ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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REGGE POLES AND PERTURBATION THEORY. III.

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Abstract

The contributions from the cuts in the ℓ-plane to the scattering amplitude are investigated. A way of separating the pole terms is discussed. These terms should understood as the main ones in the expansion in $e^2/\sqrt{E}$

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ПОЛЮСА РЕДЖЕ И ТЕОРИЯ ВОЗМУШЕНИЙ
III

В работе исследуются вклады разрезов в ℓ -плоскости в амплитуду рассеяния. Обсуждается способ выделения полюсных членов - главных в смысле разложения по $e^2/\sqrt{E}$.

Работа издаётся только на английском языке.

1. In our previous papers[1-4] we suggested a method for calculating the Regge trajectory by combining the Regge ideas and the perturbation theory.

As has been shown in[3], the obtaining of the Regge trajectory is considerably impeded by the presence of the cuts in the l-plane. Here some examples are given which illustrate the contribution from the cuts and a method for singling out the terms determining the Regge trajectory. It is indicated that these terms are main ones as far as the expansion in $e^2/\sqrt{E}$ is concerned.

2. Consider first the scattering of scalar charged particles in the Coulomb field. The interaction Lagrangian is of the form

$$L = ie(A_\nu + A_\nu^{ext})\phi^+ \Gamma_\nu \phi - e^2 (A_\nu + A_\nu^{ext})(A_\nu + A_\nu^{ext})\phi^+ \phi \qquad (1)$$

where

$$\Gamma_\nu = \frac{\vec{\partial}}{\partial x_\nu} - \frac{\overleftarrow{\partial}}{\partial x_\nu},$$

A_ν is the quantized electromagnetic field, and A_ν^{ext} is the static Coulomb field

$$\vec{A}^{ext} = 0, \quad A_0^{ext} = -\frac{e}{r}. \qquad (2)$$

The charge sign was chosen so that the particle would be attracted to the centre.

It was stated in[5] that the scattering amplitude is as follows:

$$M = \exp(\alpha B(m, \lambda, t)) M_r, \qquad (3)$$

where m and λ are the particle and photon masses, respectively, t is the transferred momentum squared. In the second order by α, we have

$$M_r = M^1 + M_r^2, \qquad (4)$$

where M^1 is the scattering amplitude in the first order, and M_r^2 is related to the scattering amplitude in the second order M^2 by

$$M_r^2 = M^2 - \alpha B M^1. \qquad (5)$$

Having calculated M^2 by the diagrams in Fig. 1 and using for B the expression from[5], we get:

$$M_r^2 = 4\pi\alpha^2 \frac{E}{\sqrt{m^2 - E^2}} \frac{1}{t} \ln t - \frac{2\pi^2\alpha^2}{E} \frac{1}{\sqrt{-t}} + \ldots \qquad (6)$$

Among the terms in (6) which give the contributions like $\frac{1}{t}\ln t$, we wrote explicitly only the first term. The other other terms are independent of E and in calculating the levels $(E/\sqrt{m^2 - E^2} >> 1)$ give only small corrections.

It should be noted that M^2 contain the terms of the form $\ln^2 \frac{t}{m^2}$. However, this term vanishes when M_r^2 is calculated with the aid of (5). As was shown in[6], the presence of the term quadratic in the field leads to the appearance of the cut in the l-plane from $-\alpha - ½$ up to $\alpha - ½$. The second term in (6) corresponding to the diagram $1c$ contains non- integer degree of t, i.e. this is a contribution from the cut in the lowest order. Thus the contribution from the Regge poles to the scattering amplitudes in the second order is

$$[M_r]_p = \frac{4\pi a}{t}\left(1 + \frac{aE}{\sqrt{m^2 - E^2}}\,\ell n ++ \dots\right). \tag{7}$$

Obviously the application of the renormalization group method gives the Regge trajectory

$$\ell(E) = -1 + \frac{aE}{\sqrt{m^2 - E^2}}. \tag{8}$$

To elucidate the character of the contribution from the cut we consider the diagrams draw in Fig. 2 and 3. Those in Fig. 2 fail to give the contribution, while the sum of the matrix elements of the diagram 1c and the diagrams in Fig. 3 is found to be

$$-\frac{1}{\sqrt{-t}}\{2\pi^2 a^2 + (2\pi^2 a^2)^2 \frac{1}{8} + [(2\pi^2 a^2)^2 \frac{1}{8}]^2 \frac{1}{8} + \dots\} = -\frac{1}{\sqrt{-t}}\frac{1}{8}\sum_{n=1}^{\infty}\left(\frac{\pi a}{2}\right)^{2^n}. \tag{9}$$

This sum is the total contribution from the cut if only the potential interaction is taken into account (the interaction with the quantized electromagnetic field is neglected).

