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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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ON A CONNECTION BETWEEN THE TOTAL CROSS SECTIONS OF REACTIONS $\frac{1}{2}+\frac{1}{2}\to 0+0$ AND THE INTRINSIC PARITIES OF PARTICLES

Дубна 1963

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Биленький С.М., Рындин Р.М. О связи полных сечений реакций $\frac{1}{2}+\frac{1}{2}\to 0+0$ с внутренними четностями частиц

Рассматриваются реакции типа $\frac{1}{2}+\frac{1}{2}\to 0+0$, где $\frac{1}{2}$ и 0 -спины частиц. Показано, что коэффициент при $\vec{P}_0\vec{P}$ в выражении для полного сечения реакции с поляризованными частицами / $\vec{P}_0$ и $\vec{P}$ поляризации пучка и мишени/ положителен, если произведение внутренних частностей равно -1, и отрицателен-в противном случае. Это свойство полных сечений может быть использовано для определения четностей странных частиц.

Работа издается только на английском языке

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Bilenky S.M., Ryndin R.M. On a Connection between the Total Cross Sections of Reactions $\frac{1}{2}+\frac{1}{2}\to 0+0$ and the Intrinsic Parities of Particles

Reactions of the type $\frac{1}{2}+\frac{1}{2}\to 0+0$ are considered, $\frac{1}{2}$ and 0 being the particle spins.

It is shown that the coefficient of $\vec{P}_0\vec{P}$ in the expression for the total cross section of the reaction with polarized particles ($\vec{P}_0$ and $\vec{P}$ being the polarizations of the beam and the target) is positive if the product of the intrinsic parities is -1 and it is negative otherwise. This property of the total cross sections can be used to determine the parities of strange particles.

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ON A CONNECTION BETWEEN THE TOTAL CROSS SECTIONS OF REACTIONS $\frac{1}{2}+\frac{1}{2}\to 0+0$ AND THE INTRINSIC PARITIES OF PARTICLES

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Recently information has appeared about the first experiments carried out on targets with polarized hydrogen [1,2]. The degree of the hydrogen polarization in these experiments amounted to 20%.

The use of polarized hydrogen targets will essentially simplify experiments on the measurement of polarization effects in elastic processes. For example, the proton polarization in $\pi - p$ scattering can be determined by measuring the left-right asymmetry resulting from the scattering on a polarized target. If the target is unpolarized, the proton polarization is determined by measuring the recoil proton asymmetry in a double scattering. A triple experiment on the determination of the correlation of polarizations in scattering is replaced by the measurement of the cross section for the scattering of polarized particles by a polarized target and so on.

The application of polarized hydrogen targets also gives a wide range of possibilities to investigate inelastic processes. In particular, in papers [3] we have shown that the investigation of inelastic reactions

$$
\begin{aligned}
&\pi + p \to \Sigma\ (\Lambda) + K, \\
&\bar{K} + p \to \Sigma(\Lambda) + \pi
\end{aligned}
\tag{1}
$$

using a polarized hydrogen target will allow one to determine unambiguously the relative parities of strange particles. The suggested method consists in the measurement of the correlation of the sign of the left-right asymmetry resulting from a polarized target with the sign of the hyperon polarization occuring in the case of an unpolarized target. This method is based on the symmetry requirements only and is valid for any reaction of the type $\frac{1}{2} + 0 \to \frac{1}{2} + 0$ where 0 and $\frac{1}{2}$ are the particles spins. If the spin of Ξ hyperon will turn out to be equal to $\frac{1}{2}$, its relative parity can be determined from the reaction $\bar{K} + p \to K + \Xi$.

II. In the present paper we shall consider other inelastic reactions on a polarized hydrogen target. We shall show that the comparison of the total cross sections for the reactions

$$
\begin{aligned}
&\bar{\Sigma}\ (\bar{\Lambda}) + p \to K + \pi \\
&\bar{\Xi} + p \to K + K
\end{aligned}
\tag{2}
$$

with polarized protons and antihyperons, and unpolarized protons and antihyperons, respectively, allows one to determine the strange particle parities as well.

A general expression for the total cross section of the reaction can be easily obtained from the requirements of the invariance under space rotations and reflexions. Besides, we should take into account the fact that the cross section depends linearly on each of the polarizations. Let $\vec{P}_0$ and $\vec{P}$ be the polarizations of the beam and the target and $\vec{k}$ be the unit vector in the direction of the relative momentum of colliding particles (c.m.s.). Using these quantities, we can construct the following general expression for the total cross section which satisfies the above requirements:

$$
\sigma = \sigma_0 + a\, \vec{P}_0 \cdot \vec{P} + \beta\, (\vec{P}_0\, \vec{k}\,)(\vec{P}\, \vec{k}\,), \tag{3}
$$

Here σ_0 is the total cross section of the reaction for unpolarized particles. The coefficients a and β depend on the initial energy and their values are determined by dynamics. We shall see, however, that independently of the assumptions about dynamics the sign of the coefficient a is unambiguously fixed by the product of the intrinsic parities of the particles involved in the reaction.

