ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

I.P.Nedialkov

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ALGEBRAIC EQUATIONS FOR MESON-NUCLEON SCATTERING IN THE APPROXIMATION OF THE TWO-PARTICLE UNITARITY

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§ 1. Introduction

Generalizing the usual dispersion relations Mandelstam /1/ considers the scattering amplitude A for the meson-nucleon collision as a function of two complex variables, e.g. $s = s' + is''$ and $t = t' + it''$, s being the square of the total energy in the c.m.s. and t - the square of the momentum transfer.

An essential step is then the hypothesis that $A(s,t)$ is holomorphic in the region $\Gamma \equiv 0 < s'' < \infty, 0 < t'' < \infty$ and $\Gamma^* \equiv 0 > s'' > -\infty;\ 0 > t'' > -\infty$.

On the basis of this hypothesis, the crossing symmetry and the condition of the two-particle unitarity Mandelstam formulates a problem for the determination of $A(s,t)$ /1,2/. This problem which we shall call the Mandelstam problem is the main topic of our investigation.

In /1/ and /2/ the Mandelstam problem is formulated as a system of singular integral equations, while in the present paper algebraic equations are preferred.

The choice of algebraic methods as a tool in the dispersion theory is not accidental. This is so because the representations of analytical functions by means of power series are usually more natural and convenient than the representation by means of the Cauchy integrals. This is demonstrated in /4,5,6,/ where algebraic representations for the scattering amplitudes derived from expansions in power series are shown to have great advantages over representations by means of the Cauchy integrals.

In § 2 expressions for the unitarity conditions in the real plane $\Pi \equiv s'' = t'' = 0$ are derived. These conditions together with the analyticity condition and the crossing symmetry define the problem we consider.

In § 3 the closed region $\Gamma + \Pi$ is mapped holomorphically on a unit bicylinder. Then the scattering amplitude is expanded in a Taylor series for the coefficients of which one deduces a system of algebraic equations describing the meson-nucleon scattering process.

In § 4 the algebraic system is generalized to the case for which $A(s,t)$ is not analytic in Γ.

§ 2. Formulation of the Mandelstam problem.

Here we shall consider only collisions of neutral scalar particles. The generalization to the case of charged non-scalar particles is trivial /1/.

The conditions of the two-particle unitarity for the first channel of the reaction $\pi N \to \pi N$ is written in the form /1/

$$A(s', z') - A^*(s', z') = \frac{i}{16\pi^2} \frac{q'}{\sqrt{s'}} \int_0^{2\pi}\int_{-1}^{+1} d\varphi' \, dz_1' \, A^*(s', z_1') \, A\left[s'; z' z_1' + \cos\varphi' \sqrt{(1-z'^2)(1-z_1'^2)}\right] \tag{1}$$

This formula is valid for $M + \mu \leq s' < \infty$ and $-1 \leq z' \leq 1$

Here $s = s' + is''$ is the total energy squared in c.m.s. determined by the four-vectors of incident nucleon and incident meson, q' is the momentum in c.m.s. which is calculated by the formula:

$$q'^2 = \frac{[s' - (M+\mu)][s' - (M-\mu)]}{4s'},$$

z' - is the cosine of the scattering angle which is found from the expression

$$z' = 1 + \frac{t'}{2q'^2}$$

where $t = t' + it''$ is the square of the invariant momentum transfer which is determined from the four-vectors of incident and scattered meson, and φ' - is as usual the angle determined by the scattering planes.

The problem will be formulated in the real plane Π. To write (1) in terms of the variables of this plane we notice that in (1) z' and s' are considered as constants and changes in z_1' and φ' lead to changes of the momentum transfers t_1' and t_2'.

Therefore the connection between new and old variables is given by the expressions

$$z_1' = 1 + \frac{t_1'}{2q'^2}$$

$$z'z_1' + \sqrt{(1-z'^2)(1-z_1'^2)}\cos\varphi' = 1 + \frac{t_2'}{2q'^2}$$

After the substitution of the variables (1) goes into

$$A(s';t') - A^*(s';t') = \iint dt_1' dt_2' F^{(I)}(s';t';t_1',t_2') A^*(s';t_1') A(s';t_2') \tag{2}$$

