ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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A MODEL EQUATION FOR NON-ANALYTICAL TRANSITION AMPLITUDES

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The Mandelstam representations are based on the hypothesis that any Feynman graphs for pion-nucleon scattering possess definite analytical properties. But already for transitions amplitudes with the next degree of complexity – one meson production amplitude – the analysis of the graphs shows that the analyticity property is violated and the use of the mandelstm integral representations becomes impossible.

There arises the question by what one may replace the usual dispersion integral representations and how to express the fact that the scattering amplitude h may be analytical but not in a so wide region as one suggested earlier.

In the present paper we shall indicate one possible solution of this problem. Owing to the fact that it is little advisable to start with a complicated case in which h is considered as a function of two or more complex variables, we restrict ourselves to one simpler problem in which h is a function of one complex variable. The more complicated cases which correspond to the Mandelstam problem are considered by the author in /11/.

This problem is a generalization of that boundary problem which is solved by the Low integral equations.

The Low equations are equivalent to the boundary problem /1/ in which the scattering amplitudes $h'_a\ (z')$, $z'=x'+iy'$, $a=1,2,\ldots N$ to be determined satisfy the three following well-known conditions: 1) analyticity on the physical sheet, 2) unitarity and 3) crossing symmetries. In this problem conditions 2) and 3) are conserved without any change and condition 1) is weakened. It is namely supposed that the analyticity is needed only on some strip containing cuts and not on the whole physical sheet. It is remarkable that the width of the strip can be reduced to zero without changing the shape of the equations, i.e. the same equations hold also for completely nonanalytical amplitudes.

In the paper it is shown that this problem is equivalent to one algebraic system of equations.

The algebraic system is convenient for investigating the solution properties and for numerical calculations. It has advantages as compared with the N/D -method /2/. With its aid one can solve equations for π-π scattering which in the framework of the simplifications made remain correct even after an eventuel disproof of the Mandelstam representations /3/.

Now we go over to the formulation of the problem. Find R -functions $h'_a\ (z')$, $a=1,2\ldots.\ N$ which satisfy the following conditions a') Let X' be the cuts of the real axis $y'=0$ determined by the inequalities $-\infty < x' < -1$; $1 < x' < \infty$ and T' be a region containing X'. Then the functions $h'\ (x')$, $a=1,2\ldots.N$ are to be continuous in T' and analytical in $T'-X'$ and $h'_a\ (x')$, $x'\in X'$ -piecewise smooth functions which at infinity decrease not slower than $\dfrac{const}{x'}$.

By h' we shall denote the vector with the components h'_a, $a=1,2,\ldots N$.

b') $\quad Im\, h'(x'+i\epsilon') = p'^{\,n}(x')\, v'^{\,2}(p')\, |h'(x'+i\epsilon')|^2 , \quad 1 \leq x' < \infty ,$

where $p'=\sqrt{x'^2-1}$ is the pion momentum $v'(p')$ is the cutoff function and n – an integer.

c') $\quad h'(x'+i\epsilon') = Ah'(-x'-i\epsilon')\ .$

Here A is the N-row matrix among the eigenvalues of which there are numbers $+1$ and -1.

The problem a'), b'), c') has been formulated by the author in papers /4,5/ The systems of algebraic equations which the solutions of the problem must satisfy have been written there. However these equations are inconvenient to find solutions of the adiabatic type. In this paper we shall obtain new algebraic equations of the problem a'), b'), c') by means of which we can investigate both the resonance and adibatic solutions.

By means of the conformal mapping

$$z' = f(z) = \frac{2z}{1+z^2} \quad . \tag{1}$$

The plane $z' = x' + iy'$ goes over to the plane $z = x + iy = re^{i\phi}$, the cuts X' to the unit circle C and T' into the closed ring region T with the outer boundary C. Then the problem a'), b'), c') goes over to the problem : Find the vector R-function $h(z)$ with components $h_a(z)$, $a = 1, 2, \dots\ N$ which satisfies the following conditions:

a) the functions $h_a(z)$ are continuous in T and analytical in $T - C$; $h_a(z)\big|_{|z|=1} = h_a(\phi)$, $a = 1,2,\dots N$ are piecewise smooth functions.

b) $$Im\, h(\phi) = F(\phi)\, |h(\phi)|^2 , \quad -\frac{\pi}{2} \le \phi \le \frac{\pi}{2} \tag{2}$$

where $$F(\phi) = p'^{\,n}\left(\frac{1}{\cos\phi}\right) v'^{\,2}(p') = \sum_{k=1}^{\infty} F_{2k} \sin 2k\phi \tag{3}$$

c) $$h(\phi + \pi) = A\, h(\phi) , \quad -\pi \le \phi \le \pi . \tag{4}$$

On the basis of a) we can expand $h(z)$ in the Loran series

$$h(z) = \sum_{n=-\infty}^{+\infty} a_n z^n \tag{5}$$

from which we obtain for $|z| = 1$

$$Re\ h(\phi) = \sum_{n=-\infty}^{+\infty} a_n \cos n\phi \tag{6}$$

$$Im\ h(\phi) = \sum_{n=-\infty}^{+\infty} a_n \sin n\phi \tag{7}$$

a_n being vectors with components $a_n^{(1)}, a_n^{(2)}, \dots\dots\dots\dots\dots\ a_n^{(N)}$.

