ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

G. Domokos

E - 1210

DIAGRAM TECHNIQUE AND REGGE POLES

Nucl. Phys., 1963, v47, n2, p 235-240.

Дубна 1963.

G. Domokos

E - 1210

DIAGRAM TECHNIQUE AND REGGE POLES

Submitted to Nuclear Physics

Дубна 1963.

Abstract

A modified 'isobar approximation' is proposed, where the 'isobar' is described by means of a Regge pole. Rules are set up for the calculation of Feynman diagrams containing Regge poles. Most properties of usual diagrams - in particular, the recipe of calculating imaginary parts-remains valid for these generalized diagrams.

Г. Домокош

ТЕХНИКА ДИАГРАММ И ПОЛЮСА РЕДЖЕ

Аннотация

В работе предлагается модифицированное "изобарное приближение", в котором "изобар" описывается с помощью полюса Редже. Устанавливаются правила для расчета диаграмм Фенмана, содержащих полюса Редже. Многие свойства обычных диаграмм, в частности, средство расчета мнимых частей остается справедливым для этих обобщенных диаграмм.

Работа издается только на английском языке.

1. Introduction

The role that Feynman's diagram technique has played in the development of field theory is well known to every - body. It allows to group the terms of perturbation theory in a suitable manner, so as to select contributions to a certain matrix element almost automatically.

In recent time, the diagram technique has been suitably amplified in order to allow writting down – at least symbolically – quantities that appear in dispersion theory /1/. These generalized diagrams are constructed in such a way that if one expands into a perturbation series the amplitudes (the 'boxes') entering Cutkosky's diagrams, one arrives at a certain class of Feynman diagrams, all of which contain a definite number of particles at a certain section. Up to now, one can deal partically with generalized diagrams, containing at most two particles in the intermediate states.

It has been proposed repeatedly (see e.g. refs. /2,3/) to approximate many particle intermediate states by considering aggregates of two, three ... particles, capable to form a bound or quasi-stationary system, as one composite 'particle', and thus reduce the problem to the calculation of diagrams with a lower number of particles. ('isobar approximation'). It is known, however, that such an approximation for the two-particle intermediate states fails to yield the correct analytic properties of the amplitudes, in particular, it is impossible to continue the expression obtained in this way to a crossed channel.

The correct analytic continuation is obtained by describing the composite system by means of Regge-poles rather than by means of propagators of the Breit-Wigner-type /4/.

It is very plausible that if one wants to calculate diagrams (either in Feynman's, or in Cutkosky's sense) in the isobar approximation for many particle intermediate states, one has to replace the 'Breit-Wigner-propagators' by 'Regge-ones'. In what follows, we describe the rules according to which interacting two-particle systems, described by means of Regge-poles, can be included into the usual diagram technique. These rules follow immediately from the partial wave decomposition of the 'truncated' two-particle Green function (the one-particle singularities corresponding to external 'legs' being split off and some other simple properties of the latter, which are briefly summarized at the beginning of Sec. 2. The use of the rules is illustrated by a simple example which, by the way, shows, how Cutkosky's rules for calculating the discontinuity of an amplitude can be extended for the case when we have Regge-poles in intermediate states. Finally, we discuss some questions concerning the existence of singularities other than poles in the angular momentum plane.

Extended Feynman rules

Regge-poles share many of the properties of ordinary particles. Consider e.g. the Fourier transform of the truncated four point Green function, G in a theory with interaction Lagrangian $g\phi^3$.

The following properties of G can be established (Cf. ref. 5):

1. The asymptotic behaviour of G is determined by a Regge-pole.
2. The trajectory of the leading pole is independent of the squares of external one particle momenta.

