ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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PARTIAL WAVES FOR PION NUCLEON SCATTERING TAKING INTO ACCOUNT PION-PION INTERACTION

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Abstract

An effect of the $\pi\pi$-interaction in state T = J = 0 on the pion-nucleon scattering partial waves is investigated by the method of dispersion relations. In the final expressions a transition to the statical limit is made which is just compared with experimental data. The most probable form of the δ_0^0 phase shift of pion-pion scattering is discussed. Relations are obtained which link the pion-pion interaction contributions to S and P waves of πN -scattering. On the basis of these relations some works on the pion–nucleon scattering are analysed. Consequences following from the abovementioned relations are considered for the statical limit.

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ПАРЦИАЛЬНЫЕ ВОЛНЫ
ПN-РАССЕЯНИЯ С УЧЕТОМ
ПП-ВЗАИМОДЕЙСТВИЯ

Аннотация

Методом дисперсионных соотношений исследуется влияние взаимодействия в состоянии с $T = J = 0$ на парциальные волны πN -рассеяния. Результаты сравниваются с экспериментальными данными в статическом пределе. Обсуждается наиболее вероятный вид δ_0^0 фазы πN -рассеяния. Получены соотношения, связывающие вклады $\pi\pi$ -взаимодействия в S - и P - волны πN - рассеяния. На основании этих соотношений проведен анализ результатов ряда работ по рассеянию π - мезонов на нуклонах.

Работа издается только на английском языке

Introduction

The present paper is devoted to the investigation of the influence of pion–pion interaction with $T = J = 0$ on the pion–nucleon scattering partial waves and is related to paper [1]. An analogous problem is discussed in papers [2–4]. Takahashi [2] investigated the problem on the basis of the Chini–Fubini one-dimensional representations. The method of obtaining the equations for partial waves used by the author was subjected to criticism [5]. The smallness of the pion–pion scattering length a_0 obtained is very likely due to this method.

Hamilton et al [3] analysed the pion–pion interaction in state with $T = J = 0$ by investigating the so-called "differences" $\Delta^{(\pm)}_{\ell, J}$ i.e. the sum of contributions from the cuts $-\infty < s \leq 0$ and $|s| = M^2 - \mu^2$ in the plane s. The combined analysis of the "differences" is facilitated by the fact that for each of them one introduces the appropriate parameters which take into account the effect of the distant singularities. The contribution from the nearest singularities (the right half of the circle $|s| = M^2 - \mu^2$) is interpreted as pion–pion interactions. Some relations are derived here (§ 4) which the pion–pion contributions to the pion–nucleon scattering must satisfy. On their basis it may be asserted that in [3] an attempt to single out the pion–pion contributions from the "differences" failed.

Atkinson [4] uses dispersion relations for the backward scattering with respect to the variable $\nu = \vec{q}^{\,2}$. With such an approach the "differences" are associated with the pion–pion interaction only, what follows from papers [6]. The pion–pion scattering phase shifts are determined by the analytical continuation of the "differences" from the region $\nu > 0$ into the cut $-\infty < \nu \leq -1$. The analytical continuation is performed by means of the conformal mapping ([4] Eq. (3.3)) which transforms the plane ν with cut $-\infty < \nu \leq -1$ into $(2n+1)$ sheet Reeman surface. Therefore for $n = 1, 2$ Atkinson's arguments are not true.

The method of taking into account the pion–pion interaction applied here enables us to choose among various forms of the energy behaviour of the δ_0^0 phase shift. It is shown that the S–dominant solution of Chew, Mandelstam, Noyes [7] does not describe the energy dependence of the pion–nucleon scattering partial waves. The approximations of the scattering length and the resonance behaviour of the δ_0^0 phase shift were also considered taken from papers of Serebriakov and Shirkov on the solution of the system of equations for the pion–pion scattering partial waves [8]. The conclusion is drawn that the variant with the δ_0^0 resonant phase shift is preferable.

1. The unitarity condition for the process $\pi\pi \to N\bar{N}$

The unitarity condition of the process $\pi\pi \to N\bar{N}$ is written by means of the states with a given helicity J_{++}, J_{+-} which are expanded in partial waves as follows *

$$J_{++} = J_{--} = \frac{p_3}{8\pi p_3^0}\left[-A^{(+)} + 4Mq_3^2 \cos^2\theta_3 \cdot \beta\right] = \frac{1}{p_3 \cdot p_3^0}\sum_\ell (\ell+\tfrac{1}{2})(p_3 q_3)^\ell f_+^\ell \cdot P_\ell(\cos\theta_3)$$

$$I_{+-} = -I_{-+} = \frac{q_3}{8\pi}\sin\theta_3 \cdot B = q_3 \sum_\ell \frac{\ell+\tfrac{1}{2}}{\sqrt{\ell(\ell+1)}}(p_3 q_3)^{\ell-1} f_-^\ell \cdot P'_\ell(\cos\theta_3)\,. \qquad (1.1)$$

* For detailed notations see [1].