3. The scattering of a spinor particle in the Coulomb field was treated in[3]. Here only the potential part of the interaction is considered, whilst the interaction with the quantized electromagnetic field is neglected. In the second order the account of the latter interaction leads to the appearance of the diagram like 1b. Its matrix element contains the terms of the form $\ell n^2 \frac{t}{m^2}$ or $\frac{1}{t}\ell n^2 \frac{t}{m^2}$. However, these terms vanish when calculating M_r^2 in (3). So, M_r^2 looks like the amplitude T in paper[3]. The terms of the form $\ell n \frac{t}{m^2}$ or $\frac{1}{t}\ell n \frac{t}{m^2}$ corresponding to the diagram 1b are now independent of E and are extremely small if compared with the terms of the form:

$$\frac{m}{\sqrt{m^2 - E^2}}\ell n \frac{t}{m^2} \text{ or } \frac{m}{\sqrt{m^2 - E^2}}\frac{1}{t}\ell n \frac{t}{m^2}.$$

4. Now let us be concerned with the electromagnetic scattering of a fermion with the mass m and the charge e on a boson with the mass M and the charge $-e$. The boson momenta in the initial and final states are denoted by q_1 and q_2. The scattering amplitude is

$$M_{inv} = \exp(aB(m,M,\lambda,s,t,u))[A + i\frac{\hat{q}_1 + \hat{q}_2}{2}C]. \tag{10}$$

Having calculated M_{inv} in the first and the second order and making use of the expression from[5] for aB, we get:

$$A = 4a^2 m \frac{(s - 2m^2)(s + M^2 - m^2)}{s} f(s) \frac{1}{t}\ell n\, t, \tag{11}$$

$$C = -8\pi a \frac{1}{t} + 4a^2[-s + M^2 + 2m^2 + m^2\frac{M^2 - m^2}{s}] f(s)\frac{1}{t}\ell n\, t, \tag{12}$$

where

$$f(s) = \int_{(M+m)^2}^{\infty} \frac{ds'}{(s' - s)\sqrt{[s' - (M+m)^2][s' - (M-m)^2]}}. \tag{13}$$

In the given order the expression A and C do not contain terms of the form $\frac{1}{t}\ell n^2 \frac{t}{m^2}$.

As has been emphasized in [4], the application of the renormalization group method to the invariant amplitudes A and C yields no information on the Regge poles as long as these amplitudes are not connected with the physical states. To determine the Regge trajectory one has to consider the amplitudes in the centre-of-mass system f_1 and f_2 connected with the total scattering amplitude by

$$M_{cm} = e^{aB}\left(f_1 + i\,\frac{\vec{\sigma}[\vec{q}_2 \times \vec{q}_1]}{q}\,f\right). \tag{14}$$

The relationships between f_1, f_2 and A, C were given in many papers (see, e.g.,[7]). Using these, we have from (11), (12)

$$f_1 = -a\,\frac{[(\sqrt{s}-m)^2-M^2](\sqrt{s}+m)}{[s-(M+m)^2][s-(M-m)^2]}[1+\eta(s)\ell n\,t]$$

$$-\frac{a}{2}\,\frac{[(\sqrt{s}-m)^2-M^2](\sqrt{s}+m)}{s}\,\frac{1}{t}[1+\eta(s)\ell n\,t] \tag{15}$$

$$-\frac{a}{2}\,\frac{[(\sqrt{s}+m)^2-M^2](\sqrt{s}-m)}{s}\,\frac{1}{t}[1+\xi(s)\ell n\,t],$$

$$f_1 - z f_2 = -\frac{a}{2}\,\frac{[(\sqrt{s}+m)^2-M^2](\sqrt{s}-m)}{s}\,\frac{1}{t}[1+\xi(s)\ell n\,t], \tag{16}$$

where s is the square of the total energy in the centre-of-mass system

$$\xi(s) = \frac{a}{2\pi}\left\{s - M^2 - 2m^2 - m^2\,\frac{M^2-m^2}{s} + \frac{m}{\sqrt{s}-m}\,\frac{(s-m^2)(s+M^2-m^2)}{s}\right\} f(s), \tag{17}$$

$$\eta(s) = \frac{a}{2\pi}\left\{s - M^2 - 2m^2 - m^2\,\frac{M^2-m^2}{s} + \frac{m}{\sqrt{s}+m}\,\frac{(s-m^2)(s+M^2-m^2)}{s}\right\} f(s). \tag{18}$$