3. The residue of the leading pole is factorized just like for amplitudes on the mass shell*.

Assuming that these relusts are true in general one is naturally led to the following rules in constructing diagrams containing Regge-poles.

a) To a Regge-pole in an intermediate state with momentum k and trajectory $a(k^2)$ there corresponds a 'propagator'

$$\frac{1}{(2\pi)^{4I}} \quad \frac{\delta_{mm'}}{(j - a(k^2))\,(j + 1 + a(k^2))} \tag{1}$$

(the sign in the nominator depends on the signature of the Regge-pole).

b) To a vertex: Regge-pole $\to n$ particles with momenta and helicities $p_1 \nu_1 \ldots p_n \nu_n$ respectively ascribe:

$$(2\pi)^{4I} g f_j (p_1 \ldots p_n) < jmk \,|\, p_1 \nu_1 \ldots p_n \nu_n > \tag{2}$$

where g is a phenomenological coupling constant, the function $f_j(p_1 \ldots p_n)$ is an essentially kinematic factor, exhibiting the behaviour of the residue near threshold. The bracket can be easily recognized to be a coupling coefficient of Clebsch-Gordan series of the Poincaré group/8/. Note in particular that the bracket contains a δ function assuring conservation of four-momentum. (If in the ket in eq. (2) one has more than two particles, additional degeneracy labels must be introduced, which are suppressed here. Correspondingly, one will have several trajectories).

c) Sum over the magnetic quantum numbers m, and integrate over j along a contour, usual in the Watson-Sommerfeld integral, with the weight:

$$(2i)^{-1} \frac{2j+1}{\sin j\pi}$$

d) Usual Feynman rules (integration over internal momenta, etc.) remain valid.

Let us remark that the continuation of the relativistic Clebsch-Gordan coefficients (eq. (2)) in j presents no difficulty, as the latter can be expressed in terms of hypergeometric functions/8,9/.

In the case if the Regge-pole is coupled to two spinless particles at both sides, the rules stated above can be considerably simplified. In fact, the Clebsch-Gordan -coefficients are then proportional to associated Legendre functions of the first kind.

One can carry out immediately the summation over the magnetic quantum numbers, make use of the addition theorem for Legendre functions and integrate over j. As a result of these operations one will be left with a Legendre function of the first kind, of index $a(k^2)$ while the factors $(2j+1)$ in the nominator and $(j+a+1)$ in the denominator compensate each other.

---

* The analogous properties of the Green function, when the external particles are on the mass shell , are known for a long time. (See e.g. ref./6/) and other literature quoted there). Although the properties 1) to 3) have been proved in ref./5/ for a certain class of diagrams, we believe that they are more general. Note in particular that one -particle singularities of Green functions show properties, closely analogous to those enumerated above/7/.

So, we obtain the following, simplified rules:

a) To a Regge-pole with momentum k and trajectory $a(k^2)$ coupled to spinless particles with four-momenta p_1, p_2 on the one side, to those with four-momenta p_3, p_4 on the other one, ascribe the 'propagator':

$$\frac{\pi}{(2\pi)^4 i} \quad \frac{1}{\sin a(k^2)\pi} \quad P_{a(k^2)}(-z) \tag{3}$$

where z is the cosine of the angle between the momenta p_1 and p_3 in the centre of mass system (c.m.s.) of the 'particles' '1' and '2'.

b) To a vertex: Regge-pole → two spinless particles with momenta p_1, p_2 ascribe the factor

$$(2\pi)^4 \, i \, \delta(k - p_1 - p_2) \, g \, q_{12}^{a(k^2)} \tag{4}$$

where q_{12} stands for the modulus of the relative momentum of 'particles' '1' and '2' in their c.m.s.

c) Apply usual Feynman rules of integrating over internal momenta etc.

The factor $q_{12}^{a(k^2)}$ corresponds to the function $f_j\,(p_1 \ldots p_n)$ in rule b) (see ref./10/).