For the isotopic index (+) the sum$_\ell$ is over even l's. The unitarity condition in the two–particle approximation expresses the partial wave phase shift $f_{\pm}^{\ell}$ in terms of s, d and other phase shifts of pion–pion scattering with $T = 0$. Since we shall be interested in the region of small q_3's then the assumption

$$\delta_0^{\ell} = 0 \qquad \ell \geq 2 \tag{1.2}$$

is natural.

From (1.1) is follows that $\beta = \frac{B}{s - \bar{s}}$ is the real function in the interval $-\infty < \nu \leq -1$. Approximating the higher partial waves by the pole terms /9/ we get for $A^{(+)}$ and β the expressions

$$A^{(+)} = 4Mq_3^2 \,Cos^2 \theta_3 \cdot \beta + \gamma \cdot e^{i\delta_0^0} - 4Mq_3^2 \,Cos^2 \theta_3 (\Delta_\beta - \Delta_2^{(+)})$$

$$\beta = \beta_1 e^{i\delta_0^2} + g^2 (\Delta_\beta - \frac{\Delta_1^{(+)}}{4pq})$$

$$\Delta_1^{(+)} = \frac{3}{2} \int_{-1}^{+1} Cos^2 \theta_3 \cdot \Delta_\beta \cdot d\,Cos\,\theta_3 \; ; \qquad \Delta_2^{(+)} = ½ \Delta_2 = ½ \Delta_1$$

(for detailed nototions see /1/)

where β_1 and γ are the unknown real functions. Comparing (1.3) with similar expressions for a and β (1.4) /1/ it may be noted that, first, the lower terms $A^{(+)}$ and β contain the s and d – waves; second, the assumption (1.2) leads to the appearance in $A^{(+)}$ of the unknown real function β_1. Therefore the method of taking into account the pion–pion interaction suggested in /6/ is to be modified.

2. Dispersion relations for $A^{(+)}$ and β. Equations for partial waves

Equations for the partial waves of the low–energy pion–nucleon scattering will be derived by combining dispersion relations for the functions $A^{(+)}$ and β when $c = Cos\,\theta = \pm 1$.

It is convenient /1/ to consider dispersion relations for the forward scattering with respect to the variable s, i.e., in the usual form:

$$\Phi(\nu, +1) = \frac{1}{\pi} \int_{(M+1)^2}^{\infty} Im\,\Phi(\nu' + 1) \left[\frac{1}{s(\nu') - s(\nu)} + \frac{1}{s(\nu') - \bar{s}(\nu)} \right] ds(\nu'). \tag{2.1}$$

In this case the functions $A^{(+)}$ and β require a different number of subtractions. Assuming, as usual, the high–energy total cross section to be constant it is easy to obtain that for $A^{(+)}$ two subtractions are sufficient, while for β there is no one required. Taking into consideration the fact that in the following use will be made of an ordinary assumption $Im\,\Phi \approx Im\,f_3^{3}$ we may restrict ourselves to one subtraction. The convergence of the integrals will be ensured. The second subtraction will not lead to the appearence of an additional constant owing to the symmetry properties of the function $A^{(+)}$. The comparison of final expressions with experimental data will serve as a criterion in choosing the number of necessary subtractions.

For the backward scattering it is advantageous to write dispersion relations with respect to the variable $\nu = \vec{q}^{\,2}$. The only nearest singularity is the cut $-\infty < \nu \leq -1$ from the reaction $\pi\pi \to N\bar{N}$. Owing to the assumption (1.2) the dispersion relation for $\beta(\nu, -1)$ is of the form:

$$\beta(\nu,-1) = \frac{1}{\pi}\int_0^\infty \frac{Im\beta(\nu',-1)}{\nu'-\nu}\,d\nu' + \text{pole term} \qquad \Delta_\beta \tag{2.2}$$