The formula is valid for all points of the region K^I which is bounded by the lines:

$$a^I \equiv s' \geq (M+\mu)^2;\ t' = 0$$

$$b^I \equiv s = M^2 + \mu^2 - \frac{t'}{2} + 2\sqrt{\left(M^2 - \frac{t'}{4}\right)\left(\mu^2 - \frac{t'}{4}\right)}$$

Here

$$F^I(s';t';t_1',t_2') = \frac{iq'}{8\pi^2\sqrt{s'}}\,\frac{D^I(z_1',\varphi')}{D^I(t_1',t_2')}$$

and the integration limits are given respectively by the expressions for t_1' : $-4q'^2$ is the lower limit, 0 is the upper limit: for t_2' : $2q'^2[z'z_1' + \sqrt{(1-z'^2)(1-z_1'^2)} - 1]$ is the lower limit, $2q'^2[z'z_1' - \sqrt{(1-z'^2)(1-z_1'^2)} - 1]$ is the upper limit.

In an analogous manner we find for the second channel

$$A(s';t') - A^*(s';t') = \iint dt_1' dt_2' F^{II}(s';t';t_1',t_2') A^*(s';t_1') A(s';t_2') \tag{3}$$

The formula is valid for $s', t' \in K^{II}$ which is bounded by the lines

$$a^{II} \equiv s' \leq (M-\mu)^2;\ t' = 0$$

$$b^{II} \equiv s' = M^2 + \mu^2 + \frac{t'}{2} - 2\sqrt{\left(M^2 - \frac{t'}{4}\right)\left(\mu^2 - \frac{t'}{4}\right)};\ 0 \leq t' < -\infty$$

Here

$$F^{II}(s;'t';t_1',t_2')=\frac{iq'(s;'t')}{8\pi^2\sqrt{2M^2+2\mu^2-s-t}}\ \frac{D^{II}(z',\varphi')}{D^{II}(t_1',t_2')}$$

$$q'^2(s;'t')=\frac{[s'+t'-(M-\mu)^2][s'+t'-(M+\mu)^2]}{4[2M^2+2\mu^2-s'-t']}$$

and the functional determinant is calculated from the equations

$$z_1'=1+\frac{t_1'}{2q'^2}$$

$$z'z_1'+\sqrt{(1-z'^2)(1-z_1'^2)}\cos\varphi'=1+\frac{t_2'}{2q'^2}$$

The limits of integration are determined from the expressions for t_1' : $-4q'^2$ is the lower limit, 0 is the upper limit, for t_2' : $2q'^2[z'z_1'+\sqrt{(1-z'^2)(1-z_1'^2)}-1]$ is the lower limit, $2q'^2[z'z_1'-\sqrt{(1-z'^2)(1-z_1'^2)}-1]$ is the upper limit.

In the third channel the unitarity condition is of the form

$$A(s;'t')-A^*(s;'t')=\iint ds_1'ds_2'\,F^{III}(s;'t';s_1',s_2')A^*(s_1';t')A(s_2';t') \qquad (4)$$

$$s;'t'\in K^{III}$$

which is bounded by the lines:

$$a^{III}\equiv s'=M^2-\mu^2-2q'^2-2q'\sqrt{q'^2+\mu^2-M^2}\,;\quad 4M^2\le t'<\infty$$

$$b^{III}\equiv s'=M^2-\mu^2-2q'^2+2q'\sqrt{q'^2+\mu^2-M^2}\,;\quad 4M^2\le t'<\infty$$

where

$$q'^2=\frac{t'}{4}-\mu^2,\quad t'\ge 4M^2$$

$$F^{III}(s;'t';s_1',s_2')=\frac{i}{8\pi^2}\frac{q'}{\sqrt{t'}}\frac{D^{III}(z_1',\varphi')}{D^{III}(s_1',s_2')}$$

The connection between z_1', φ' on the one hand and s_1', s_2' on the other is determined by the equations

$$z_1'=\frac{s_1'-M^2+\mu^2+2q'^2}{2q'\sqrt{q'^2+\mu^2-M^2}}$$

$$z'z_1'+\sqrt{(1-z'^2)(1-z_1'^2)}\cos\varphi'=\frac{s_2'-M^2+\mu^2+2q'^2}{2q'\sqrt{q'^2+\mu^2-M^2}}$$

The limits of integration are given by the formula

for s_1' : $-2q'\sqrt{q'^2+\mu^2-M^2}-s_1'+M^2-\mu^2-2q'^2$ is the lower limit

$2q'\sqrt{q'^2+\mu^2-M^2}-s_1'+M^2-\mu^2-2q'^2$ is the upper limit.

