ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ
ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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RELATIVISTIC CAUSALITY IN THE QUANTUM MECHANICS OF A SCALAR PARTICLE

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Introduction

The relativistic causality is usually discussed in the framework of theories of interacting particles. However, one may require it to be fulfilled in free motion of isolated particles as well, in the following sense : Let we have initially a wave packet localized in a local region (so that the probability of detecting the particles anywhere outside of this region is zero). During free motion and spreading of this packet we must have vanishing probability of observing the particle outside of the light cone, i.e.out of the front of the light wave radiating from the bouudary of the above regiou.

However, the quantum-mechanical packet spreads instantly over all space[1,2] even in the relativistic consideration. This fact means that it is possible to transfer a signal faster than light although I should make a reservation just from the very start that this effect is practically unobservable (at least, in direct experiments) and therefore in most applications of the theory (stationary problems, problems of scattering) it may be neglected. However, this effect is directly related to the problem of the local microcausality of the theory: the packet spreading may be considered in the framework of field theory, satisfying the usual causality condition (see, e.g.[3])*.

It is shown that the above effect is allowed by ordinary quantal postulates.

One can attempt to find a way out from this trouble by forbidding the treatment of such observables as the coordinate operators which we use to describe the packet localization. There is another form of the scalar particle theory, which is similar in some respect to the Dirac theory of electron[5]. The investigation of a spreading Dirac particle reveals, however, analogous difficulties and , apparently, more drastic attempts are to be made to overcome them (see §4).

1. Spreading of the scalar particle packet.

The evolution of a physical system consisting of one isolated scalar particle is exhausted by the free motion. This problem is solved in terms of the field theory (although in our case such a theory is equivalent to the non-second-quantized one, see[7], ch. 7) . For the sake of simplicity only one scalar neutral field $\varphi(\vec{x}, t)$ is borne in mind which interacts with nothing. However, we might consider any model of interacting field (scalar field with spinor one, e.g.) satisfying the usual causality condition[3].

1. Owing to the vacuum definition

$$\varphi^{(+)}(\vec{x}, t)\ |0\rangle = \langle 0|\varphi^{(-)}(\vec{x}, t) = 0$$

and therefore the wave function of our particle (one-particle Fock amplitude) in the coordinate representation

$$\Phi(\vec{x}, t) = \langle 0 |\ \varphi(\vec{x}, t)\ | \Psi\rangle = \langle 0 |\ \varphi^{(+)}(\vec{x}, t)\ | \Psi\rangle \tag{1}$$

is a positive frequency solution of the Klein-Gordon equation $(\Box + m^2)\ \Phi(\vec{x}, t) = 0$, see[7], 7 c. It can be expressed in terms of Φ at the moment t_0 by the formula (see [7], ch. 7 , (81)) :

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* The relativistic covariance of the theory by itself does not ensure the relativistic causality. Theories with relativistic form factors or nonlinear ones may be an example (see, e.g.[4]).

$$\Phi(\vec{x},t)=\int d^3x_0\,[\frac{\partial\Delta^{(+)}(\vec{x}-\vec{x}_0,t-t_0)}{\partial t_0}\,\Phi_0(\vec{x}_0,t_0)-$$

$$-\Delta^{(+)}(\vec{x}-\vec{x}_0,t-t_0)\frac{\partial}{\partial t_0}\Phi_0(\vec{x}_0,t_0)\,] \tag{2}$$

where

$$\Delta^{(+)}(\vec{x}-\vec{x}_0,t-t_0)=-\frac{i}{2(2\pi)^3}\int\frac{d^3k}{\sqrt{k^2+m^2}}\,e^{i\vec{k}(\vec{x}-\vec{x}_0)}\,e^{-i\sqrt{k^2+m^2}(t-t_0)} \tag{3}$$

We express $\Phi(\vec{x},t)$ in terms of the initial wave function in the momentum representation. Let

$$\Phi_0(\vec{x}_0,t_0)=\frac{1}{(2\pi)^{3/2}}\int\frac{m\,d^3k_0}{\sqrt{k_0^2+m^2}}\,e^{i\vec{k}_0\vec{x}_0}\,e^{-i\sqrt{k_0^2+m^2}\,t_0}\,\phi_0(\vec{k}_0) \tag{4}$$