$$(Im\ \beta\ (\nu<0,-1) = 0\ ).$$

Consider the function $-\frac{8\pi p^0}{p} J_{++} = A^{(+)} - 4Mq_3^2 \cos^2\theta_3 \cdot \beta$ According to (1.3) on the cut $-\infty<\nu<-1$ it has a simple structure: the first term is the s-wave amplitude f_+^0, the second one is the sum of all the higher partial waves in the pole approximation. Therefore the unphysical cut can be calculated in just the same way as it has been done in/1/, i.e. considering instead of $-\frac{8\pi p^0}{p_3} J_{++}$ the function $-\frac{8\pi p^0}{p_3}\frac{J_{++}}{F_0(\nu)}$. The function $F_0(\nu)$ has no zeros in the complex plane ν. The phase shift $F_0(\nu)$ on the cut $-\infty<\nu<-1$ coincides with δ_0^0.

Then, the dispersion relation for $A^{(+)}$ can be written in the form:

$$A^{(+)}(\nu,-1) = A^{(+)}(0,-1)\,F_0(\nu) + 4M\beta(0,-1)[F_0(\nu)-1] +$$

$$+ \Sigma\ Res\ \frac{4Mq^2[1-F_0(\nu)]}{[M^2-s(0)][M^2-\bar{s}(0)]} + \frac{\nu}{\pi}\int_0^\infty \frac{Im\,A^{(+)}(\nu',-1)}{\nu'(\nu'-\nu)}\,d\nu' + \tag{2.3}$$

$$+ \frac{\nu}{\pi}\int_0^\infty \frac{Im\,[A^{(+)}(\nu',-1) + 4M\omega'^2\beta(\nu',-1)][\frac{F_0(\nu)}{F_0(\nu')}-1]}{\nu'(\nu'-\nu)}\,d\nu'.$$

The quantity $A^{(+)}(0,-1)$ is expressed in terms of the scattering lengths:

$$\frac{1}{4\pi} A^{(+)}(0,-1) = \frac{2M+1}{2M} a^+ - 2M(a_1^+ - a_3^+). \tag{2.4}$$

The function $F_0(\nu)$ is determined as follows:

$$F_0(\nu) = \exp\{\frac{\nu}{\pi}\int_0^\infty \frac{\delta_0^0(k)}{(k^2+1)(k^2+1+\nu)}\,dk^2\}. \tag{2.5}$$

An arbitrary polynom can be introduced into Eq. (2.5), but it is not defined by the phase shift δ^0. $F_0(\nu)$ is the only function whose behaviour at $\nu\geq 0$ is completely defined only by δ_0^0 on the physical (for pion-pion scattering) cut $-\infty<\nu<-1$. In this case it is not very important whether the phase shift δ_0^0 satisfies the crossing symmetry relations of pion-pion scattering or not: $F_0(\nu)$ enters (2.3) when $\nu>0$. Therefore the details in the behaviour of δ_0^0 at $\nu<0$ are of no importance. If δ_0^0 well approximates the true function δ_0^0 then this is sufficient for determining $F_0(\nu)$ in the region $\nu>0$. The true form of δ_0^0 is unknown, hence, in what follows use will be made of various concrete assumptions on the form of the function $tg\ \delta_0^0$.

The transition to dispersion relations for the partial waves is performed by means of the relations

$$f_s^{(\pm)}(\nu) = \tfrac{1}{2}[f_1^{(\pm)}(\nu,+1) + f_1^{(\pm)}(\nu,-1)]$$

$$f_{p_{3/2}}^{(\pm)}(\nu) = \frac{1}{6}[f_1^{(\pm)}(\nu,+1) - f_1^{(\pm)}(\nu,-1)] \tag{2.6}$$

$$f_{p_{1/2}}^{(\pm)} - f_{p_{3/2}}^{(\pm)} = \tfrac{1}{2}[f_2^{(\pm)}(\nu,+1) + f_2^{(\pm)}(\nu,-1)].$$

The connection between functions $f^{(+)}_{1,2}$ and $A^{(+)}$ and β is of the form

$$f^{(+)}_1(\nu, C) = \frac{p_0 + M}{8\pi W}\left[A^{(+)}(\nu, C) + 2(W - M)\beta(\nu, C)[2p^0 q^0 + \nu(1 + C)]\right]$$

$$f^{(+)}_2(\nu, C) = \frac{p_0 - M}{8\pi W}\left[- A^{(+)}(\nu, C) + 2(W + M)\beta(\nu, C)[2p^0 q^0 + \nu(1 + C)]\right] \qquad (2.7)$$

