ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ
Лаборатория теоретической физики

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ON INTERACTING FIELDS WITH A DEFINITE SPIN

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Abstract

The notion of the spin of interacting field is discussed. For the interaction of high spin fields such requirements are stated under which every interacting field transfers a single angular momentum, i.e., it has one definite spin. The corresponding conditions select a certain limited class of interactions (theories of the class A). Examples of such theories are given.

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О ВЗАИМОДЕЙСТВУЮЩИХ ПОЛЯХ С ОПРЕДЕЛЕННЫМ СПИНОМ

Аннотация

Обсуждается понятие спина взаимодействующего поля. Сформулированы требования к взаимодействию частиц высших спинов, при соблюдении которых каждое взаимодействующее поле переносит только один момент количества движения, т.е. имеет один определенный спин. Соответствующие условия выделяют некоторый ограниченный класс взаимодействий /теории класса А/, примеры которых приведены.

1. General Considerations

1. In the theory of interacting fields the spin of a field is defined, in fact, as the number of dynamically independent components/1, 2/. As a rule, the Heisenberg field operators have superfluous components. The restriction of the number of degrees of freedom and thereby the selection of some definite spin of a field is achieved by imposing a supplementary condition (referred hereafter as S.C.). Generally speaking, these S.C. are dependent on interaction and are different from S.C. for free fields.

It is worth while to note that for fields with spins 0 and ½ S.C. are simply absent (the number of components is equal to that of degrees of freedom) in any case with or without interaction.

The question arises as to which physical consequences the differences in the S.C. lead .

We consider, as an example, the process corresponding to the diagram in Fig. 1, where the initial and final states are connected by one virtual line of the field we are investigating (in the general case with all the radiative corrections).

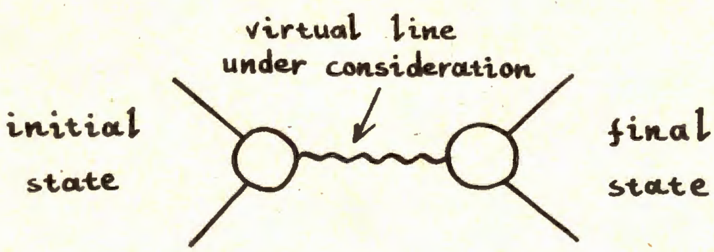

Fig. 1.

It is well-known that if an intermediate particle possesses spin 0 (or ½) it is able to transfer the only angular momentum 0 (or ½): the amplitude of such a process contains the partial waves corresponding to the total angular momentum 0 (or ½) only.

In the simplest theory of the neutral vector field, where the S.C. have the same form as in the free case

$$\partial_\mu A_\mu (x) = 0 \qquad (1)$$

the quanta of the vector field can also transfer single angular momentum equal to unity.

Generally speaking, this is not so: in a number of theories the virtual quanta are able to trasfer several angular momenta. So, in the simplest pseudovector theory the S.C. is

$$\partial_\mu A_\mu = C g \bar{\psi} \gamma_5 \psi \qquad (2)$$

(g is the coupling constant with the spinor field ψ , C is the constant expressed in terms of the masses of both fields). In spite of the fact that (2) reduces the number of the dynamically independent field components A_μ to 3 , this field, nevertheless, transfers the angular momenta 0 and 1: the amplitude of the process of Fig.1 has the partial waves corresponding to both values of the total angular momentum. This is due to the difference between S.C. (2) and S.C. (1).

1.2. In the general case the field transfers several angular momenta. However, any field (for instance, the tensor one) with a finite number of components is able to transfer only the limited number of angular momenta.

The answer to the question as to which set of the angular momenta can be transferred by a ceratin field is given by the operator of the square of the spin $\hat{s}^2$ for the given field *

$$\hat{s}^2 = ½\, m_{\rho\sigma} m_{\rho\sigma} - p^{-2} m_{\lambda\rho} m_{\lambda\sigma} p_\rho p_\sigma = \tag{3a}$$

$$= ½\, s_{\rho\sigma} s_{\rho\sigma} - p^{-2} s_{\lambda\rho} s_{\lambda\sigma} p_\rho p_\sigma \tag{3b}$$

It has been built up of the generators of the transformation for the field operator

$$p_\lambda = -i\partial_\lambda \tag{4}$$

$$m_{\rho\sigma} = x_\rho p_\sigma - x_\sigma p_\rho + s_{\rho\sigma} \tag{5}$$

where $s_{\rho\sigma}$ are the generators of the Lorentz rotations of the field components.