For the sake of convenience, let us quote the invariant expressions of q_{12} and z .

$$4\, s\, q_{12}^2 = \lambda\,(s, p_1^2, p_2^2) \tag{5}$$

where $s = (p_1 + p_2)^2$ and the function λ is defined as follows:

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \tag{6}$$

For z , the cosine, we of course obtain:

$$z = \frac{t - (p_1^\circ - p_3^\circ)^2 + q_{12}^2 + q_{34}^2}{2\, q_{12}\, q_{34}} \tag{7}$$

Here $t = (p_3 - p_1)^2$, $p_1^\circ = (q_{12}^2 + p_1^2)^{1/2}$; $p_3^\circ = (q_{34}^2 + p_3^2)^{1/2}$

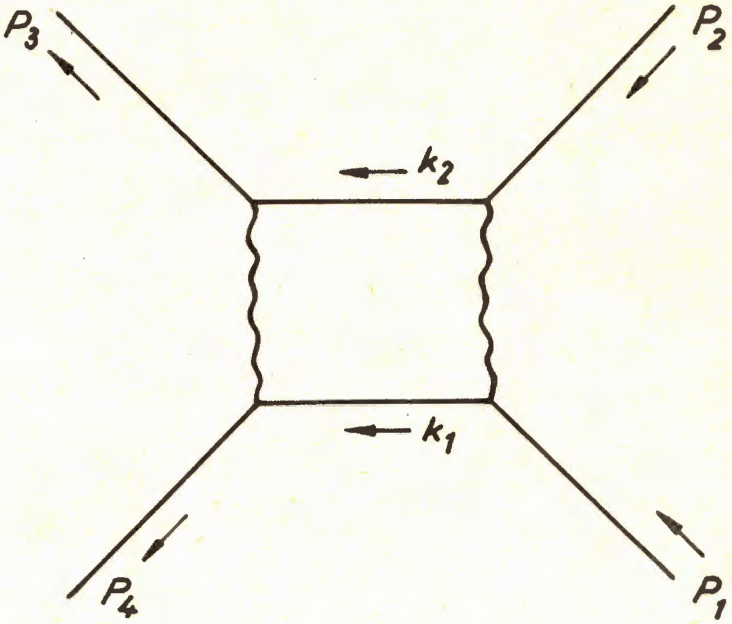
Let us remark finally that 'propagator' for a Regge-pole in the form of eq. (3) has been conjectured already by Frautschi et al. and by Gribov and Pomeranchuk/11/, for the special case, when all the external particles are on the mass shell.

3. Example

Let us agree in the following to denote ordinary particles by straight lines, Regge-poles by wavy ones. For the sake of simplicity, we consider spinless particles of unit mass, one Regge-pole with trajectory $a\ (s)$, coupled to two particles at both sides.

The reader can at once verify hinself that for the simple pole diagram (scattering of particles with the exchange of a Regge-pole) one obtains the familiar expression, commonly used now in high energy physics (cf. Ref. 11).

Let us go over to a somewhat less trivial example and calculate the contribution of the diagram of Fig. 1.



For the sake of simplicity we calculate its imaginary part in the s -direction. (For the notation see Fig. 1.). Our simplified rules are applicable, so we find for F_s , the imaginary part in the s -direction /13/

$$F_s(s,t) = \frac{1}{16\pi^2} \iint \frac{dz_1\, dz_2}{\sqrt{-k(z, z_1, z_2)}} A(s, z_1) A^*(s, z_2) \times$$

$$\times \left(\frac{s-4}{s}\right)^{1/2} \theta(-k(z, z_1, z_2)) \quad . \tag{8}$$

where

$$z = 1 + 2\frac{(p_4 - p_1)^2}{s-4} \equiv 1 + 2\frac{t}{s-4}$$

$$z_1 = 1 + 2\frac{(p_1 - k_1)^2}{s-4} \equiv 1 + 2\frac{t_1}{s-4} \tag{9}$$

$$z_2 = 1 + 2\frac{(p_4 - k_1)^2}{s-4} \equiv 1 + 2\frac{t_2}{s-4}$$

The functions $A(s, z_1)$, $A(s, z_2)$ are given by (cf. eq. (3)) :

$$A(s, z_1) = g^2\pi \left(\frac{t_1 - 4}{4}\right)^{\alpha(t_1)} P_{\alpha(t_1)}\left(-1 - 2\frac{s}{t_1 - 4}\right) \frac{1 \pm e^{-i\alpha(t_1)}}{\sin \alpha(t_1)\, \pi} \tag{10}$$

and correspondingly for $A(s, z_2)$ by writing t_2 instead of t_1 .

