the choice of the other levels there is a certain arbitrariness. In order to decrease this arbitrariness we have taken into account all the orbits of those subshells in which the location of at least one singleparticle level was proved experimentally. The calculations are made for the deformation $\delta = 0.3$ for nuclei in the region $152 \leq A \leq 186$ and for $\delta = 0.2$ for nuclei in the region $228 \leq A \leq 254$ with wave functions given in[11]. In each region for all nuclei the same single-particle energies $E(s)$ of the average field have been used and the changes of the deformation for different nuclei have been disregarded. In order to see how strongly the results of calculations depend on the wave functions in the region $228 \leq A \leq 254$ ω's were calculated with wave functions for $\delta = 0.3$ but with unchanged values of $E(s)$. The values of ω obtained in this case differ little from those of ω in case $\delta = 0.2$. The octupole-octupole interaction constant $\varkappa^{(3)}$ is taken to be equal to $\varkappa^{(3)} = \frac{1.3}{A}$ Mev, i.e. the values of $\varkappa^{(3)}A$ are identical in both regions of strongly deformed nuclei.

The results of calculations of the energies of the octupole states with $K\pi = 0-$ in the region $228 \leq A \leq 254$ for $\delta = 0.2$ when $\varkappa_n^{(3)} = \varkappa_p^{(3)} = \varkappa_{np}^{(3)} = \varkappa^{(3)}$ are given in Fig. I. The best agreement with all the experimental data is obtained at $\varkappa^{(3)} = 0.00083\,\hbar\mathring{\omega}_0 \approx 0.0055\,\text{Mev} \approx \frac{1.3}{A}\,\text{Mev}$. However for the isotopes of Th best are $\varkappa^{(3)} = 0.00081\,\hbar\mathring{\omega}_0$ and for the isotopes of U $\varkappa^{(3)} = 0.00085\,\hbar\mathring{\omega}_0$. As was noted in[12], the smallest calculated values of ω are in satisfactory agreement with the experimental values of energies for states with $K\pi = 0-$. The tendency of the lowering of the energies of these states in the light isotopes of Th and U is represented correctly. In the isotopes of Th, U and Pu the states with $K\pi = 0-$ are to a large degree collectivized, the values of ω are by 0.8 - I.0 Mev smaller than those of the energies of the nearest poles in (I). As to Fm^{254} the contribution to the state with $K\pi = 0-$ of the proton two-quasi-particle state with configuration $633\downarrow - 514\uparrow$ amounts to 96 % so that this state is with good accuracy two-quasi-particle one. Its energy is I.4 Mev. It is lower only by 26 Kev than the energy of the nearest pole. According to the calculations[10] the energy of the two-quasi - particle proton state $633\downarrow - 514\downarrow$ in Fm^{254} is I.08 Mev taking into account the blocking effect but for $\varkappa^{(3)} = 0$. If in investigating the collective effects we took into account the blocking effect then we would obtain the energy of this state equal to I Mev.

Thus some states with $K\pi = 0-$ possess the collective properties and others are two-quasi-particle. The average field defines the structure of the given state — whether it is collective or quasi- particle.

The results of calculations of the energies of states with $K\pi = 0-$ in the region $152 \leq A \leq 186$ at $\delta = 0.3$ when $\varkappa_n^{(3)} = \varkappa_p^{(3)} = \varkappa_{np}^{(3)} \equiv \varkappa^{(3)}$ are given in Fig. 2. As is seen from Fig. 2, a satisfactory agreement is obtained between calculated energy values of states with $K\pi = 0-$ and corresponsing experimental data. In this region of strongly deformed nuclei all the lowest states with $K\pi = 0-$ are collective. So in Er^{166} the contributions to state 0- with an energy I.66 Mev are given by one two-quasi-particle state 30 %, two states I6 % each, and by nine states (I-5) % each. The energy of ω in Er^{166} is by 0.55 Mev lower than the energy of the nearest pole and by 0.34 Mev lower than the energy of the two-quasi-particle state taking into account the blocking effect.