for s_2': $s_2' = \left[z' z_1' + \sqrt{(1-z'^2)(1-z_1'^2)}\right] 2q' \sqrt{q'^2 + \mu^2 + M^2} + M^2 - \mu^2 - 2q'^2$ is the lower limit

$s_2' = \left[z' z_1' - \sqrt{(1-z'^2)(1-z_1'^2)}\right] 2q' \sqrt{q'^2 + \mu^2 - M^2} + M^2 - \mu^2 - 2q'^2$ is the upper limit

This determines the formulas for the unitarity conditions for all channels.

We write the hypothesis about the holomorphicy of the scattering amplitude:

$$A(s,t) \quad \text{is holomorphic in } \Gamma \tag{5}$$

The crossing symmetry is expressed by the relation

$$A(2M^2 + 2\mu^2 - s - t;\, t) = A(s,t) \tag{6}$$

If we take into account the fact that in the dispersion approach no more information is available we are led to the following definition of the Mandelstam problem.

Find a function A of the two complex variables s and t, which satisfies the condition (5) and equations (2), (3), (4) and (6)

§3. Derivation of the algebraic system for the Mandelstam problem

By means of the holomorphic transformation

$$z = r e^{i\varphi} = \frac{s-i}{s+i} \tag{7}$$

$$Z = R e^{i\Phi} = \frac{t-i}{t+i} \tag{8}$$

the real plane Π transforms into the hypersurface C:

$$C \equiv r = 1;\ R = 1$$

and eq. (2) into the equation

$$A(\varphi, \Phi; 1, 1) - A^*(\varphi, \Phi; 1, 1) = \iint d\Phi_1\, d\Phi_2\, F^I(\varphi, \Phi; \Phi_1, \Phi_2) A^*(\varphi, \Phi_1; 1, 1) A(\varphi, \Phi_2; 1, 1) \tag{9}$$

$$\varphi, \Phi \in \varkappa^I$$

in doing so, we put

$$\mathcal{A}\left(\frac{r e^{i\varphi} + 1}{r e^{i\varphi} - 1}, \frac{R e^{i\Phi} + 1}{R e^{i\Phi} - 1}\right) = \mathcal{A}(\varphi, \Phi; r, R)$$

The boundaries of the region $\varkappa^I$ and the limits of integration of the variables Φ_1 and Φ_2 are obtained by inserting (7) and (8) into the corresponding formulas from §2.

Further it is advisable to put

$$A(\varphi, \Phi; r, R) = B(\varphi, \Phi; r, R) + \Delta(\varphi, \Phi; r, R) \tag{10}$$

$$\Delta(\varphi, \Phi; r, R) = \Delta_1(\varphi, \Phi; r, R) + \Delta_2(\varphi, \Phi; r, R)$$

$\Delta_1(\varphi,\phi;r,R)$ is the sum of the pole singularities and singularities for $s\to\infty$ and $t\to\infty$ which after the holomorphic transformation passed from Π to C. $\Delta_2(\varphi,\phi;r,R)$ is a known rough solution of the problem we are interested.

In general $\Delta(\varphi,\phi;r,R)$ should be considered as a known function and $B(\varphi,\phi;r,R)$ - as a correction to be determined.

Owing to (5) it may be asserted that B satisfies the condition:

$B(\varphi,\phi;r,R)$ is holomorphic in the bicylinder K, (11)

$$K\equiv\begin{cases}1>r\geq 0\\ 1>R\geq 0\end{cases}$$

Taking into consideration the fact that the singularities of A on Π and consequently on C are taken into account by $\Delta(\varphi,\phi;r,R)$ we may assume that $B(\varphi,\phi;r,R)$ is piecewise smooth in $C+K$

Then $B(\varphi,\phi;r,R)$ in K can be expanded in the Taylor series

$$B(\varphi,\phi;r,R)=\sum_{m,n=0}^{\infty}X_{mn}z^m Z^n=\sum_{m,n=0}^{\infty}X_{mn}(re^{i\varphi})^m(Re^{i\phi})^n$$

which on C goes over into the Fourier series

$$B(\varphi,\phi)=B(\varphi,\phi;1,1)=\sum_{m,n=0}^{\infty}X_{mn}e^{i(m\varphi+n\phi)} \tag{12}$$