By inserting (4) and (5) into (2), we get:

$$\Phi(\vec{x},t)=\frac{1}{(2\pi)^{3/2}}\int\frac{m\,d^3k}{\sqrt{k^2+m^2}}\,e^{i\vec{k}\vec{x}}\,e^{-i\sqrt{k^2+m^2}\,t}\,\phi_0(\vec{k}) \tag{5}$$

Formula (5) can be also obtained in the following way: $\phi_0(\vec{k})$ beiug given, one can find $\phi(\vec{k},t)=\exp[-i\sqrt{k^2+m^2}\,t]\,\phi_0(\vec{k})$, see/7/, ch. 7 (41), and go over to the coordinate representation for obtaining $\Phi(\vec{x},t)$.

By means of (5) we can check that $\Phi(\vec{x},t)$ obeys in fact the first order equation (see/7/, ch. 3)

$$i\frac{\partial}{\partial t}\Phi(\vec{x},t)=[(-i\frac{\partial}{\partial\vec{x}})^2+m\,]^{1/2}\,\Phi(\vec{x},t) \tag{6}$$

Therefore, we need not specify separately $\frac{\partial}{\partial t_0}\Phi(\vec{x}_0,t_0)$ (it is sufficient to know $\phi_0(\vec{k})$ at $t=t_0$). Eq. (6) is relativistic covariant in just the same sense as the Dirac equation in the form $i\frac{\partial}{\partial t}\psi=(\vec{\alpha}\vec{p}+\beta m)\psi$ or the Maxwell equations.

Till now the presentation was made in the representation of the covariant coordinate x. It will be used in §2

However, the operator x is non-Hermitian/6,7/. For the scalar particle we know only one satisfactory coordinate operator – the Newton and Wigner q coordinate/12, 7/. It is non-covariant but only in the sense that it is transformed in accordance with the non-tensor representation of the Lorentz group (a known example of like quantity is the usual three dimensional velocity).

In the q - space we have as before $k_x=-i\frac{\partial}{\partial q_x}$ /13/ and therefore Eq. (6) and formula (5) are of the same form as in the x -space:

$$\Phi(\vec{q},t)=\frac{1}{(2\pi)^{3/2}}\int\frac{m\,d^3k}{\sqrt{k^2+m^2}}\,(k^2+m^2)^{1/4}\,e^{i\vec{q}k}\,e^{-i\sqrt{k^2+m^2}\,t}\,\phi_0(\vec{k}) \tag{5'}$$

$$i\frac{\partial}{\partial t}\Phi(\vec{q},t)=[(-i\frac{\partial}{\partial\vec{q}})^2+m^2]^{1/2}\,\Phi(\vec{q},t) \tag{6'}$$

The norm in the q -space is of the form/13,14/

$$\int d^3q\, \rho\, (\vec{q}, t) = \int d^3q\, \Phi^* (\vec{q}, t)\, \Phi (\vec{q}\, t) = 1$$

2. Consider the particular form of the initial state

$$\Phi_0\, (\vec{q}, 0) = N_1 \iiint \frac{d^3k}{E} \sqrt{E}\; e^{i\vec{k}\vec{q}} \left(\frac{\sin^2 ka}{k^2} \sqrt{E} \right) = \tag{7}$$

$$= \frac{N_2}{ir} \int_{-\infty}^{+\infty} dk\; e^{ikr} \frac{\sin^2 ka}{k}$$

N_1 and $N_2 = (2\pi^3 a)^{-1/2}$ are the normalizing factors; $E = \sqrt{k^2 + m^2}$ $\Phi_0(\vec{q}, 0)$ is the localized function of $\vec{q}$ (see, e.g.[8], page 131);

$$\Phi_0(\vec{q}, 0) \equiv \Phi_0(r) = \begin{cases} \dfrac{\pi N_2}{2r} & \text{for } r < 2a \\ 0 & \text{for } r > 2a \end{cases}$$

and describes a wave packet which is localized inside the $2a$ -radius sphere and has zero average momentum. The center of the packet is at rest and its spreading is described by formula (5) with $t_0 = 0$:

$$\Phi\, (\vec{q}, t) = \frac{N_2}{ir} \int_{-\infty}^{+\infty} dk\; e^{ikr}\; e^{-iEt}\; \frac{\sin^2 ka}{k} \equiv \Phi\, (r, t) \tag{8}$$

There are no such integrals in the tables of Fourier transforms[9]. But according to general reasons Φ cannot be zero at any point during the finite interval of time[2,10]. This holds for the points $|\vec{q}| > t + 2a$ too. The geometric locus of the points $r = t + 2a$ is called by us a light front. This is a sphere whose radius increases at the velocity of light (we use the system of units $c = 1$ and $h = 1$).