Calculations have been made for more general cases $\varkappa_n^{(3)} \neq \varkappa_p^{(3)} \neq \varkappa_{np}^{(3)}$ which, however, differ little from calculations with $\varkappa_n^{(3)} = \varkappa_p^{(3)} = \varkappa_{np}^{(3)} \equiv \varkappa^{(3)}$. In solving (I) the conservation of particle number on the average was controlled, for what the following quantity was calculated

$$\Delta n = \langle Q \sum_{s\sigma} a^+_{s\sigma} a_{s\sigma} Q^+ \rangle - \langle \sum_{s\sigma} a^+_{s\sigma} a_{s\sigma} \rangle \tag{3}$$

i.e. the difference of the average number of neutrons (protons) in the excited collective and ground states. In most cases it was obtained $\Delta n < 0.3$. However, states with $\Delta n \div 0.3-0.6$ occur. The reduced probabilities of $E3$ transitions calculated with $e_p = e + e_{eff}$, $e_n = e_{eff}$, $e_{eff} = 0.5e$ as in[4] are larger than the single-quasi-particle ones by a factor I.2 - 3.6 for the exception of Cm, Cf, Fm, where they are somewhat smaller than the single-particle ones. The reduced probabilities of the $E1$ transitions with $e_p = \frac{N}{A}e$, $e_n = -\frac{Z}{A}e$ are smaller then the single-particle by about a factor 10^2. Thus the behaviour of the energies of the collective octupole states with $K\pi = 0-$ is explained by introducing one new constant $\varkappa^{(3)}$, all the other parameters were fixed earlier in[8,9]. It should be noted that the microscopic treatment of the states on the basis of the superfuid nuclear model strongly differs from the phenomenological treatment of the unified nuclear model. So, according to the treatment of the superfluid nuclear model the octupole states in some nuclei lie comparatively low (below beta and gamma vibrational states) and possess the pronounced collective properies and in other ones such states may have high energy and in their nature will be close to the two-particle exoited states.

In conolusion we express our deep gratitude to N.N. Bogoliubov for interesting discussion, and to G.Jungklaussen for help in performing numerical calculations.

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Received by Publishing Department
on July,16,1963.

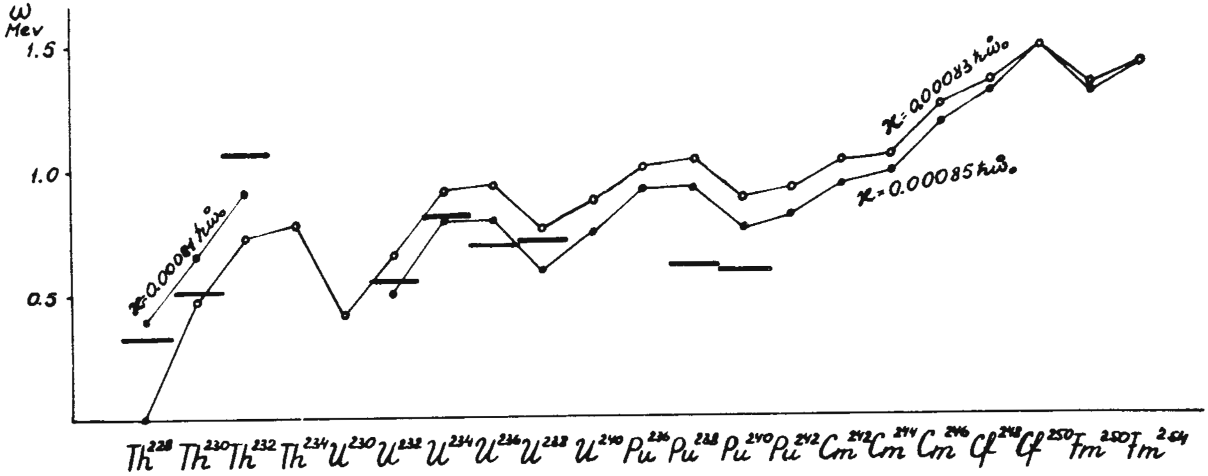

Fig. I The energies of the octupole collective states with $I\pi K = 1\text{-}0$ in the region $228 \le A \le 254$. The lines — denote the experimental data, taken from [13], the open circles (joined by the straight lines for the sake of illustration) are the calculated values of energies for $\varkappa^{(3)} = 0.00083\,\hbar\omega_0$, the dark circles denote the energies for Th when $\varkappa^{(3)} = 0.00081\,\hbar\omega_0$ and for other nuclei when $\varkappa^{(3)} = 0.00085\,\hbar\omega_0$.