In the integrands, the functions $Im\, A^{(+)}$ and $Im\, \beta$ are expressed in terms of the amplitude $Im\, f^{(+)}_{p_{3/2}}$:

$$Im\, A^{(+)}(\nu, C) = \left\{ \frac{W + M}{p^0 + M} 3C + \frac{W - M}{p^0 - M}\right\} Im\, f^{(+)}_{p_{3/2}}$$

$$Im\, \beta(\nu, C) = \frac{1}{4p^0\omega + 2\nu(1 + C)}\left[\frac{1}{p^0 + M} 3C - \frac{1}{p^0 - M}\right] Im\, f^{(+)}_{p_{3/2}} \qquad (2.8)$$

Eqs. (2.1) – (2.8) enable us to express the real parts of the partial waves s, $p^{(+)}_{1/2}$, $p^{(+)}_{3/2}$ in terms of the coupling constant, the $\sin^2 a_{33}$, subtraction parameters and the function $F_0(\nu)$.

Until recently the expansion in powers of $1/M$ was not used anywhere.

3. The Static Limit ($1/M \to 0$)

Expanding Eqs. (2.1) – (2.8) in powers of $1/M$ we get

$$Re\, f^{(+)}_s(\nu) = \frac{a^+}{2}[1 + F_0(\nu)] - \frac{\nu}{\pi} P \int_0^\infty \frac{Im\, f^{(+)}_{p_{3/2}}(\nu')}{\nu'(\nu' - \nu)}\left[\frac{F_0(\nu)}{F_0(\nu')} - 1\right] d\nu'$$

$$Re\,(f^{(+)}_{p_{1/2}} - f^{(+)}_{p_{3/2}}) = -2\frac{\nu}{\omega} f^2 - \frac{\nu\omega}{\pi} P \int_0^\infty \frac{Im\, f^{(+)}_{p_{3/2}}(\nu')}{\nu'\omega'} \frac{d\nu'}{\nu' - \nu} \qquad (3.1)$$

$$Re\,(f^{(+)}_{p_{1/2}} + 2f^{(+)}_{p_{3/2}}) = \frac{a^+}{2}[1 - F_0(\nu)] + \frac{2\nu}{\pi} P \int_0^\infty \frac{Im\, f^{(+)}_{p_{3/2}}(\nu')}{\nu'(\nu' - \nu)} d\nu' + \frac{\nu}{\pi} P \int_0^\infty \frac{Im\, f^{(+)}_{p_{3/2}}(\nu')}{\nu'(\nu' - \nu)}\left[\frac{F_0(\nu)}{F_0(\nu')} - 1\right] d\nu'.$$

Eqs. (3.1) contain a single parameter, the scattering length $a^{(+)}$. They satisfy the crossing symmetry properties /1/:

$$f^{(+)}_s(\omega) - f^{(+)}_s(-\omega) = 0 ,$$

$$[f^{(+)}_{p_{1/2}}(\omega) - f^{(+)}_{p_{3/2}}(\omega)] + [f^{(+)}_{p_{1/2}}(-\omega) - f^{(+)}_{p_{3/2}}(-\omega)] = 0 \qquad (3.2)$$

$$[f^{(+)}_{p_{1/2}}(\omega) + 2 f^{(+)}_{p_{3/2}}(\omega)] - [f^{(+)}_{p_{1/2}}(-\omega) + 2 f^{(+)}_{p_{3/2}}(-\omega)] = 0$$

Note that the pion-pion and pion-nucleon terms of Eqs. (3.1) apart satisfy Eqs. (3.2).

An interesting feature of dispersion relations (3.1) is the absence of pion-pion terms in the expression $Re\,[f^{(+)}_{p_{1/2}} - f^{(+)}_{p_{3/2}}]$. The same holds for the difference $Re\,[f^{(-)}_{p_{1/2}} - f^{(-)}_{p_{3/2}}]$ which follows from Eqs. (5.4) – (5.6) /1/. The above feature follows from a certain symmetry of pion-pion contributions to pion-nucleon scattering.