The eigenvalue spectrum of the operator $\hat{s}^2$ for the given field (i.e., when $s_{\rho\sigma}$ are fixed) gives the spectrum of the angular momenta it transfers (§ 3). Generally speaking, the field can possess several spins (≡ transfer several angular momenta). Using $\hat{s}^2$ one can write down a condition under which the field has one definite spin: it must be an eigenfunction of $\hat{s}^2$ with the given eigenvalue $s(s+1)$. **

So, when we speak about the quantum number 'the spin of the field' we confront it, in accordance with the principles of quantum mechanics, with a certain operator, that is, operator (3).

1.3. It appears reasonable to divide the theories of particles with higher spins into two classes:

A) The theories in which each interacting field, just as the free one, has the only spin.***

B) The theories in which no definite spin can be ascribed to the interacting fields.

It will be proved in the following that theories belong to the class A if and only if the interaction is chosen so that it does not change the form of the S.C. in comparison with the free case (It is meant, of course, that in the free case the S.C. single out only one spin. ****).

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* The components of the spin operator can be defined, for instance, as

$$\hat{s}_{\rho\sigma} = m_{\rho\sigma} + p^{-2}(p_\rho m_{\sigma\lambda} p_\lambda - p_\sigma m_{\rho\lambda} p_\lambda) = s_{\rho\sigma} + p^{-2}(p_\rho s_{\sigma\lambda} p_\lambda - p_\sigma s_{\rho\lambda} p_\lambda)\ (\hat{s}^2 = ½ s_{\rho\sigma} s_{\rho\sigma})$$

** Such an approach to single out spin 1 of the vector field was discussed in our paper /3-5/

*** In a complete analogy with the fields with spins 0 and ½.

**** Note also, that if no S.C. singling out one spin are imposed, we are faced with the difficulties concerning either indefinite metric, or the positiveness of energy.

How can we account for the fact that in the theories of the class B, the interacting field, though it has the same number of degrees of freedom as the free one, transfers, nevertheless, a greater number of angular momenta?

It is this problem (in a slightly different form) which was raised by Byers and Peierls /6/ and elucidated by Kemmer /7/ for the theories of the vector field. The field components transferring the extra angular momenta are expressed in terms of the independent field components. For instance, S.C. (2) implies that a part of the pseudovector field with spin 0 is a combination of ψ. It was pointed out by Kemmer that there, apparently always exists a certain canonical transformation leading to new fields which obey the same S.C. as in the free case. However, after such a transformation the theory takes on a rather complicated, non-local form. Thus, the theory of the class B can be reduced to the theory of the class A by introducing the non-localities and other complications.

1.4. The operator of the square of the spin for the closed quantum mechanical system or for the system of fields is one of the invariants of the inhomogeneous Lorentz group. It has been discussed in different aspects, in connection with the classification of the representations of this group /8-11/. The application of the operator of the squared spin to free fields turns out to be rather helpful, as far as it unites all the S.C. and allows to get them in a uniform fashion (§ 2). The free field with a definite mass and a definite spin is transformed according to the irreducible representation of the inhomogeneous Lorentz group. It is an eigenfunction of the invariants of this group.

1.5. Unlike the operators of the free field, the operators of the interacting field are no longer transformed under one irreducible representation of the inhomogeneous group. Indeed, the interacting field corresponds to some eigenvalue spectrum of the invariant of this group—of the operator of the squared mass $-p^2 = \Box$. Indeed, the interacting field operator in the momentum space is different from zero for time-like, for space-like, for isotropic, and for zero 4-momenta*. Actually it is this fact which makes the interaction between the fields, their transmutation one into another possible.

As far as the second invariant—spin— is concerned, in the existing theories the spectrum of its values for the interacting field turns out artificially restricted in practice because use is made of the fields transforming according to the finite dimensional irreducible representations of the homogeneous Lorentz group**. However, with such restrictions the spin spectrum of the interacting field has, as a rule, several values***.

In the theories of the class A the spectrum of the spins of each field is the simplest: just as in the free case it consists of only one value. At the same time each field will be always an eigenfunction of its own operator $\mathcal{S}^2$. In this sense one can speak that the spin of each field is conserved.

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* In particular, there appear non-zero masses for the interacting electromagnetic field. This is the reason for the appearance of the Coulomb interaction.