Estimate the behaviour of $\Phi\, (r, t)$ at $r > t + 2a$ (out of the light front). The integrand of (8) is analytical in the upper half plane with the cut from im to $i\infty$, see Fig. 1 (one might draw the cut $(im_1\, - im)$ but only with the specified cut one has such a branch of the function $\sqrt{k^2 + m^2}$ which equals $+ |\sqrt{k^2 + m^2}|$ for all real k). In this case on the arc Bb $Jm\, [\sqrt{k^2 + m^2} > 0$ and on the arc Aa $Jm \sqrt{k^2 + m^2} < 0$

On the line (b, im) where k is purely imaginary $k = iq$ we have

$\exp(-i\sqrt{k^2 + m^2}\, t) = \exp|\sqrt{q^2 - m^2}|\, t$ and on the line (im, a) we have

$\exp\,(-i\sqrt{k^2 + m^2}\, t) = \exp\,(-|\sqrt{q^2 - m^2}|\; t)$.

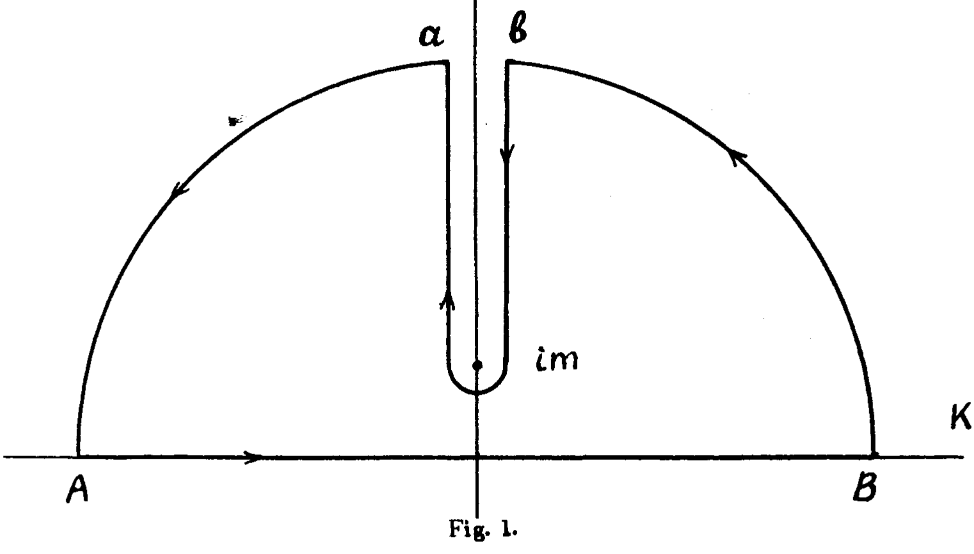

Fig. 1.

According to the Jordan lemma, at $r > t + 2a$ the integral along the arc Bb tends to zero (the more the integral over Aa) and

$$I \equiv \int_{-\infty}^{\infty} dk\, e^{ikr} e^{-t\sqrt{k^2+m^2}\, t} \frac{\sin^2 ka}{k} = \qquad (9)$$

$$= -2 \int_{m}^{\infty} dq\, e^{-qr} \frac{sh^2 qa}{q} \quad sh\sqrt{q^2-m^2}\, t$$

Owing to the fact that the integrand of I is positive I and $\Phi(r, t) = N_2 I/ir$ are non-vanishing at $r > t + 2a$ .Let us estimate I from below. By replacing $\sqrt{q^2 - m^2}$ by a larger function $q - m$ I is reduced to the sum of integral exponential functions. Using the asymptotic representation of these functions (see, e.g./11/) at $r - t - 2a >> \lambda_m$ ($\lambda_m = 1/m$ is the particle Compton wavelength) we get

$$I > L \cong \frac{t}{2m} e^{-mr} \cdot \left\{ \frac{e^{2ma}}{(r-2a)^2 - t^2} + \frac{e^{-2ma}}{(r+2a)^2 - t^2} - \frac{2}{r^2 - t^2} \right\} \qquad (10)$$

Thus, out of the light front the probability density $|\Phi(r, t)|^2$ is non-vanishing and the particle can be observed there.