Like in the extreme case $M \to \infty$ treated in [3] the firest term in f_1 is the contribution from the cut from $-a$ up to $+a$. According to the results of this paper the contribution from the cut from $-a-1$ up to $+a-1$ in $f_1 - z f_2$ is equal to :

$$[f_1 - z f_2]_c = -a\,\frac{[(\sqrt{s}-m)^2-M^2](\sqrt{s}+m)}{2s}\,\frac{1}{t}[-1+\eta(s)\ell n\,t]. \tag{19}$$

Subtracting (19) from (16) we obtain the following expression for the contribution of the Regge poles in $f_1 - z f_2$

$$[f_1 - z f_2]_p = -a\,\frac{s-M^2-m^2}{\sqrt{s}}\,\frac{1}{t}\left[1 + \frac{a}{\pi}\,\frac{m}{\sqrt{s}}(s+M^2-m^2)f(s)\right]. \tag{20}$$

The renormalization group method gives the Regge trajectory in the form

$$\ell^{+}(s) = -1 + \frac{a}{\pi}\frac{m}{\sqrt{s}}(s + M^2 - m^2) f(s). \qquad (21)$$

For $(M-m)^2 \le s \le (M+m)^2$, we have

$$f(s) = \frac{2}{\sqrt{[(M+m)^2 - s][s-(M-m)^2]}} \operatorname{arc\,tg} \frac{s-(M-m)^2}{(M+m)^2 - s}. \qquad (22)$$

In the extreme case exps. (15), (16), (19), (20) and (21) go over into (3), (4), (9), (10) and (11) respectively in paper/3/.

5. In all previous examples the contributions from the cuts has either 'not that' asymptotic behaviour in t, or it was known beforehand that this was the contribution from the cut. Consider now the electromagnetic scattering of scalar particles with the masses m and M. The interaction Lagrangian in this case is

$$L = ieA_\nu(\phi^{+}\Gamma_\nu \phi - \Phi^{+}\Gamma_\nu \Phi) + e^2 A_\nu A_\nu(\phi^{+}\phi + \Phi^{+}\Phi). \qquad (23)$$

Here ϕ, ϕ^{+} are the field operators with the mass m, Φ, Φ^{+} are the fields with the mass M, the particles charges are equal to $+e$ and $-e$, respectively.

The 'main' diagrams for determining the Regge trajectory will be the diagrams drawn in Fig. 4, where p_1, q_1 are the initial and p_2, q_2 are the final momenta, $p^2 = -m^2$, $q^2 = -M^2$.

The scattering amplitudes in the first and the second orders in e^2 at $t \to \infty$ are

$$T_2 = e^2 \frac{(q_1+q_2)_\nu(p_1+p_2)_\nu}{t} = -e^2\frac{s-u}{t} = -e^2\frac{2(M^2+m^2-s)+t}{t}. \qquad (24)$$

$$T_4 = \frac{e^4}{16\pi^2} 4(p_1 q_1)[(p_1+p_2)_\nu(q_1+q_2)_\nu - (p_1+p_2-q_1-q_2)_\nu(p_1-q_1)_\nu \frac{t}{2u}] J_0 = \frac{e^4}{16\pi^2}[2(M^2+m^2-s)]^2 J_0, \qquad (25)$$

where

$$t = -(p_1-p_2)^2,$$

$$s = -(p_1+q_1)^2,$$

$$u = -(p_1-q_2)^2, \qquad (26)$$

$$J_0 = -\frac{\ell n\, t}{t} f(s).$$

Here we are faced with the situation : the amplitude in the lowest order has a bad asymptotic behaviour, but in the next order it becomes better. At the same time one cannot reject 'bad' terms in T_2 in this way we shall not obtain the amplitudes of the Regge type. So, both the pole terms and the terms whose appearance is due to the cuts have the same asymptotic behaviour in t.