** If this purely mathematical postulate is rejected, then theories are conceivable in which the free field has only spin 0 (or ½) but the interacting field possesses some others spins in addition.

*** Nevertheless, one can consider that the wave functions of the physical one-particle states are transformed according to the irreducible representation of the inhomogeneous group. Let us emphasize that all the rest quantum numbers (for instance, charge, isotopic spin, parity etc) coincide both for the free and interacting fields and are the same as the one-particle states have. Only the mass is diffused with the necessity.

1.6. It is practically more convenient and mathematically more correct to carry out all the considerations concerning the interacting field not with the field itself (say, $A(x)$), but to consider the matrix elements

$$\langle 0 \mid A(x) \mid \Phi_j \rangle \tag{6}$$

where Φ_j are the physical states which are the eigenfunctions of the operator of the square of the spin for a system of interacting particles (of the total angular momentum in the centre–of–mass system). (§ 3).

This matrix element corresponds to the diagram Fig. 2.

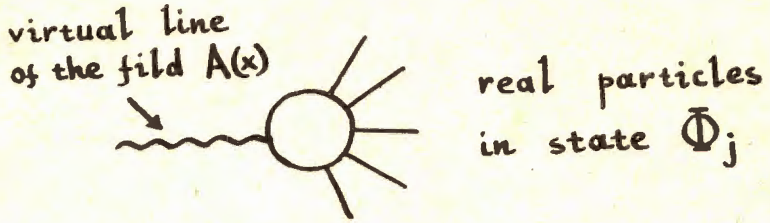

Fig. 2.

The field $A(x)$ transfers, by definition, those angular momenta j for which the matrix element (6) is different from zero. In the theories of the class A there is one such value (the field has one spin). Note that if the real particle of the field $A(x)$ with a definite spin decayed into a system of particles in the state Φ_j, then the matrix element would be different from zero only for j coinciding with the spin of the free field.

Obviously, the analysis of the diagrams of Figs.1 and 2 lead to the same conclusions about the spin of the field, since the diagram like that drawn in Fig.2 is a part of the diagram of Fig.1.

1.7. In Sec.4 it was shown that the class A is not empty Examples are given for the theories of particles with spin 1 belonging to this class. The close connection between the notion of the spin in the conventional space and the isotopic invariance is also indicated there.

1.8. As for zero masses, we make only the following remark. The found limitations on the matrix elements cannot be expressed in terms of the operators (14) or (17) if there are physical states with zero mass. In this case the interacting field will have a definite spin if for it the S.C. are fulfilled for the matrix elements between the physical states. For instance, such a situation takes place in the Fermi electrodynamics. In all the gauge–invariant theories of the vector fields one can impose any restriction on $\partial_\mu A_\mu$, as this quantity is quite arbitrary and is not determined by the equations of motion. The restriction $\langle \Psi_{Phys} \mid \partial_\mu A_\mu \mid \Phi_{Phys} \rangle = 0$ allows to carry out the quantization according to Fermi. Therefore, the gauge–invariant theories of the vector fields are those which describe only the quanta with spin 1.These problems were discussed in our previous papers /4,5/, and the present work arised as the development of such ideas.

2. Formalism of Higher Spins on the Basis of the Operator $\hat{s}^2$ (Free Fields)

2.1. The application of the operator of the square of the spin seems expedient even in the case of the free field since this leads to the unified standpoint on all the S.C. and brings them together.

The field A is transformed according to the irreducible representation of the inhomogeneous Lorentz group, if

$$\Box \; A = m^2 A \tag{7}$$

$$\hat{S}^2 A = s(s+1)\, A \tag{8}$$

where the m (mass) and s (spin) are the given numbers.

2.2. We start with the integer spins. We shall work in the tensor formalism /12/, where the tensor of the s–th rank $\phi_{\mu_1\mu_2\dots\mu_s}$ is used to describe the spin s. In this case matrices $s_{\rho\sigma}$ in Eq. (5) has the form

$$\begin{aligned}(s_{\rho\sigma})_{\mu_1\dots\mu_s;\nu_1\dots\nu_s} &= -i(\delta_{\rho\mu_1}\delta_{\sigma\nu_1} - \delta_{\rho\nu_1}\delta_{\sigma\mu_1})\,\delta_{\mu_2\nu_2}\delta_{\mu_3\nu_3}\dots\delta_{\mu_s\nu_s} - \\ &- i\delta_{\mu_1\nu_1}(\delta_{\rho\mu_2}\delta_{\sigma\nu_2} - \delta_{\rho\nu_2}\delta_{\sigma\mu_2})\,\delta_{\mu_3\nu_3}\dots\delta_{\mu_s\nu_s} - \\ &- \dots\dots\dots\dots\dots\dots - \\ &- i\,\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}\dots\delta_{\mu_{s-1}\nu_{s-1}}(\delta_{\rho\mu_s}\delta_{\sigma\nu_s} - \delta_{\rho\nu_s}\delta_{\sigma\mu_s}).\end{aligned} \tag{9}$$