3. We state some generalizations of the particular problem solved. Of course, besides $\frac{\sin^2 ka}{k^2} \sqrt{E}$ many other functions $\phi_0(k)$ may be indicated which describe in the momentum representation the localized packets (in particular correspon ding to smoother $\Phi_0(\vec{q}, 0)$, e.g. $\frac{\sin^4 ka}{k^4} \sqrt{E}$). See the tables of Fourier transforms/8,9/. For all of them $\Phi(\vec{q}, t)$ will tend to zero out of the light front more rapidly than $exp(-mr)$.

If instead of the initial $\phi_0(\vec{k})$ for which $\vec{k}_{av} = 0$ we take $\phi_\Delta(\vec{k}) = \phi_0(\vec{k} + \vec{\Delta})$ then we get initially the localized packet (with the zero average coordinate $\vec{q}_{av} = 0$ *), with the average momentum $\vec{\Delta}$.

We emphasize that the relativistic consideration ensnre the motion of the packet center at a velocity not greater than that of light.

2. The Hypothetical Experiment on the Signal Transfer at a Superluminary Velocity.

We saw that out of the light front the probability density is appreciable only at a distance of several Compton wavelengts $\lambda_m = 1/m$ from the front edge. The lightest particle, electron, has $\lambda_m = 3.86 \cdot 10^{-11}$ cm. (In the present paper we consider non-zero rest masses only). It seems therefore that the effect cannot be observed in real (at least direct) experiments. It is reasonable to put the question, if there exists this trouble at all (perhaps it is totally masked by the uncertainty relation, for example). We show that the quantal postulates do not forbid the fol lowing hypothetical experiment to be made.

From point A (see Fig. 2) to point G and D two signals are transmitted simultaneously at equal velocity.

* The generalization to the case $\vec{q}_{av} \neq 0$ is trivial: instead of $\phi_0(\vec{k})$, $\phi_0(\vec{k}) \exp(\pm i\vec{k}\vec{q}_{av})$ should be taken. To such an initial function there will correspond $\Phi_c(\vec{q}, t) = \Phi(\vec{q} + \vec{q}_{av}, t)$ if $\Phi(\vec{q}, t)$ corresponded to $\phi_0(\vec{k})$

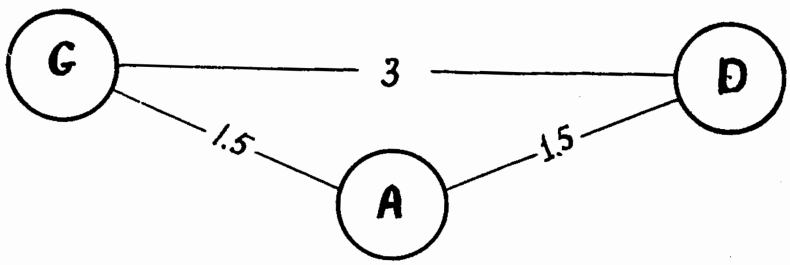

Fig. 2

Having received the signal, the generator G produces one particle during one second. This particle has λ_m equal to one million km (we call it a ν -particle). As a result, by the end of this second, there is one free ν -particle in some region near G . The dimensions of the region cannot exceed 0.3 million km, since according to the theory of relativity, during one second the particle cannot move off from the production point by more than 0.3 million km. The second signal puts in motion detector D for a period of one second .It gives a pulse, if the ν -particle falls into its volume. Let the distances AG , AD and GD be equal to 1.5; 1.5 and 3 mil lion km, respectively.

According to the theory of relativity D must not give any record during its work. However, according to the solution of the problem of a spreading of a localized packet under the given conditions there is the non-vanishing probability of the record and such a record will be a signal transmitted at a velocity greater than the light one. Such a result was obtained in spite of the fact that at all the stages we took into account the theory of relativity (namely 'preparing' the initial state and using the relativistic theory of a spreading of this state).