Let us suppose that we reconstructed somehow the amplitude of the Regge type by T_2 and T_4. Then we have to get the values for the energy in the bound states up to e^2. However, in this approximation one should deal with the Coulomb interaction only. We pass to the centre-of-mass system and single out in (24) the products of the fourth components of the vectors $p_1, \dots q_2$:

$$(q_1 + q_2)_4 (p_1 + p_2)_4 = s - \frac{(m^2 - M^2)^2}{s}, \tag{27}$$

$$s = (E_1 + E_2)^2$$

E_1 and E_2 are the particle energies in the c.m.s. This corresponds to the breaking up of $u - s$ into

$$u - \frac{(m^2 - M^2)^2}{s} \quad \text{and} \quad s - \frac{(m^2 - M^2)^2}{s}$$

the first expression vanishes at the threshold, the second one is finite there. Having distinguished in T_4 the term proportional to the square of $s - \frac{(m^2 - M^2)^2}{s}$ and having found the Regge trajectory corresponding to these terms, we can get the correct value for the Coulomb levels of a two- particle system.

A similar situation arises in calculating the energy levels of positronium. It has been already pointed out in /4/, that none of the invariant amplitudes of the electron-positron scattering in the fourth order has a 'good' asymptotic behaviour. Having separated the singlet part of the scattering amplitude, we got the amplitude with the asymptotic behaviour appropriate to obtain the levels. The triplet part of the scattering amplitude contains 'bad' terms. Having made the phase shift analysis, one can see that the appearance of these 'bad' terms is due to the cuts. If we remember that in the basis of the diagrams calculated in /4/ it is possible to get the energy levels up to e^2, while the spin effects give the contribution to the energy of order of e^4, we may hope, (if the parts independent of spin particle states are singled out of the invariant amplitudes) to obtain the terms with a correct asymptotic behaviour. We followed this procedure, and as a result we got an expression coinciding with the singled scattering amplitude what is consistent with the spin independence of the energy levels in the e^2– approximation. (The rejected terms in the resonance region are small).

6. What do the results of the 'theoretical experiment' we have made speak about. First and foremost, the terms determining the energy levels may be not main ones at $t \to \infty$. The asymptotic behaviour we are expecting to get is spoiled by the terms due to the cuts.

The separation of the terms caused by the cuts becomes still much more difficult for they not only spoil the asymptotic behaviour, but also give the terms 'quite decent by their appearance' and having a asymptotic behaviou good in t. The help comes from paper[3] where it was pointed out that to get the energy levels it is necessary to expand in $e^2/\sqrt{E}$ rather than in e^2. Calculating the perturbation theory diagrams we get, as a rule

$$e^n \Phi(s, t) f(s, t),$$

where $f(s, t) \to \infty$ approaching s the threshold. Having divided $\Phi(s, t)$ into the sum $\phi_1(s, t) + \phi_2(s, t)$ where the first term is finite, and the second one is zero at the threshold, we single out the term which is the main one in the sense of the expansion in $e^2/\sqrt{E}$.

The reconstruction of the Regge trajectory by this term yields correct results.

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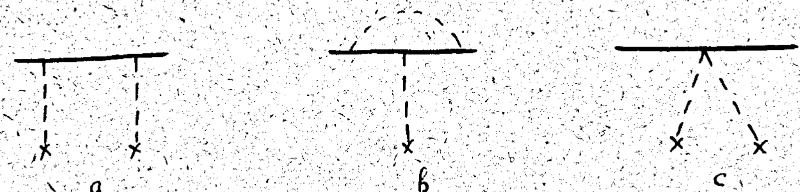

Fig. 1.

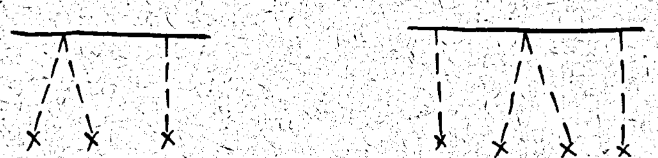

Fig. 2.

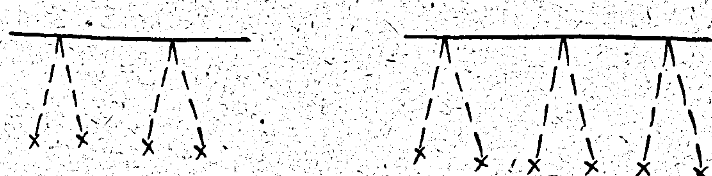

Fig. 3.

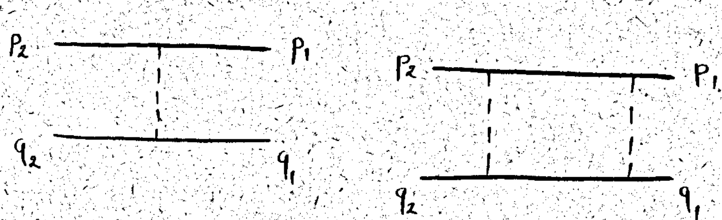

Fig. 4.