Then the operator $\hat{S}^2$ is written down as

$$\begin{aligned}(\hat{S}^2)_{\mu_1\dots\mu_s;\nu_1\dots\nu_s} &= \\ = \tfrac{1}{2}(s_{\rho\sigma})_{\mu_1\dots\mu_s;\lambda_1\dots\lambda_s}(s_{\rho\sigma})_{\lambda_1\dots\lambda_s;\nu_1\dots\nu_s} &- p^{-2}(s_{\rho\tau})_{\mu_1\dots\mu_s;\lambda_1\dots\lambda_s}(s_{\rho\sigma})_{\lambda_1\dots\lambda_s;\nu_1\dots\nu_s}\, p_\tau p_\sigma .\end{aligned} \tag{10}$$

As long as the representation we are considering is reducible, $\hat{S}^2$ is not a unit operator, and its eigenvalue spectrum consists of the numbers $n(n+1)$ where $n = 0, 1, 2, \dots, s$. We are interested in the eigenfunctions with the maximum eigenvalue

$$(\hat{S}^2)_{\mu_1\dots\mu_s;\nu_1\dots\nu_s}\, \phi_{\nu_1\dots\nu_s} = s(s+1)\,\phi_{\mu_1\dots\mu_s} \tag{11}$$

It is convenient to work in the p–representation, where, by (7), $p^2 = -m^2 \neq 0$, and to go over to the rest system. In this system the operator of the square of the spin (3b) looks like

$$\hat{S}^2 = \tfrac{1}{2}\, s_{rs}\, s_{rs} \quad (r, s = 1,2,3,) \tag{12}$$

and for the tensor field it is written down as

$$\begin{aligned}(\hat{S}^2)_{\mu_1\mu_2\dots\mu_s;\nu_1\nu_2\dots\nu_s} &= 2(s-v)\,\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}\dots\delta_{\mu_s\nu_s} + \\ &+ \left\{ \left[\begin{matrix} -2(\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2} - \delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})\,\delta_{\mu_3\nu_3}\dots\delta_{\mu_s\nu_s} & \text{if } \mu_1\mu_2\nu_1\nu_2 \neq 4 \\ 0 & \text{otherwise} \end{matrix} \right] + \right. \\ &\left. + \text{ all the terms with different choice of } \mu_i\,\nu_i \text{ and } \mu_k\,\nu_k \right\}\end{aligned} \tag{13}$$

where v is the number of pairs $(\mu_i\ \nu_i)$ in which there is, at least, one four.

The analysis of Eq. (11) leads to the well–known /12,2/ s. c.

$$\begin{aligned}&\text{a) } \phi_{\mu_1\dots\mu_s} \text{ totally symmetrical tensor} \\ &\text{b) } \phi_{\mu\mu\mu_3\dots\mu_s} = 0 \\ &\text{c) } \partial_\mu \phi_{\mu\mu_2\dots\mu_s} = 0\end{aligned} \tag{14}$$

2.3. Now we consider the half-integer spins in the γ – formalism suggested by Rarita and Schwinger /13/, and Tamm (see /14, 15/). In this formalism the spin $s = k + ½$ is described by the tensor of the k –th rank which has one Dirac spinor index a $\psi_{a \mu_1 \dots \mu_k}$ or, briefly, $\psi_{\mu_1 \dots \mu_k}$. The matrices $s_{\rho\sigma}$ for $\psi_{\mu_1 \dots \mu_k}$ read.