4. The Symmetry Properties of Pion-Pion Terms in Pion-Nucleon Scattering

We denote by $G^{(\pm)}_{\ell J}(W)$ the contribution of pion-pion terms to the pion-nucleon scattering partial wave with given ℓ, J and the isotopic index $(\pm)$. Since pion-pion terms enter dispersion relations for the backward scattering only, then from (2.6) it follows that

$$G^{(\pm)}_{s}(W) = -3\, G^{(\pm)}_{p_{3/2}}(W) \tag{4.1}$$

The relation (4.1) is true in the s and p approximation, i.e. in the low-energy region. For the backward scattering the replacement $s \to \bar{s}$ means the transition from the variable $W = p^{\circ} + \omega$ to that $W' = p^{\circ} - \omega$ so that

$$W \cdot W' = M^2 - 1 \tag{4.2}$$

In this case $\nu(W) \equiv \nu(W')$ and $A^{(+)}[\nu(W)] = A^{(+)}[\nu(W')]$. In the approximation (1.2) terms responsible for pion- pion contributions to the functions $A^{(+)}$ and β are of the form:

$$A^{(+)}_{\pi\pi}(\nu) = -\frac{1}{\pi}\int_{-\infty}^{-1} \frac{\gamma\, Sin\, \delta^{\circ}_{o}}{\nu' - \nu}\, d\nu' \tag{4.3}$$

$$\beta_{\pi\pi}(\nu) = 0$$

what leads to an additional relation

$$G^{(+)}_{p_{3/2}}(W) - G^{(+)}_{p_{1/2}}(W) = \frac{p^{\circ} - M}{p^{\circ} + M}\, G^{(+)}_{s}(W) \tag{4.4}$$

Using Eqs. (4.3) and (2.6), (2.7) it is easy to get that

$$W\,[\,G^{(+)}_{s}(W) - 3\,G^{(+)}_{p_{3/2}}(W)\,] = W'\,[\,G^{(+)}_{s}(W') - 3\,G^{(+)}_{p_{3/2}}(W')\,] \tag{4.5}$$

$$W\,[\,G^{(+)}_{p_{1/2}}(W) - G^{(+)}_{p_{3/2}}(W)\,] = W'\,[\,G^{(+)}_{p_{1/2}}(W') - G^{(+)}_{p_{3/2}}(W')\,]$$

Eqs. (4.1) and (4.5) lead to symmetry properties of the functions $G^{(+)}$ found first by Lovelace/10/ :

$$W\, G^{(+)}_{s}(W) = W'\, G^{(+)}_{s}(W')$$

$$W\, G^{(+)}_{p_{1/2}}(W) = W'\, G^{(+)}_{p_{1/2}}(W')$$

$$W\, G^{(+)}_{p_{3/2}}(W) = W'\, G^{(+)}_{p_{3/2}}(W') \tag{4.6}$$

They result immediately from the crossing symmetry properties and from the second of equalities (4.3).

Similar relations for the functions $G^{(-)}_{\ell J}(W)$ are more complicated because in this case $A^{(-)}_{\pi\pi}$ and $B^{(-)}_{\pi\pi}$ are different from zero. They reflect only the crossing symmetry properties of the functions $A^{(-)}$, $B^{(-)}$

Since the crossing symmetry properties for the partial waves are the simplest in the static approximation we go over to the limit $M \to \infty$.

Let us introduce the following notation:

$$g_{\ell J}^{(\pm)}(\omega) = \lim_{M \to \infty} G^{(\pm)}(W) \, . \tag{4.7}$$

For the functions $g_{\ell J}^{(\pm)}(\omega)$ it is easy to find that

$$g_{\ell J}^{(\pm)}(\omega) = \pm\, g_{\ell J}^{(\pm)}(-\omega) \, . \tag{4.8}$$

The comparison of (4.8) with the crossing symmetry relations (5.1)/1/ shows that the same functions $g_{P_{1/2}}^{(\pm)}(\omega) - g_{P_{3/2}}^{(\mp)}(\omega)$ must be simultaneously symmetrical and antisymmetrical in ω i.e. it is equal to zero, or

$$g_{P_{1/2}}^{(\pm)}(\omega) \equiv g_{P_{3/2}}^{(\mp)}(\omega) \, . \tag{4.9}$$

Eq. (3.1) satisfies the conditions (4.1) , (4.8), (4.9). Eqs. (4.1), (4.4), (4.6), (4.8) and (4.9) are convenient for checking the non-contradictory of pion-pion contributions to different patrial waves, if the latter is calculated independently. In papers of Hamilton et al the relations (4.6) are fulfilled.However, the equality (4.1) is valid for the 'differences' $\Delta^{(\pm)}$ themselves, which besides of pion-pion contributions contain, e.g., the integrals from the pion-nucleon scattering crossing reaction. If one accepts the suggested breaking down of $\Delta^{(+)}$ into pion-pion terms and the contribution of distant singularities then (4.1) for pion-pion contributions is fulfilled neither in magnitude nor in sign. Threfore an attempt to single out from the differences $\Delta_{\ell J}^{(+)}$ pion-pion terms failed.