Thus, if we choose the target polarization $\vec{P}$ ortogonal to $\vec{k}$, the comparison of the total cross sections σ and σ_0 allows one to determine the sign of a and, consequently, the relative parity of the particles involved.

III. Now we proceed to the proof of this statement. The amplitude of the process (2) is written in the form:

$$M_{\sigma\sigma'}(\vec{k}', \vec{k})\, \phi(\sigma)\, \chi(\sigma') = \phi^T M(\vec{k}', \vec{k})\, \chi \, , \tag{4}$$

where $\chi(\sigma')$ and $\phi(\sigma)$ are spin functions of the nucleon and the antihyperon, $\vec{k}$ and $\vec{k}'$ are the unit vectors in the direction of the initial and the final relative momenta in the c.m.s. and T denotes transposition.

Averaging the module squared of the amplitude (4) over the initial spin states, we get the following expression for the differential cross section

$$\frac{d\sigma}{d\omega} = Sp\, M(\vec{k}', \vec{k})\, \rho\, M^+(\vec{k}', \vec{k})\, \rho_0^T \, . \tag{5}$$

Here $\rho_0 = ½(1+\vec{\sigma}\, \vec{P}_0)$ and $\rho = ½\,(1 + \vec{\sigma}\, \vec{P})$ are the density matrices of the beam and the target.

From the invariance under space rotations and space inversion we get:

$$M(\vec{k}', \vec{k}) = R^T M(\vec{k}'_R, \vec{k}_R)\, R, \tag{6}$$

$$M(\vec{k}', \vec{k}) = I\, M(-\vec{k}' \; -\vec{k}), \tag{7}$$

where R is the spin rotation operator, $I = I_P I_{\bar{Y}} I_K I_\pi$ is the product of the intrinsic partities of all four particles, $\vec{k}'_R$ and $\vec{k}_R$ are the vectors obtained as a result of a rotation from $\vec{k}'$ and $\vec{k}$.

If we introduce the matrix* $N(\vec{k}', \vec{k}) = \sigma_2\, M(\vec{k}', \vec{k})$ then due to the relation $\sigma_2 R^T \sigma_2 = R^+$ from (6) and (7) we get

$$N(\vec{k}', \vec{k}) = R^+ N(\vec{k}'_R, \vec{k}_R)\, R \, , \tag{8}$$

$$N(\vec{k}', \vec{k}) = I\, N(-\vec{k}', -\vec{k}) . \tag{9}$$

Hence it follows that for $I = 1$ the matrix $N(\vec{k}', \vec{k})$ is a scalar:

$$N(\vec{k}', \vec{k}) = a + b\, \vec{\sigma}\, \vec{n} \, , \quad \vec{n} = \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|} . \tag{10}$$

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* We use the standard representation of Pauli matrices.

In the case $I = -1$ the matrix $N(\vec{k}', \vec{k})$ is a pseudoscalar:

$$N(\vec{k}', \vec{k}) = c\,\vec{\sigma}\,\vec{k} + d\,\vec{\sigma}\,\vec{\kappa}, \quad \vec{\kappa} = \frac{\vec{k}' - (\vec{k}.\vec{k}')\vec{k}}{(1 - (\vec{k}'.\vec{k})^2)^{1/2}} . \tag{11}$$

The expression of the cross section (5) in terms of the matrix N reads.

$$\frac{d\sigma}{d\omega} = Sp\, N(\vec{k}', \vec{k})\, \rho\, N^+(\vec{k}', \vec{k})\, \bar{\rho}_0 , \tag{12}$$

where $\bar{\rho}_0 = \sigma_2\, \rho_0^T\, \sigma_2 = \frac{1}{2}(I - \vec{\sigma}\,\vec{P}_0)$.

By means of formulas (10), (11) and (12) it is easy to get the following expressions for the differential cross sections of the reaction under consideration for both cases:

1. $I = 1$

$$\frac{d\sigma}{d\omega} = \frac{1}{2}\{ |a|^2 + |b|^2 + 2Re\, ab^* [\vec{P}\vec{n} - \vec{P}_0\vec{n}] - |a|^2\vec{P}_0\vec{P} - |b|^2 [2(\vec{P}\vec{n})(\vec{P}_0\vec{n}) - \vec{P}_0\vec{P}] - 2\, Im\, ab^*\, \vec{P}\times\vec{P}_0\, \vec{n} \} . \tag{13}$$

2. $I = -1$

$$\frac{d\sigma}{d\omega} = \frac{1}{2}\{ (|c|^2 + |d|^2)(1 + \vec{P}_0\vec{P}) + 2\, Im\, cd^* [\vec{P}.\vec{n} + \vec{P}_0.\vec{n}] - 2|c|^2(\vec{k}.\vec{P})(\vec{k}.\vec{P}_0) - 2|d|^2(\vec{\kappa}\,\vec{P})(\vec{\kappa}.\vec{P}_0) - 2\,Re\, cd^* [(\vec{k}.\vec{P})(\vec{\kappa}.\vec{P}_0) + (\vec{\kappa}.\vec{P})(\vec{k}.\vec{P}_0)] \} . \tag{14}$$