$$
\begin{aligned}
(s_{\rho\sigma})_{\mu_1 \dots \mu_k; \nu_1 \dots \nu_k} &= ½ \sigma_{\rho\sigma} \delta_{\mu_1 \nu_1} \dots \delta_{\mu_k \nu_k} - \\
&- i (\delta_{\rho \mu_1} \delta_{\sigma \nu_1} - \delta_{\rho \nu_1} \delta_{\sigma \mu_1}) \delta_{\mu_2 \nu_2} \dots \delta_{\mu_k \nu_k} - \\
&- \dots\dots\dots\dots - \\
&- i \delta_{\mu_1 \nu_1} \delta_{\mu_2 \nu_2} \dots \delta_{\mu_{k-1} \nu_{k-1}} (\delta_{\rho \mu_k} \delta_{\sigma \nu_k} - \delta_{\rho \nu_k} \delta_{\sigma \mu_k})
\end{aligned}
\qquad (15)
$$

$$
| \sigma_{\rho\sigma} = - i (\gamma_\rho \gamma_\sigma - \delta_{\rho\sigma}) |
$$

Again it is simpler to make the calculations in the rest system, where

$$
\begin{aligned}
(\hat{s}^2)_{\mu_1 \dots \mu_k; \nu_1 \dots \nu_k} &= [3/4 + 2 (k - v)] \delta_{\mu_1 \nu_1} \dots \delta_{\mu_k \nu_k} + \\
&+ \left\{ \begin{bmatrix} - i \sigma_{\mu_1 \nu_1} & \text{if } \mu_1 \nu_1 \neq 4 \\ 0 & \text{otherwise} \end{bmatrix} \delta_{\mu_2 \nu_2} \dots \delta_{\mu_k \nu_k} + \right. \\
&\left. + \text{the terms in which } \mu_1 \nu_1 \text{ are replaced by } \mu_2 \nu_2 , \mu_3 \nu_3 \dots \mu_k \nu_k \right\} + \\
&+ \left\{ \begin{bmatrix} - 2 (\delta_{\mu_1 \mu_2} \delta_{\nu_1 \nu_2} - \delta_{\mu_1 \nu_2} \delta_{\mu_2 \nu_1}) \delta_{\mu_3 \nu_3} \dots \delta_{\mu_k \nu_k} & \text{if } \mu_1 \mu_2 \nu_1 \nu_2 \neq 4 \\ 0 & \text{otherwise} \end{bmatrix} + \right. \\
&\left. + \text{all the terms with different choice of } \mu_i \nu_i \text{ and } \mu_j \nu_j \right\}
\end{aligned}
\qquad (16)
$$

where v is again the number of pairs (μ_i ν_i) in which there is, at least, one four. In the present case $\hat{s}^2$ has the eigenvalues $r(r+1)$ where $r = ½, 3/2, \dots, k + ½$. We are seeking again for the eidenfunctions $\hat{s}^2$ with the maximum eigenvalue. The analysis of Eq. (11) gives the well-known S.C.. for half-integer spin field:

a) $\psi_{\mu_1 \dots \mu_k}$ totally symmetric

b) $\gamma_{\mu_1} \psi_{\mu_1 \mu_2 \dots \mu_k} = 0$

c) $\partial_{\mu_1} \psi_{\mu_1 \mu_2 \dots \mu_k} = 0$,

In case of the half-integer spins, use is usually made of the Dirac equation instead of the Klein–Gordon one (7). It is quite natural that the same interpretation of the S.C. on the basis of the operator of the square of the spin is also possible for the field operators in the Gelfand–Yaglom formalism. /16/

3. Interacting Fields with Higher Spins.

3.1. Now we proceed to the analysis of the interacting Heisenberg fields $\phi_{\mu_1 \dots}$ and $\psi_{\mu_1 \dots}$ with any spins. Let Φ_{jP} be the state vector of the many particle system which has the total 4–momentum P_μ ($P^2 < 0$, $P_0 > 0$) and the spin (the total angular momentum in the c.m.s.) j

$$\hat{S}^2 \Phi_{jP} = j(j+1)\Phi_{jP} \tag{18}$$

(we are not interested in the spin projection). The operator $\hat{S}^2$ has the form

$$\hat{S}^2 = ½ \hat{M}_{\rho\sigma} \hat{M}_{\rho\sigma} - \hat{P}^{-2} \hat{M}_{\rho\sigma} \hat{M}_{\rho\tau} \hat{P}_{\sigma} \hat{P}_{\tau} \tag{19}$$

where $\hat{P}_{\lambda}$ and $\hat{M}_{\rho\sigma}$ are the generators of the transformations and the Lorentz rotations for the state vectors (they are constants of motion of the system of interacting fields). The relationship of these operators with the operators p_{λ} and $m_{\rho\sigma}$ (4) and (5) for some field $A(x)$ follows from the Lorentz invariance and is well-known