We discuss possible objections conserning such an experiment. We note first that the quantal postulates (see, e.g., /7/, ch. 1) do not yield the particle mass spectrum and do not forbid, consequently, the existence and the treatment of ν -particles.

1. Does the quantum mechanics allow the localization of a particle in a region smaller than λ_m (and also the detection of the coordinate with an accuracy not worse than λ_m) ?

We would remind of the well known interpretation of the wave function: $|\psi(a)|^2$ is the density of probability that the device measuring the physical quantity $\hat{a}$ gives the value a for it . Thus, the use of the symbol q implies the existence of some abstract device which imparts to q an exact physical meaning. The problem of the realization of such an device bears no relation to the hypothetical experiment since one discusses the intrinsic contradiction in the relativistic quantum mechanics, i.e. contradiction which exists in the abstract 'physical world' of this mechanics ('world' which is constructed according to the quantal relativistic theory postulates).

The examples of Gedanken experiments are known in which coordinates of some particles cannot be measured with accuracy better than λ_m /16/. However this does not prove the existence of this limit of the measurement accuracy: 'Ob ferner dieser Grenze für die erstgenannten Teilchen eine prinzipielle Bedeutung zukommt/16/ oder ob sie durch indirecte Methoden umgehbar ist, lässt sich durch elementare Überlegungen nicht von vornherein entscheiden' * ... We may attach to this limit a principle importance only after constructing such a mechanics which results automatically in the impossibility of exact measurement of the coordinate, i.e. in the ideal physical world of this mechanics, by definition, no device could exactly measure the coordinate.

Thus, the problem of localization of a particle in a region less than λ_m is exhausted by the concrete construction (7) of such a state from the positive – frequency functions (referred to the moment $t = 0$)

The localized state may be referred to a certain moment of time. Even if time is considered as an operator (see Pauli[17] p. 1, §8) this possibility follows from the fact that $\vec{q}$ and $\vec{p}$ commute with t . The fact that the state with definite q coordinate in the x representation is non-local does not mean that it is impossible to prepare and measure the localized states in the q- space[13]. They can be constructed as some superpositions of plane waves, for example. Assuming language of the q coordinate we must express the packet localization and the measurement in D in terms of this coordinate.

2. In the hypothetical experiment use is made of only those devices which localize or detect coordinates. A simultaneous measurement of momenta or energy is not required. Therefore the uncertainty relations are of no importance in this experiment.

We notice that the instantaneous transfer of a signal is due to the stage of the quantal free motion but not to the preparing and recording stage.

3. Fierz[19] and Ma[20] state that the non-vanishing of the causal function Δ_c outside the light cone does not lead practically to the trouble of an instantaneous transfer of a signal. We will show that the authors give examples in which this trouble is absent (or is almost absent). It is natural that these examples do not exclude another example (our hypothetical experiment).

Fierz, in his approach, instead of the initial state specifies the photon source $j(x_\mu)$ in a certain space-time region V_x . The coordinate x is used as a parameter characterizing the space position of the (external) current.

Let particles be produced during the finite time in the finite volume:

$$j(x_\mu) = F(\vec{x})\, f(t) \qquad f(t) = \begin{cases} const & \text{in the interval } -T<t<+T \\ 0 & \text{outside} \end{cases} \tag{11}$$

For example, $f(t) = \int_{-\infty}^{+\infty} \exp iEt \, \frac{\sin ET}{E} \, dE$

For the sake of simplicity of calculations we may assume $F(\vec{x})$ to be proportional to $\delta(\vec{x})$. If a region V_y is situated as is shown in Fig. 3 the current $j(y_\mu)$ in this region can only absorb particles. The integral

$$\int_{V_y} d^4y \int_{V_x} d^4x \; j(y)\,\Delta_c(y-x)\, j(x) = \int_{V_y} d^4y \int_{V_x} d^4x \; j(y)\, \Delta^{(+)}(y-x)\, j(x) \tag{12}$$

* This remark of Pauli ([17], p. 1, §2) may be illustrated by the following notes. Using the proton microscope we are not able to measure the coordinate of a slow electron even with the accuracy 1836 λ_m (see[19]; 5.12). On the other hand, photons emitted b by atoms or neutrinos and electrons emitted by nuclei are localized in regions far smaller than their wave length.