After inserting (10) and (12) into (9) and integrating over ϕ_1 and ϕ_2 we get

$$\mathcal{D}^I(\varphi,\phi)=\sum_{m,n=0}^{\infty}X_{mn}E^I_{mn}(\varphi,\phi)+\sum_{m,n=0}^{\infty}X^*_{mn}G^I_{mn}(\varphi,\phi)+$$

$$+\sum_{mn,pq=0}^{\infty}X^*_{mn}X_{pq}H^I_{mn;pq}(\varphi,\phi)+K^I(\varphi,\phi)=0$$
$$\varphi,\phi\in\mathfrak{x}^I$$

where

$$E^I_{mn}(\varphi,\phi)=e^{i(m\varphi+n\phi)}-\iint d\phi_1 d\phi_2\,\mathcal{F}^I(\varphi,\phi;\phi_1,\phi_2)\Delta^*(\varphi,\phi_1)e^{in\phi_2}$$

$$H^I_{mn;pq}(\varphi,\phi)=-e^{i(-m\varphi+n\phi)}\iint d\phi_1 d\phi_2\,e^{-in\phi_1+iq\phi_2}\mathcal{F}^I(\varphi,\phi;\phi_1,\phi_2)$$

$$G^I_{mn}(\varphi,\phi)=-e^{-im\varphi+in\phi}-\iint d\phi_1 d\phi_2\,\mathcal{F}^I(\varphi,\phi;\phi_1,\phi_2)\Delta(\varphi,\phi_2)e^{-in\phi_1}$$

$$K^I(\varphi,\phi)=\Delta^*(\varphi,\phi)-\Delta(\varphi,\phi)-\iint d\phi_1 d\phi_2\,\mathcal{F}^I(\varphi,\phi;\phi_1,\phi_2)\Delta^*(\varphi,\phi_1)\Delta(\varphi,\phi_2)$$

It is advisable to unify the indices m and n in k and the indices p and q in κ :*

$$\mathcal{D}^{I}(\varphi,\phi)=\sum_{k=0}^{\infty}\mathcal{X}_k E_k^{I}(\varphi,\phi)+\sum_{k=0}^{\infty}\mathcal{X}_k^{*} G_k^{I}(\varphi,\phi)+\sum_{k,\kappa=0}^{\infty}\mathcal{X}_k^{*}\mathcal{X}_\kappa H_{k,\kappa}^{I}(\varphi,\phi)+K^{I}(\varphi,\phi)=0 \quad (13)$$

Eq. (13) is convenient form for expressing the analytic properties of the unknown solution and the unitarity condition(2).

We can make the same calculations for the second and the third channels so that we can write

$$\mathcal{D}^{(\ell)}(\varphi,\phi)=\sum_{k=0}^{\infty}\mathcal{X}_k E_k^{(\ell)}(\varphi,\phi)+\sum_{k=0}^{\infty}\mathcal{X}_k^{*} G_k^{(\ell)}(\varphi,\phi)+\sum_{k,\kappa=0}^{\infty}\mathcal{X}_k^{*}\mathcal{X}_\kappa H_{k,\kappa}^{(\ell)}(\varphi,\phi)+K^{(\ell)}(\varphi,\phi)=0 \quad (14)$$

$$\ell=1,2,3;\ \varphi,\phi\in\varkappa^{(\ell)}$$

where $\ell=1$ corresponds to the first channel, $\ell=2$ - to the second and $\ell=3$ - to the third.

The equation of the crossing symmetry (6) goes over in

$$\tilde{\mathcal{D}}(\varphi,\phi)=\sum_{k=0}^{\infty}\mathcal{X}_k M_k(\varphi,\phi)+N(\varphi,\phi)=0 \quad (15)$$

$$\varphi,\phi\in\varkappa^{(1)}$$

where

$$M_k(\varphi,\phi)=M_{mn}(\varphi,\phi)=e^{i2m\,\mathrm{artg}\left[2i(M^2+\mu^2)+\mathrm{ctg}\frac{\varphi}{2}+\mathrm{ctg}\frac{\phi}{2}\right]+in\phi}-e^{i(m\varphi+n\phi)}$$

and

$$N(\varphi,\phi)=\Delta\left\{2i\,\mathrm{artg}\left[2i(M^2+\mu^2)+\mathrm{ctg}\frac{\varphi}{2}+\mathrm{ctg}\frac{\phi}{2}\right],\phi\right\}-\Delta(\varphi,\phi)$$

Problem (2), (3), (4), (5) and (6) is reduced to the problem (14), (15) which is formulated as follows.