$$p_{\lambda} A = [A, \hat{P}_{\lambda}] \tag{20}$$

$$m_{\rho\sigma} A = [A, \hat{M}_{\rho\sigma}] \tag{21}$$

3.2. It can be easily seen that the matrix elements

$$< 0 \mid \phi_{\mu_1\mu_2\cdots\mu_s}(x) \mid \Phi_{jP} > \text{ and } < 0 \mid \psi_{\mu_1\mu_2\cdots\mu_k}(x) \mid \Phi_{jP} > \tag{22}$$

are generally different from zero for $j = 0, 1, \ldots s$ and $j = ½, 3/2, \ldots k+½$ respectively.

Indeed, with account of the translational invariance, viz.,

$$< 0 \mid \phi_{\mu_1\cdots\mu_s}(x) \mid \Phi_{jP} > = e^{iPx} < 0 \mid \phi_{\mu_1\cdots\mu_s}(0) \mid \Phi_{jP} > \tag{23}$$

Eq.-ns (3),(21) and the vacuum property $\hat{M}_{\rho\sigma} \mid 0 > = 0$ we see that

$$(\hat{s}^2)_{\mu_1\cdots\mu_s;\nu_1\cdots\nu_s} < 0 \mid \phi_{\nu_1\cdots\nu_s}(x) \mid \Phi_{jP} > = \tag{24}$$

$$= < 0 \mid \phi_{\mu_1\cdots\mu_s}(x) \hat{S}^2 \mid \Phi_{jP} > = j(j+1) < 0 \mid \phi_{\mu_1\cdots\mu_s}(x) \mid \Phi_{jP} >$$

In virtue of (23), the operators p_{λ} in $\hat{s}^2$ are replaced by the c-numbers P_{λ}. The problem concerning the eigenvalues of the operator $\hat{s}^2$, if applied to such matrix elements, is solved just as in the free case. (see § 2). In particular, the spectrum of the eigenvalues has the form $n(n+1)$, where $n = 0, 1, \ldots, s$. Hence it follows, that $j = 0, 1, \ldots, s$. Similarly in case of the half-integer spin $j = 1/2, 3/2, \ldots k + 1/2$.

3.3. The aim of this paper is to get the conditions under which the field transfers only one angular momentum, i.e., for j the only value must be possible. Just as in the free case we choose as this only value the maximum one, i.e., s *.

The conditions which single out the matrix element corresponding to this maximum value are written down for the integer spins as follows

$$\begin{aligned} &\text{a)} \quad < 0 \mid \phi_{\mu_1\cdots\mu_s}(x) \mid \Phi_{jP} > \quad \text{totally symmetric} \\ &\text{b)} \quad < 0 \mid \phi_{\mu\mu\mu_3\cdots\mu_s}(x) \mid \Phi_{jP} > = 0 \end{aligned} \tag{25}$$

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* Other possibilities will lead to the equivalent representations for the fields with the definite spin in which the Heisenberg operator will have more vector indices than it is necessary for describing the given spin.

c) $< 0 \mid \partial_{\mu_1} \phi_{\mu_1\mu_2\ldots\mu_s}(x) \mid \Phi_{jP} > = 0$ (25)

Since the vectors of the physical states Φ_{jP} form the complete system, then, from the hypothesis about the locality, covariance and positive definiteness, by the Federbush-Johnson theorem/17,18/ it follows, that conditions (25) must be satisfied for the field operators themselves. In other words, the interacting field describing one integer spin s must obey the same S.C. as the free one, i.e., the S.C. (14).

Of course, the reversed statement is always correct: from conditions (14) follow conditions (25).

Thus, if and only if the interacting field $\phi_{\mu_1\ldots\mu_s}(x)$ obeys the same S.C. as the free one does, we can (and should) speak that it has a definite spin s .

By making a reference to Federbush-Johnson's theorem we get rid of the necessity to consider the matrix elements.*

$$< \Phi_{j_1 P_1} \mid \phi_{\mu_1\ldots\mu_s}(x) \mid \Phi_{j_2 P_2} > = e^{i(P_2 - P_1)x} < \Phi_{j_1 P_1} \mid \phi_{\mu_1\ldots\mu_s}(0) \mid \Phi_{j_2 P_2} > \qquad (26)$$

In this case the analysis will essentially depend on whether the vector $P_2 - P_1$ is a time-like, isotropic, space-like or a zero one. These four possibilities correspond to the four different classes of the represetations of an inhomogeneous Lorentz group. Each time they require a special approach.