Using (2) and (3) we transform the well-known formula for the Fourier components:

$$a_n = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \sin n\phi\ Im\, h(\phi)\, d\phi + \frac{1}{\pi} \int_{+\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin n\phi\ Im\, h(\phi)\, d\phi .$$

The transformed formula has the form

$$a_n = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \sin n\phi\ F(\phi)\, |h(\phi)|^2 d\phi + \frac{1}{\pi} (-1)^n \int_{+\frac{\pi}{2}}^{+\frac{3\pi}{2}} \sin n\phi\, F(\phi) A\, |h(\phi)|^2 d\phi .$$

By replacing $F(\phi)$ and $|h(\phi)|^2 = Re^2 h(\phi) + Im^2 h(\phi)$ by means of expansions (3), (6) and (7) we get

$$a_n = \sum_{m,k=-\infty}^{+\infty} \Gamma_{nk}\, a_m * a_{m+k} , \tag{9}$$

where

$$\Gamma_{nk} = \frac{1 + (-1)^n A}{2\pi} \sum_{\ell=-\infty}^{+\infty} (f_{\ell-k} + f_{\ell+k}) \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi\ \sin n\phi\ \sin \ell\phi$$

$$f_h = -f_{-h} = \frac{F_h}{2} \quad for \quad h = 2, 4, 6, \dots .$$

$$f_h = 0 \quad for \quad h = 0, 1, 3, 5, 7, \dots .$$

The symbol "$*$" is to be understood as: if vectors a and b have the components a_i, b_i, $i = 1, 2, \ldots N$ respectively, then $a * b$ is a vector with components $a_i b_i$, $i = 1, 2, \ldots N$.

We emphasize that in some respect eq. (9) is generalizing. It is equally applicable to amplitudes which are throughout non-analytical as well as to fully analytical ones. In the first case the region of analycity T reduces to a line-the circumference C, and in the second T goes over to the whole unit circle. It is convenient to represent a_n in the form

$$a_n = \Delta_n + X_n ,$$

where Δ_n is a known vector by means of which the asymptotic behaviour of the solution is taken into account, and X_n the vector to be determined. Inserting the expression $\Delta_n + X_n$ in (9) we get:

$$\sum_{h=-\infty}^{+\infty} E_{hl} * X_h + \sum_{h=-\infty}^{+\infty} \left(H_{hkl} \; X_h * X_k \right) + K_l = 0 \ldots . \qquad l = -\infty, \ldots, -1, 0, 1, \ldots + \infty \tag{10}$$

where

$$E_{hl} = \sum_{k=-\infty}^{+\infty} \Gamma_{lk} \left(\Delta_{h+k} + \Delta_{h-k} \right) - 1$$

$$H_{hkl} = \Gamma_{l,\, k-m}$$

$$K_l = \sum_{h,k=-\infty} \Gamma_{lk} \; \Delta_h * \Delta_{h+k} - \Delta_l .$$

Putting in (9) $a_n = 0$ for $n = -\infty, \ldots -2, -1,$ we get an algebraic system corresponding to the equation of Chew and Mandelstam[2] and Shirkov and al[6]. If in (9) we put $a_n = 0$ for $n = -\infty, \ldots -3, -2$ and $a_{-1} = \frac{\lambda}{2}$ then we obtain an algebraic system[1] corresponding to the Chew-Low equation[7,8]. To the residue $\frac{\lambda}{2}$ in the plane z there corresponds the residue λ in the initial plane z'. λ is the three dimensional vector

$$\lambda = \frac{2}{3} f^2 \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} .$$

Using (9) we have made the calculations for the adiabatic solution of the Chew-Low equation at $f^2 = 0{,}02$ and $f^2 = 0{,}04$, $v'^2(p')$ being taken in the form[9]

$$v'^2(p') = \frac{1}{1 - 0{,}027\, p'^2} .$$

The comparison of the results of the numerical solution with the corresponding results of[10] shows that the system (9) can be solved with a desired accuracy (the error amounts to 0,5% if the infinite system is replaced by algebraic system with 40 ununowns).

In[8] the calculations have been made for $f^2 = 0{,}087$ as well. However as it was shown in[10] the possibility of finding the numerical solutions for so large values of f^2 is doubtful.

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R e f e r e n c e s

1. П.Х.Бырнев, В.А.Мещеряков, И.П.Недялков. Об одной краевой задаче дисперсионных соотношений. Препринт ОИЯИ Р-949, Дубна, 1963.

2. C.Chew and S.Mandelstam. Phys. Rev. 119, 467-477 (1960).

3. А.А.Логунов, Лю И-чень, И.Т.Тодоров, Н.А.Черников. Дисперсионные соотношения и аналитические свойства парциальных амплитуд в теории возмущений. Препринт ОИЯИ Р-1043, Дубна, 1962.

4. I.P.Nedelkov. Darstellungen der Strenamplituden Annalen der Physics (to be printed).

5. И.П.Недялков. Определяне На амплитудите на разсейване.
Izwestia Akad. Bolg. Akademii Nauk,seria fizitcheskaja,1963 (to be printed).

6. А.В. Ефремов, Чжу Хун-юань, Д.В. Широков. Препринт ОИЯИ Д-757, Дубна, 1961.

7. G.F.Chew and F.E.Low. Phys. Rev. 101, 1570 (1956).

8. G.Salzman and F.Salzman. Phys. Rev. 108, 1619 (1957).

9. W.M.Layson. Nuovo Cimento 20, 1207 (1961).

10. I.P Nedelkov and G.I.Pentchev. Numerical solution of the non regularized equations of Low (to be printed).

11. I.P.Nedelkov. Algebraic Equations for Meson-Nucleon Scattering in the approximation of the Two-Particle Unitarity. Preprint E-1262, Dubna, 1963.

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