Let functions $E_k^{(\ell)}(\varphi,\phi),\ \ldots\ K^{(\ell)}(\varphi,\phi),\ M_k(\varphi,\phi),\ N(\varphi,\phi)$ be known which are given in the regions $\varkappa^{(\ell)},\ \ell=1,2,3$ and $\varkappa^{(1)}$ respectively. We seek the complex numbers $\mathcal{X}_k,\ k=0,1,\ldots\infty$ by means of which eq. (14) and (15) would be satisfied in $\varkappa^{(\ell)},\ \ell=1,2,3$ and $\varkappa^{(1)}$ respectively.

The problem of such atype are not investigated yet. To solve this problem we shall use methods which have been applied successfuly for numerical solution of the Low equation and other similar problem.

The first method for numerical solution of (14), (15) resembles the method from the paper[7]. This method consists in the following: by means of some weight multipliers $\lambda^{(\ell)}(\varphi,\phi),\ \ell=1,2,3$ and $\tau(\varphi,\phi)$ the functional is first calculated

$$\mathcal{J}(\mathcal{X}_k)=\sum_{\ell=1}^{3}\iint_{\varkappa^{(\ell)}}d\varphi\, d\phi\,\lambda^{(\ell)}(\varphi,\phi)\left[\mathcal{D}^{(\ell)}(\varphi,\phi)\right]^2+\iint d\varphi\, d\phi\,\tau(\varphi,\phi)\left[\tilde{\mathcal{D}}(\varphi,\phi)\right]^2$$

Then it is advisable to seen for values of $\mathcal{X}_k$ which minimize $|\mathcal{J}(\mathcal{X}_k)|$.

The second method[4,6] reduces the problem (14), (15) to an algebraic system:

The boundaries of $\varkappa^{(\ell)}$ are piecewise smooth and this is sufficient for a complete orthonormal system of functions $L_q^{(\ell)}(\varphi,\phi),\ q=0,1,2,\ldots\infty$ to exist by means of which in $\varkappa^{(\ell)}$ the expansions

* This is possible because, $B(\varphi,\phi)$ being piecewise smooth, the series is absolutely convergent.

$$E_k^{(\ell)}(\varphi,\phi)=\sum_{i=0}^{\infty} E_{ki}^{(\ell)} L_i^{(\ell)}(\varphi,\phi)$$

$$G_k^{(\ell)}(\varphi,\phi)=\sum_{i=0}^{\infty} G_{ki}^{(\ell)} L_i^{(\ell)}(\varphi,\phi)$$

$$H_{k,\kappa}^{(\ell)}(\varphi,\phi)=\sum_{i=0}^{\infty} H_{k\kappa i}^{(\ell)} L_i^{(\ell)}(\varphi,\phi)$$

$$K^{(\ell)}(\varphi,\phi)=\sum_{i=0}^{\infty} K_i^{(\ell)} L_i^{(\ell)}(\varphi,\phi)$$

$$M_k(\varphi,\phi)=\sum_{j=0}^{\infty} M_{kj} L_j^{(1)}(\varphi,\phi)$$

$$N(\varphi,\phi)=\sum_{j=0}^{\infty} N_j L_j^{(1)}(\varphi,\phi)$$

are formed.

The insertion of these expressions into (14) and (15) leads to the algebraic system of equations.

$$\sum_{k=0}^{\infty} E_{ki}^{(\ell)} X_k+\sum_{k=0}^{\infty} G_{ki}^{(\ell)} X_k^{*}+\sum_{k,\kappa=0}^{\infty} H_{k\kappa i}^{(\ell)} X_k^{*} X_\kappa + K_i^{(\ell)}=0 \tag{16}$$

$$\sum_{k=0}^{\infty} M_{kj} X_k + N_j = 0 \tag{17}$$

§4. Generalization of the equations for non-analytical scattering amplitudes.

There are two reasons which make the Mandelstam hypothesis probable. First , it is the simplest and natural generalization of the usual dispersion relations. Second, it is checked by the Feynmann diagrams up to the fourth order inclussively and recently it becomes obvious that it will stand the check of the sixth order diagrams too.