By arguing in a similar manner in the case of the fields $\psi_{\mu_1\mu_2\ldots\mu_k}(x)$ which have, apart from the vector indices, one more Dirac spinor index, we are led to the conclusion that for the field $\psi_{\mu_1\ldots\mu_k}$ to describe the only value of the spin equal to $k + ½$ it must obey S.C. (17). The theories of the class A are those in which every field satisfies conditions (14) or (17) determining the fields with the only value of the spin.

4. Theories of the Class A

The equations for the free fields which contain all the S.C. singling out one value of the spin are well-known**. These are all the equations of Gelfand and Yaglom/16/, in the tensor formalism- Proka's equation for spin 1, Rarita-Schwinger's equation for spin 3/2. One can also point out the following equation for spin 2

$$\begin{aligned} &½(\Box \phi_{\mu\nu} - \partial_\mu \partial_\lambda \phi_{\lambda\nu} - \partial_\nu \partial_\lambda \phi_{\mu\lambda}) + ½(\Box \phi_{\nu\mu} - \partial_\nu \partial_\lambda \phi_{\lambda\mu} - \partial_\mu \partial_\lambda \phi_{\nu\lambda}) + \\ &+ \delta_{\mu\nu} \partial_\lambda \partial_\rho \phi_{\lambda\rho} - 1/3(\delta_{\mu\nu}\Box - \partial_\mu \partial_\nu) \phi_{\rho\rho} - m^2 \phi_{\mu\nu} = 0 \end{aligned} \qquad (27)$$

We know the following examples of theories of the interaction belonging to the class A:

1) All mutual interactions of spinor and scalar fields,, and their electromagnetic interactions also, because conventional manner switching on interaction with electromagnetic field guarantees spin 1 for the latter ***

2) The theory of the neutral vector field interacting with the conserved current (see, e.g., /3/)

$$\Box A_\mu - \partial_\mu \partial_\nu A_\nu - m^2 A_\mu = - j_\mu ; \quad \partial_\mu j_\mu = 0 \qquad (28)$$

3) Yang-Mills's theory/19/ and all its generalizations indicated by Gell-Mann and Glashow/20/

$$\partial_\mu G^i_{\mu\nu} - m^2 A^i_\nu = - j^i_\nu \quad , \quad \partial_\mu j^i_\mu = 0 \qquad (29)$$

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* In the case of the field with spin s , these matrix elements are non-zero if $\vec{j}_1 + \vec{j}_2 + \vec{s} = 0$ (according to the conventional rule of adding the angular momenta).

** Such equations are equivalent to the conditions determining the irreducible representation of the inhomogeneous Lorentz group (for instance, (7) and (8)).

*** It is worth while to note, that due to gauge invariance virtual photon has spin 1 even in theories of the class B .

where

$$j^i_\nu = 2\gamma_0 \, [\, i\,\psi\, \gamma_\nu M^i \psi + c_{ikj} G^j_{\nu\mu} A^k_\mu \,]$$

(the notations are the same as in /20/).

With account of the current conservation, the S.C. (1) singling out spin 1 is the consequence of Eq. (28) and (29) /5/. Thus, the class A is not empty.

This problem can be reversed: how to swich on the interaction so that the theory would belong to the class A, i.e., so that not only the fields with spin 0 and ½, but all the rest ones would possess a definite spin.

For the fields with spin 1, we succeeded in solving an inverse problem /21/ and in proving that the only theories of the class A with the dimensionless coupling constants are those in which the equations of motion look like (29). In other words, only for the following cases we can switch on the interaction with the vector field so that the spin of this field would be equal to unity

a) if the vector field is neutral and it is coupled with a some conserved current

b) if the three vector fields form the isotopic triplet, while the theory as a whole is isotopically invariant:

c) If there are some other higher symmetries.

The possibility b) reveals a close relation of the isotopic invariance with the notion of spin in the usual space.

We are making a similar analysis for higher spins.

It should be stressed that there exist interactions which do not belong, certainly, to the class A. The electromagnetic interactions of charged particles of higher spins first of all, refer, to them. These interactions violate the isotopic invariance and do not allow, therefore, to ascribe a definite spin to the interacting fields with spins ≥ 1 .

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