The equality (4.9) is fulfilled for $\Delta_{P_{1/2},\,3/2}^{(+)}$ within the statistical errors. For the quantities $\Delta_{\ell,J}^{(-)}$ none of the aforementioned relations is fulfilled, what may not be put down to the neglect of d waves, since $\Delta_{\ell,J}^{(-)}$ are calculated up to 100 MeV only. If we explain the non-fulfilment of (4.9) and (4.8) by terms of the order $\frac{1}{M}$ then the equality (4.1) must however be fulfilled.

5. Comparison with Experimental Data

The set of equations (3.1) contains one subtraction parameter a^+. It is small $(a^+ = -0.005)$ /11/ therefore in calculations we put $a^+ = 0$. Since the subtraction parameters take into account the behaviour of functions at high energies then in the present case the low-energy region weakly depends on the behaviour of the scattering amplitudes at high energies.

To obtain the energy dependence of $\mathrm{Re} f_{\ell J}^{(\pm)}$ it is necessary to know the functional form of δ_0^0. In what follows use will be made of the following variants

$$\text{a)} \quad tg\,\delta_0^0 = a_0 k$$

$$\text{b)} \quad tg\,\delta_0^0 = \frac{a_0 k}{1 + \frac{k^2}{3}} \tag{5.1}$$

$$\text{c)} \quad tg\,\delta_0^0 = \frac{a_0 k}{1 + \frac{k^2}{3}} \; \frac{1}{1 - b_0 k^2} \, .$$

The method of calculating the function $F_0(\nu)$ described in [1] gives the general formula

$$F_0(\nu) = \prod_{i,j} \frac{i\omega - k_j}{i\omega + k_i} \cdot \frac{i + k_i}{i - k_j} ,$$

where $k_{i,j}$ are the roots of the equation

$$1 + i\, tg\, \delta_0^0 = 0 ; \quad Im\, k_i > 0 ; \quad Im\, k_j < 0 , \quad \omega = \sqrt{1 + \nu} .$$

The variants a) and b) differ from one another by the asymptotic behaviour at $k \to \infty$. The calculation shows that for $1 < a_0 < 3$ and $1 < \omega < 3$ the ratio $F_0^{(a)}/F_0^{(b)}$ lies in the interval $(1 ; 0.75)$. The same may be said about the pion- pion contribution: in the low-energy region $(\omega \le 3)$ it changes by 20% only. Thus, the assumption about the asymptotic behaviour of the phase shift δ_0^0 affects little the low-energy region $\omega \le 3$.

The variant b) is more realistic than a) because in the scattering length approximation $tg\, \delta_0^0 = a_0 k / \sqrt{1 + k^2}$. The presence of the root $\sqrt{1 + k^2}$ makes difficult the calculation of $F_0(\nu)$ and therefore it is approximated by the quantity $1 + \frac{k^2}{3}$ in the region $k^2 \le 10$.

To explain the energy behaviour of $Re\, f_s^{(+)}$ in b) it is necessary that $a_0 > 3$. So large values of the scattering length are unlikely, we consider therefore the variant c).

From the comparison of the solutions of the pion-pion scattering equations with experimental data it follows that the parameters a_0 and b_0 are in the intervals $0,5 \le a_0 \le 1$ and $0.05 \le b_0$ [8]. Therfore the extreme values of the scattering length a_0 were determined using these values. The following intervals:

$$a_0 = 0.5 ; \quad 0.04 \le b_0 \le 0.08 \quad (1030\ Mev < t_r^{1/2} < 1430\ Mev)$$

$$a_0 = 1 \quad 0.07 \le b_0 \le 0.11 \quad (890\ Mev \le t_r^{1/2} \le 1095\ Mev)$$

do not contradict the experimental data on pion-nucleon scattering. The variant $a_0 = 1$, $b_0 = 0.05$ should be assumed as the best one.* The results of calculations are given in Fig. 1.

It is interesting to analyse the S-dominant solution of Chew, Mandelstam, Noyes [7]. Up to $k^2 = 9$ the phase shift δ_0^0 can be approximated with good accuracy by the expressions

$$tg\, \delta_0^0 = 0.63 \frac{k}{1 + 0.7k^2} \quad (\lambda = -0.1)$$

$$tg\, \delta_0^0 = 4.2 \frac{k}{1 + 3.08k^2} , \quad (\lambda = -0.3) . \tag{5.2}$$

The corresponding curves for $Re\, f_s^{(+)}$ lie lower than all the experimental points (see Fig. 1). Notice that after the assumption $a^+ = 0$ has been made the quantity $Re\, f_s^{(+)}$ is defined only by the pion-pion contribution. Therefore the $s^{(+)}$ wave is most sensitive to the pion-pion interaction parameters.