vanishes, only if $\Phi(y) = \int d^4x \; \overset{(+)}{\Delta}(y-x)\, j(x) = 0$ in the region V_y. It may be shown that $\Phi(y) \neq 0$ in this region. Using the device of the type of Fig. 2 we are therefore able to transmit signals with the superluminary velocity (V_x corresponds to G and V_y to D)

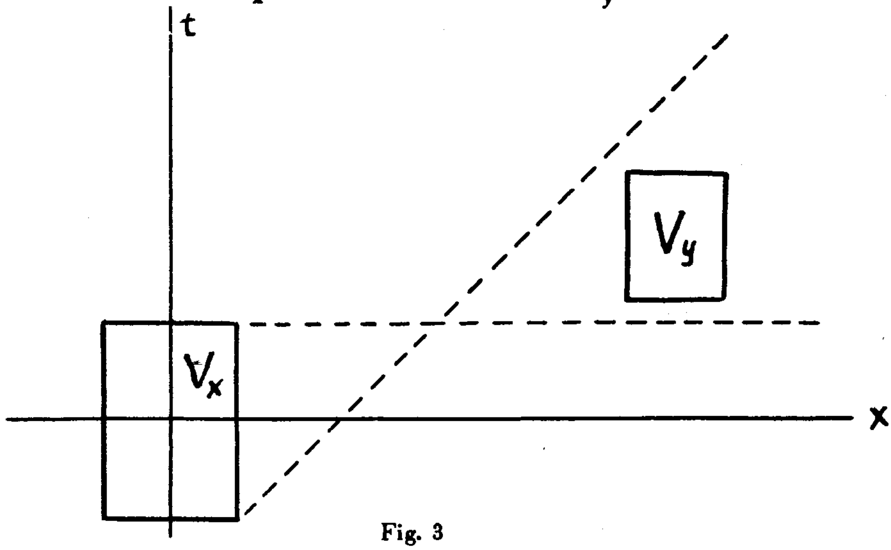

Fig. 3

However it may occur that the Δ_c non-causal properties do not exhibit themselves. S.T.Ma transforms the integral $\Phi(y) = \int d^4x \; \Delta \; (y-x)\, j(x)$ so that the non-causal properties of Δ_c turn out to be carried to a redetermined source: $j(x)$ is replaced by $j_+(x)$, $j_+(x)$ is the positive-frequency part of $j(x)$. If the source is such that $j(x) = j_+(x)$ then Δ_c can be simply replaced by a trully causal function Δ_{ret}. Thirring in his book[21] has shown that if $j(x_\mu) = \delta(\vec{x})\, f_+(t)$ then

$$\int d^4x \; \Delta_c(y-x)\, j(x) = \frac{f_+(t-|\vec{y}|)}{|\vec{y}|} \qquad (13)$$

like for the classical electromagnetic wave. If the line width of the source is small compared with the main frequency (just as in all practical cases of emission) then the source may be considered with a large degree of accuracy to be a positive-frequency current. The spreading effect will be very small[19].

Nevertheless, strictly speaking, it is impossible to transfer a signal by means of $j_+(x)$. It cannot be a localized function of time*. For the signal to be transmitted one must locally change the time part of the current

$$j_+(x) \to j_\ell(x) = j_+(x) + \ell(x)$$

but then $j_\ell(x)$ will not be a positive-frequency function*. (The time dependence of $j_+(x)$ can be changed only for all times simultaneously).

3. Dirac particles packet spreading.

As in §1 we start with the presentation in terms of the field theory. The Fock amplitude

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* If it were so, then its frequency Fourier component would be an analytical function of the frequency and would vanish in the frequency interval ($-\infty$, 0) only when it is identically zero.

$$\Psi_a^{(1,0)}(\vec{x},t) = (\Phi_0, \psi_a(\vec{x},t)\Psi), \qquad \psi_a = \psi_a^{(+)} + \psi_a^{(-)} \tag{14}$$

satisfies the Dirac equation and since $\psi_a^{(+)} \Phi_0 = 0$ then

$$\Psi_a^{(1,0)}(\vec{x},t) = (2\pi)^{-3/2} \sum_{r=1}^{2} \int d^3k \sqrt{m/E}\, (\Phi_0, b_r(\vec{k})\Psi)\, w_a^r(k)\, e^{-iEt} e^{i\vec{k}\vec{x}} \tag{15}$$