Inserting eq. (10) into eq. (8) and taking into account eqs. (9), one obtains an expression, which for $s \to \infty$ goes over to that derived by Amati et al[14] in a somewhat different context. In particular, if one takes the partial wave projection of $\xi(s, t)$ in the t -channel, one finds a cut in the angular momentum[14]. In order to calculate $F_{st}(s, t)$ the spectral function, we can proceed as in ref.[13] . We remark that $A(s, t)$ as given by eq. (13), satisfies a dispersion relation in t (or equivalently in z). Ignoring subtractions, we write for $s > 4$:

$$A(s, z) = 1/\pi \int \frac{dz'}{z'-z} A_z(s, z') \qquad (11)$$

with

$$A_z(s, z) = (2i)^{-1} [A(s, z+i0) - A(s, z-i0)]$$

Inserting eq. (14) into the expression of $A(z)$ (eq. (11)) and finding its jump across the cut in the t- plane, we finally arrive at the familiar expression:

$$F_{st}(s, t) = \frac{1}{16\pi^2} \iint \frac{dz_1 dz_2 \, \theta(k)}{\sqrt{k(z, z_1, z_2)}} A_z(s, z_1) A_z^*(s, z_2) \qquad (12)$$

3. Discussion

In our opinion, the lesson one can learn from the foregoing calculations is that a Regge-pole is in no way worse from the point of view of diagram technique than a 'particle'. One can, of course, object that the diagram technique developed here is of no practical use, because , we do not know the trajectories of Regge-poles in field theory. However, if we use any approximate expression for the trajectory (e.g. a semiempirical formula, or a perturbation expansion possessing the correct analytic properties, then all our considerations remain valid. In particular, we see that the jump of a diagram across a cut can be calculated essentially according to Cutkosky's reciple[1] if only the Regge-trajectory in the intermediate state has a correct spectral representation.

A possible field of application of the diagram technique seems to be the search for cuts in the angular momentum plane. After the work of Amati et al[14] it seems probable that cuts (or possibly other singularities as well) do exist, although they do not necessarily follow from general principles, like unitarity[15,16]. The application of diagram technique may give useful hints, where such singulatities may appear.

R e f e r e n c e s

1. R.E.Cutkosky. Jour. Math. Phys. 1, 429 (1960).
2. S.Mandelstam, F.E.Paton, R.F.Peierls and A.G.Sarker. Ann. Phys. (New York), 18, 198 (1962).
3. I.S.Ball, W.R.Frazer and M.Nanenberg. Phys. Rev. 129, 478 (1962).
4. G.F.Chew, S.C.Frautschi and S.Mandelstam. Phys. Rev. 126 1202 (1962).
5. J.C.Polkinghorne, preprint (1962).
6. G.Domokos, Nuovo Cim. (to be published).
7. K.Symanzik. Nuovo Cim. 13, 503 (1959).
8. A.S.Wightman, Relations de Dispersion et Particulas Elementaires, p. 159 (Hermann, Paris, 1960).,
   M.Jacob and G.C.Wick. Ann. Phys. (New York) 7, 404 (1959).
   G.C.Wick. Ann. Phys. (New York) 18, 65 (1962).
9. U.Fano and G.Racah, Irreducible Tensorial Sets, p. 144 (Academic Press, New York, 1955).
10. V.N.Gribov and I.Ja.Pomeranchuk. Nucl. Phys. 38, 516 (1962).
11. S.C.Frautschi, M.Gell-Mann and F.Zachariasen. Phys. Rev. 126, 2204 (1962).
    V.N.Gribov and I.Ja.Pomeranchuk. preprint (1962).
12. S.D.Drell. Proc. 1962, Intern. Conference on High Energy Phys. Geneva, p. 897.
13. S.Mandelstam. Phys. Rev. 112, 1344 (1958).
14. D.Amati, S.Fubini and A.Stanghellini, Proc. 1962. Conf. on High Energy Phys. Geneva, p. 560.
15. D.Y.Wong, preprint (1962).
16. S.Okubo, preprint (1962).

Received by Publishing Department
on February 26, 1962.