The available experimental data on the $p_{1/2}^{(+)}$ wave have large errors. The study of this wave yields no new information about the parameters a_0 and b_0 although there is an agreement with experiment in the low-energy region. The behaviour of the wave $p_{3/2}^{(+)}$ is, in the main explained by the resonance wave, the pion-pion terms are not so important.

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* The χ^2 test allowed to divide two minima in the region $b_0 < 0.09$. The former one with the boundaries $0.05 < b_0 < 0.07$ and $0.7 < a_0 < 1.3$ corresponds to the resonance form of the phase shift. The second one ($b_0 < 0.05$, $a_0 > 1.4$) is wide and has the variant b) with $a_0 \approx 3$ and $b_0 = 0$. The parameters a_0 and b_0 are strongly correlated. For $b = 0.07$, one has $a_0 = 1 \pm 0.12$

Relations (4.1), (4.9) connect the pion-pion contributions to s and p wave of pion-nucleon scattering. Using them and knowing pion-pion terms in s waves we can calculate the quantity $g^{3/2}_{3/2}(\omega)$. As it should be expected it is small. (see Table).

Thus, the low-energy data on the pion-nucleon scattering do not contradict the parameters $a_0 = 1$ and $b_0 = 0.05$ ($t_r^{1/2} \approx 1250$ MeV). The scattering length $a_0 = 1$ is in agreement with the results obtained by Hamilton et al.[3]

Conclusion

The Mandelstam double dispersion representations associate the pion – nucleon scattering problem with the pion – pion interaction. Eqs. (3.1) and (5.6)[1] exhibit this connection. They are obtained by the same method which has been suggested in [5]. This same method is applied to the pion – pion scattering analysis [8]. The way of obtaining Eqs. (3.1), (5.6)[1] allows one to use different assumptions about the form of the pion–pion scattering phase shifts in order to explain the pion–nucleon scattering. It turns out that the best description of the s and p waves at low energies is reached when using the solutions from [8]. The S – dominant solution of Chew, Mandelstam, Noyes [7] does not correspond to experimental data on pion–nucleon scattering. The obtained result, from our point of view, provides evidence of the self–consistency of the method in describing phenomena in the low – energy region.

The comparison with experimental data is made in the static approximation. The following conclusions are drawn:

1. To describe correctly the pion–nucleon scattering it is necessary to take into account the pion –pion interaction.

2. The pion–pion contributions to the pion–nucleon scattering satisfy the conditions (4.1), (4.4), (4.6).(4.8), (4.9).

3. The s and p waves of pion-pion scattering wave were taken into account independently. The assumption about the resonance character of these phase shifts yields a satisfactory description of experimental data on pion–nucleon scattering.

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Table

q	$g^{3/2}_{3/2}$	$Re f^{3/2}_{3/2}$
0.5	+0.00025	0.05
1.0	−0.0006	0.23
1.5	−0.0054	0.24
2.0	−0.0120	−0.24
2.5	−0.0216	−0.18
3.0	−0.043	−0.095