The Schweber notations are used/7/, ch. 8b; $\psi_a^{(+)}$ is the fermion destruction operator; $(\Phi, b_r(\vec{k})\Psi)$ is the fermion Fock amplitude in the momentum representation. $\Psi_a^{(1,0)}(\vec{x},t)$ describes the state with positive energy. In the field theory this follows from the definition of the vacuum Φ_0. In the non-second quantized theory of a fermion one may directly require that the wave function describe a fermion rather than a superposition of a fermion and antifermion.

It may be checked that $\psi_a^{(1,0)}(x,t)$ satisfies Eq. (6). However, the demonstration of the contradiction with the theory of relativity is now more difficult than in the scalar case because there is no initial state $\psi_a^{(1,0)}(x,0)$ such that the Dirac density $\rho(\vec{x},0) = \sum_{a=1}^{4} \psi_a^*(\vec{x},0)\,\psi_a(\vec{x},0)$ (indices (1,0) being omitted) is a localized function of $\vec{x}$ (vanishing outside the finite volume). Not dwelling on the proof of this fact we indicate cases when the number of fermions in a certain region, at a certain moment of time, turns out to be larger than the maximal number of particles allowed by the theory of relativity in the situation under consideration. We explain this by the example of the one-dimensional case

$$(\Phi_0, b_r(\vec{k})\Psi) \equiv \Phi_r(k) = \delta_{r,1}\, \delta(k_x)\, \delta(k_y)\, c(k_z) \tag{16}$$

so that $\psi_a(\vec{x},0) = \psi_a(z,0)$. Let the fermion be, at first, near the point $z = 0$. The number of particles which we have at the moment t in the region $2\Delta z$, see Fig. 4, cannot exceed the number that we have had at the moment $t=0$ in the region $2\Delta z + 2t$. More exactly, the quantity

$$\delta = \int_{z_0-\Delta z}^{z_0+\Delta z} \rho(z,t)\,dz - \int_{z_0-\Delta z-t}^{z_0+\Delta z+t} \rho(z,0)\,dz \tag{17}$$

must be negative. Indeed, according to the theory of relativity the value of a physical quantity in a certain region at a certain moment of time is to be defined only by a physical situation in near regions, at near preceding times.

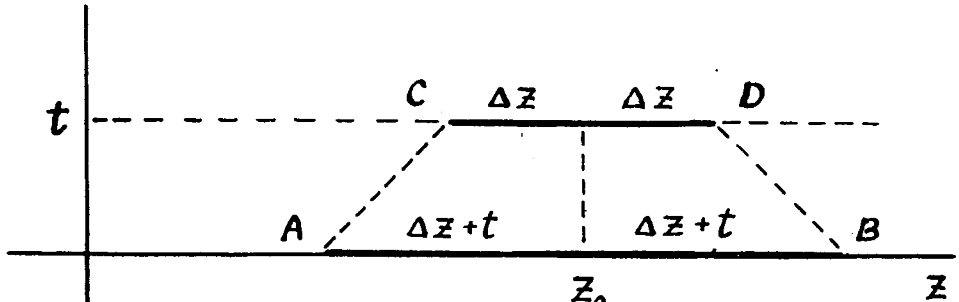

Fig. 4

It is clear that this criterion of the relativistic causality is rather rough. Let

$$c\ (k_z) = \sqrt{\frac{E}{m}} \cdot \sqrt{\frac{2E}{E+m}}\ \{\ \frac{\sin^2 k_z a}{k_z} - \frac{1}{i}\ \frac{\sin^3 k_z a}{k_z} . (\frac{1}{ma} - 1)\ \} \tag{18}$$