Nevertheless unless the hypothesis is not strictly proved it will be useful to do without it, if it is possible.

In the present section we shall not assume that the scattering amplitude $A(s,t)$ in Γ is an analytical function of the complex variables s and t . In this case the problem formulated in §2 passes to the problem: find the function $A(s,t)$ of the two-complex variables s and t which satisfies the conditions (2), (3), (4). and (6).

For the exception of one point, the derivation of the algebraic system for the problem (3), (2), (4) and (6) is identical to the derivations of the corresponding system in §3. The difference is due to the fact that in §3 the function $B(\varphi,\phi)$ is holomorphic in the unit bicylinder and is expanded in the Taylor series since here we refused the analyticity and can only assume that on C it is a piecewise smooth function which is periodic with the period 2π both with respect to φ and ϕ . Therefore on C we have the Fourier series expansion:

$$B(\varphi,\phi)=\sum_{m,n=0}^{\infty}\left[B_{mn}^{*'}\cos m\varphi\cos n\phi+B_{mn}^{*''}\cos m\varphi\sin n\phi+B_{mn}^{*'''}\sin m\varphi\cos n\phi+\right.$$

$$\left.+B_{mn}^{*\,\mathrm{IV}}\sin m\varphi\sin n\phi\right]+i\sum_{m,n=0}^{\infty}\left[B_{mn}^{**'}\cos m\varphi\cos n\phi+B_{mn}^{**''}\cos m\varphi\sin n\phi+\cdots\right]$$

which is equivalent to the series

$$B(\varphi,\phi)=\sum_{m,n=-\infty}^{+\infty} X_{mn}\, e^{i(m\varphi+n\phi)} \tag{18}$$

The series (18) corresponds to (12) and the calculations from §3 can be repeated literally, by replacing (12) by (18).

As a result we have that eq. (14), (15), (16) and (17) pass into the equations:

$$\mathcal{D}^{(\ell)}(\varphi,\phi)=\sum_{k=-\infty}^{+\infty} X_k E_k^{(\ell)}(\varphi,\phi)+\sum_{k=-\infty}^{+\infty} X_k^* G_k^{(\ell)}(\varphi,\phi)+\sum_{k,\kappa=-\infty}^{+\infty} X_k^* X_\kappa H_{k,\kappa}^{(\ell)}(\varphi,\phi)+K^{(\ell)}(\varphi,\phi)=0 \tag{19}$$

$$\ell=1,2,3;\ \varphi,\phi\in \mathscr{X}^{(\ell)}$$

$$\tilde{\mathcal{D}}(\varphi,\phi)=\sum_{k=-\infty}^{+\infty} X_k M_k(\varphi,\phi)+N(\varphi,\phi)=0 \tag{20}$$

$$\varphi,\phi\in \mathscr{X}^{(4)}$$

$$\sum_{k=-\infty}^{+\infty} E_{ki}^{(\ell)} X_k+\sum_{k=-\infty}^{+\infty} G_{ki}^{(\ell)} X_k^*+\sum_{k,\kappa=-\infty}^{+\infty} H_{k\kappa i}^{(\ell)} X_k^* X_\kappa+K_i^{(\ell)}=0 \tag{21}$$

$$i=-\infty,\dots -1,0,1,2,\dots +\infty$$

$$\ell=1,2,3$$

$$\sum_{k=-\infty}^{+\infty} M_{kj} X_k+N_j=0 \tag{22}$$

$$j=-\infty,\dots -1,0,1,2,\dots +\infty$$

5. Conclusion

Algebraic systems have been obtained which describe the meson-nucleon scattering in the two-particle unitarity approximation. The system (16), (17), is derived under the assumption that the scattering amplitude is analytical in complex points and the system (21), (22) without this assumption. In general to construct the algebraic systems the analyticity properties are not obligatory. Therefore the algebraic systems can be used where the integral equations are not applied.

In systems of the type (16), (17) there is a small information concerning the analyticity, unitarity and the crossing symmetry and in the system (21), (22) - smaller. Therefore one believes that the solutions, if they exist, are multivalued.

Mandelstam has used the integral equations for summing the series of the perturbation theory /8/. The same can be made, in principle, by means of the algebraic systems, in this case due to the absence of the Cauchy integrals the calculation process will be more effective.

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