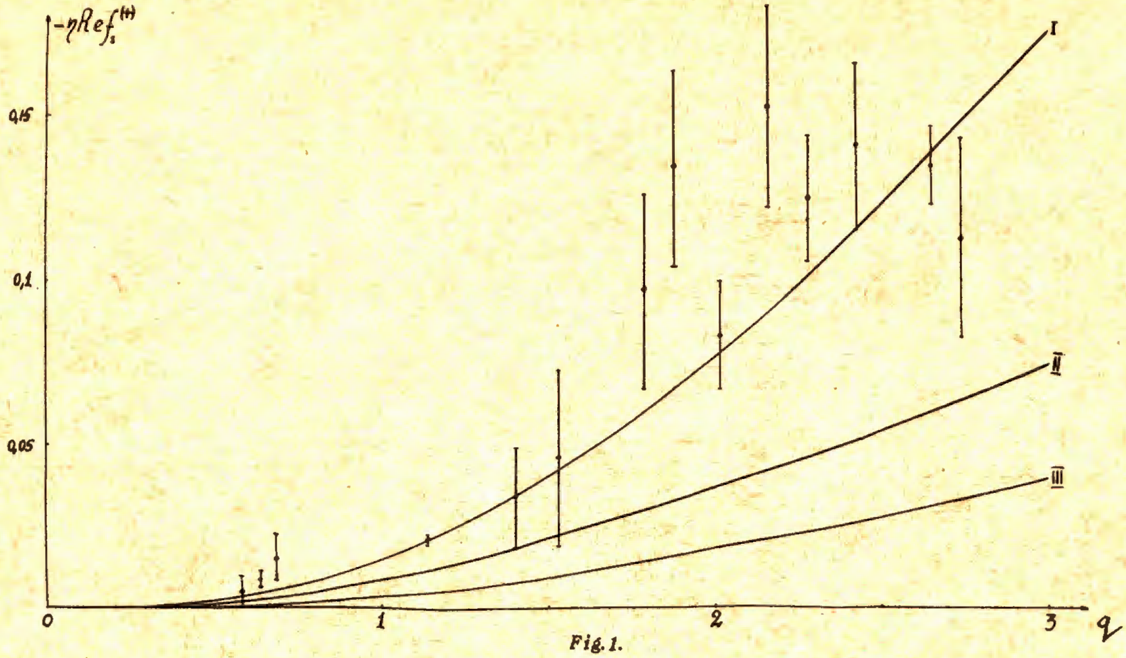
$-\eta Re f_s^{(+)}$
0,15
0,1
0,05
0
1
2
3
I
II
III
q

Fig. 1.

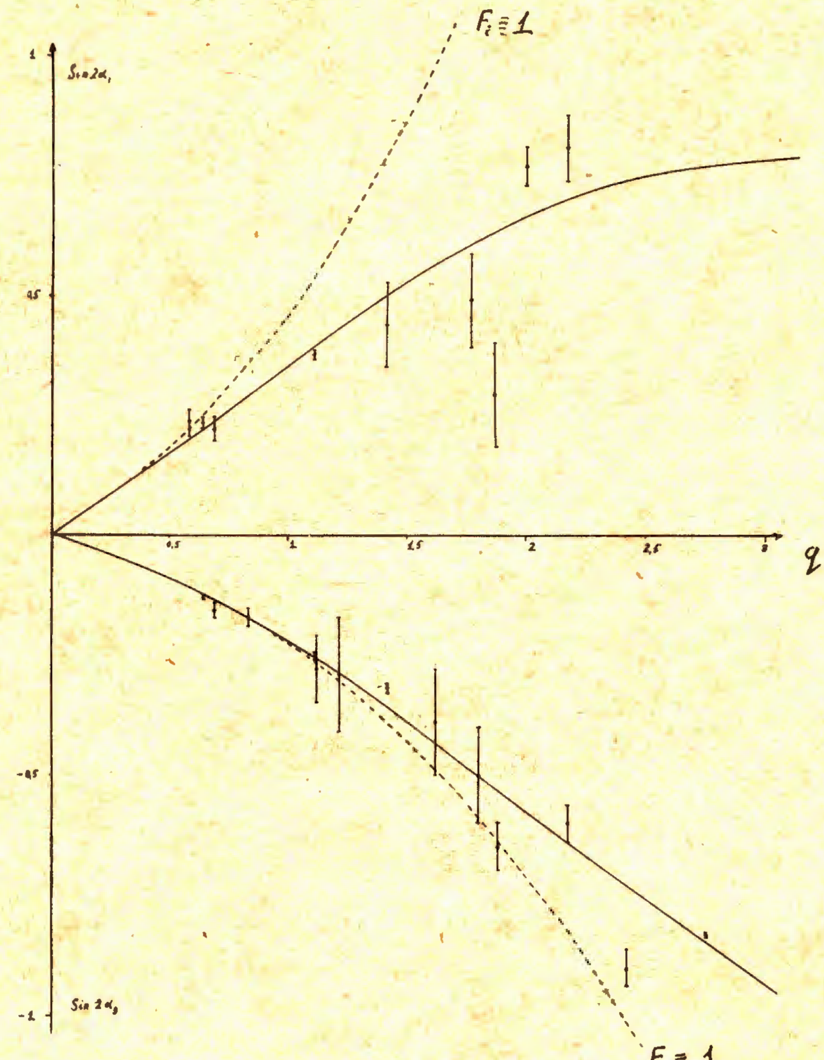
$F_i \equiv 1$
1
$\sin 2\alpha_1$
0,5
-0,5
-1
$\sin 2\alpha_3$
q
$F_i \equiv 1$

Fig. 2.