The norm factor is not written down. We assume that $ma << 1$ (the first component $\psi_1\ (z, 0)$ is localized in the region $<< \lambda_m$) As the point z_0 we take the point in which $\rho\ (z, 0) = 0$ so that to render the second integral in (17) a rather small quantity (z_0 is equal approximately to $3/2\, (\frac{1}{ma})\, \lambda_m >> \lambda_m$) When $z - z_0$ are large we can calculate δ approximately. When $t = 2\Delta z << \lambda_m$ we have

$$\delta \approx a^6\, e^{-2mz_0}\ (\Delta z)^3\ /\ (m z_0)^5 > 0$$

Besides this example, we calculated numerically the quantity

$$\Delta = \int_{z_0+t}^{\infty} \rho\ (z, t)\ dz - \int_{z_0}^{\infty} \rho\ (z, 0)\ dz \tag{19}$$

with the initial function

$$c\,(k_z) = \sqrt{\frac{E}{m}}\ \sqrt{\frac{2E}{E+m}}\ (\ 2i\ \frac{\sin 5\lambda_m k_z}{k_z} - e^{-5}\ \frac{\sin^2 5\lambda_m \cdot k_z}{k_z}\ ) \tag{20}$$

Then $\rho\ (z, 0)$ vanishes at the point $z = z_0 = 13.1\lambda_m$

The only difference between Δ– and δ -criterion is that the points B and D (Fig. 4) in case of Δ–criterion are moved to infinity (it is easy to understand that the Δ criterion is less rough). The results of calculations carried out by the collaborator of the Mathematical Department of the Laboratory of Theoretical Physics of the JINR Om Sang Ha are given in Table. The accuracy of calculation is the following: the first figure after point is correct. The normalization of the initial state is such that $|\Delta|$ is of the order of unity. Δ is positive for all chosen t .

Table

$\tau = t/\lambda_m$	0.1	0.2	0.5	1	2
Δ	0.1	0.26	0.72	0.88	0.45

4. Attempts Made to Overcome the Causality Trouble

1. Till now we consider the localized initial states. Let us show that the causality trouble cannot be avoided by introducing a postulate that only non localized states are permissible. This postulate would restrict the superposition principle but would not prevent from describing sufficiently exactly all the physical real states.

If the propagation velocity of physical processes is finite then the nnmber of records in D (Fig. 2) during one second must depend on the initial physical situation in the sphere V_D only (with the center at the point D) with the radius not exceeding 0.3 million km. Therefore two non-localized states which have identical initial wave functions in the region V_D, but differ from one another outside V_D, should yield the same number of records in D. To show that the postulate does not mend matter, it is sufficient to give examples in which the numbers of records are different. Let one of the states be described by a certain function Φ_1 and the second one by $\Phi_2 = \Phi_1 + L$ where L is the state localized near G at the moment $t=0$ ($L=0$ in the sphere V_D).

For Φ_1 the number of records at the moment t will be proportional to $|\Phi_1(\vec{q}_D, t)|^2$ and for Φ_2 to $|\Phi_1|^2 + |L|^2 + 2\,Re\, L^* \Phi_1$. The numbers will be different of course if, e.g., $|L(\vec{q}_D, t)| \gg |\Phi_1(\vec{q}_D, t)|$ (so that the interference term may be neglected).

2. In the present paper we have considered the theory of scalar particles (in the form represented in the book by Schweber/7/). The results of §1 and §2 are directly applicable to the Foldy-Wouthuysen equation for an electron (see/7/, 4f). Besides, Eq. (6) can be written for the particle of any spin/22/. It has turned out that in the case of the Dirac equation there is an analogous causality trouble as well. Therefore it is worthwhile to mention here possible attempts to avoid it.

Knight/2/ defines the localized states in another way (so that in particular, the number of particles in such states is not definite). The spreading of such states does not lead to a contradictory with the theory of relativity. However Knight did not change the definition of vacnum and the current-current interaction is carried out as before by the function Δ_c, i.e. in a non-causal manner, see § 2. We emphasize that it is just due to the usual definition of vacuum that only a positive-frequency part of the field operators survives in definition (1) and as a result $\Phi(\vec{x}, t)$ spreads according to the 'non-local' equation (6).

There must be quite a different situation in a theory in which it would be possible to measure (or localize) the coordinate only with an accuracy λ_m, see § 2, sect 1. But we have no such a theory. There is no realization yet for far more radical attempts to eliminate totally the microscopic notion of coordinate/23/ aud to retain it as a statistical notion only, see/24/.

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