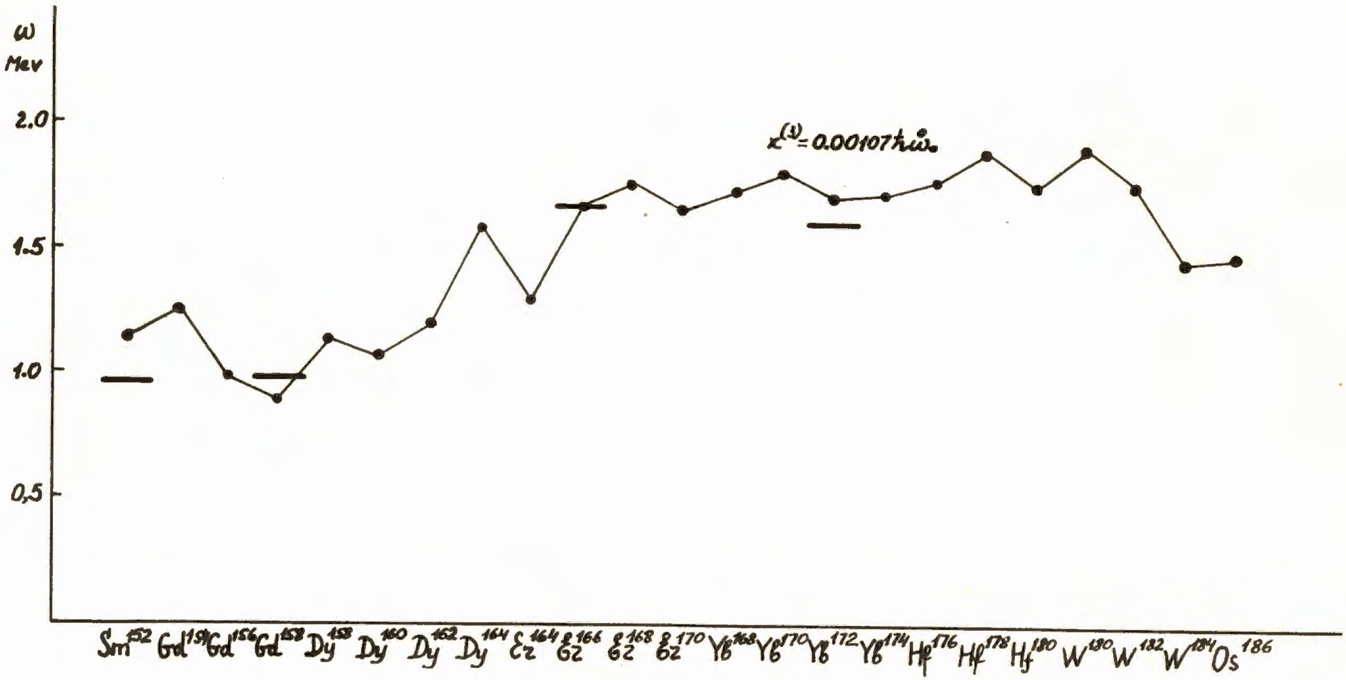

Fig.2 The energies of the octupole collective states with $I\pi K = 1{-}0$ in the region $152 \leq A \leq 186$. The lines —— denote the experimental data, taken from[14], the dark circles (joined by the straight lines for the sake of illustration) are the calculated values of energies for $\varkappa^{(s)} = 0.00107\ \hbar\omega_0$.