Integrating (13) and (14) over the directions of $\vec{k}'$, we obtain the following expressions for the total cross sections for the reaction (2):

1. $I = 1$

$$\sigma = \frac{1}{2}\int [|a|^2 + |b|^2]\, d\omega - \frac{1}{2}\int |a|^2 d\omega\, \vec{P}_0 . \vec{P} + \frac{1}{2}\int |b|^2 d\omega\, (\vec{k}.\vec{P})(\vec{k}\,\vec{P}). \tag{15}$$

2. $I = -1$

$$\sigma = \frac{1}{2}\int [|c|^2 + |d|^2]\, d\omega + \frac{1}{2}\int |c|^2 d\omega\, \vec{P}_0 . \vec{P} + \frac{1}{2}\int [|d|^2 - 2|c|^2]\, d\omega\, (\vec{k}.\vec{P}_0)(\vec{k}.\vec{P}). \tag{16}$$

The coefficients of $\vec{P}_0\,\vec{P}$ and $(\vec{P}.\vec{k})(\vec{P}_0.\vec{k})$ in the expressions for the cross sections have a simple meaning. If the quantization axis is chosen along the vector $\vec{k}$, then in the case $I = 1$ the amplitude of the reaction M_0^t from the triplet state with zero projection vanishes and the other amplitudes are

$$M^t_{\pm 1} = b e^{\pm i\phi} , \quad M^s = -i\sqrt{2}\, a . \tag{17}$$

In the case $I = -1$ the reaction from the singlet state is forbidden and the amplitudes of the reaction from the triplet states are

$$M^t_{\pm 1} = -ide^{\pm i\phi} , \quad M^t_0 = i\sqrt{2}\ c . \tag{18}$$

If the target polarization is perpendicular to the vector $\vec{k}$, according to (17) and (18) the expressions for the total cross sections can be written in the form:

1. $I = 1$

$$\sigma = \sigma_0 - \tfrac{1}{4}\,\sigma^s\,(\vec{P}_0 . \vec{P}\,) . \tag{19}$$

2. $I = -1$

$$\sigma = \sigma_0 + \tfrac{1}{4}\,\sigma_0^t\,(\vec{P}_0\ \vec{P}\,) , \tag{20}$$

where σ^s and σ^t_0 are the total cross sections for the reactions from the singlet state and the triplet state with zero projection, respectively.

Thus, the coefficient of $\vec{P}_0\ \vec{P}$ in the expression for the total cross section is negative if the product of the intrinsic parities is $+1$, and is positive in the case $I = -1$. Accordingly, for $\vec{P}_0\ \vec{P} > 0$ the total cross section for the reaction σ is smaller than that σ_0 with unpolarized particles in the case $I = 1$, and σ is larger than σ_0 provided that $I = -1$. If the polarizations $\vec{P}$ and $\vec{P}_0$ are known, this property of the cross sections may be used for the determination of the intrinsic parities of strange particles.

In the general case of the reaction with two particles with spin ½ in the initial state it is not difficult to get ($\vec{P}$ is ortogonal to $\vec{k}$) the following relation between the total cross sections

$$\sigma = \sigma_0 + \tfrac{1}{4}\,(\sigma_0^t - \sigma^s)(\vec{P}_0\ \vec{P}\,) .$$

From this relation it is evident that in the general case there is no connection between the sign of the coefficient of $(\vec{P}_0\ \vec{P})$ and the intrinsic parity, independent of dynamics.

It should be noted that the relative parities can be determined also in investigating the differential cross sections for the reaction (2). As it is seen from (13), for $I = 1$ the ratio of the left-right asymmetry ϵ_{P_0} in the reaction with a polarized beam and an unpolarized target to the asymmetry ϵ_P in the reaction with an unpolarized beam and a polarized target is

$$\frac{\epsilon_{P_0}}{\epsilon_P} = -\frac{P_0}{P} . \tag{21}$$

In the case $I = -1$ this ratio is

$$\frac{\epsilon_{P_0}}{\epsilon_P} = \frac{P_0}{P} . \tag{22}$$

Here we have supposed that P_o and P are directed along the normal to the plane of the reaction.

IV. We note that our conclusion about the behaviour of the total cross sections is valid for any reaction of the type $\frac{1}{2} + \frac{1}{2} \to 0 + 0$, e.g. the reactions $p + He^3 \to He^4 + \pi^+$, $\bar{p} + p \to K + \bar{K}$.

The product of the intrinsic parities $I_p\, I_Y\, I_K$ can be in particular determined from the comparison of the total cross sections of the reactions

$$p + He^3 \to He^4_\Lambda + K^+ ,$$

$$\Sigma^- + He^3 \to He^4 + K^-$$

with polarized and unpolarized particles.

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References

1. A.Abragam et al. Phys. Lett., 2, 310 (1962).
2. O.Chamberlain, C.D.Jeffries et al. Bull. Am. Phys. Soc., 8, 38 (1963).
3. С.М.Биленький, Р.М.Рындин. ЖЭТФ, 35, 827 /1958/.
4. S.M.Bilenky. Nuovo Cim., 10, 1049 (1958).

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