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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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REGGE POLES AND FIRE BALLS

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Abstract

The behaviour of the amplitude of inelastic processes is calculated assuming the existence of Regge poles. The asymptotic behaviour of the cross section of the emission of n particles is shown to be $\sigma_n \approx \frac{Cn}{\log(\frac{s}{s_0})}$. The two centre emission of particles is found to give the largest contribution in the asymptotic region.

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ПОЛЮСА РЕДЖЕ И МОДЕЛЬ ОГНЕННЫХ ШАРОВ

Аннотация

Определяется поведение амплитуды неупругих процессов, предполагая существование полюсов Редже. Получается поведение сечения образования n частиц: $\sigma_n \approx \frac{Cn}{\log(\frac{s}{s_0})}$. Показывается, что асимптотически наибольший вклад дает рождение частиц в двух центрах.

Several authors [1,2,3] have investigated recently the behaviour of inelastic scattering amplitudes in the Regge pole approximation.

Miyazawa and Suzuki [1] proposed a model of inelastic interactions using a single vacuum pole exchange approach (Fig. 1).

Using the optical theorem they obtained the behaviour of the imaginary part of the amplitude for forward scattering:

$$Im\,A(s,\,0) \approx \int ds_1\, ds_2\, f(s_1, s_2)\, \frac{\left(\frac{s}{s_0}\right)^{2\alpha(t_{max})-1}}{\log\left(\frac{s}{s_0}\right)} \tag{1}$$

where $t_{max} \approx -\frac{s_1 s_2}{s}$, s_1 and s_2 are the total four momenta squared of the particles exchanged in the upper and lower vertices correspondingly.

The upper limit of the integrations is given by the condition $\sqrt{s} > \sqrt{s_1} + \sqrt{s_2}$. If we wish to obtain a constant total cross section, we have to choose a mass distribution of the following type (apart from logarithmic factors):

$$f(s_1, s_2) \approx \frac{1}{s_1\, s_2} \tag{2}$$

In the paper of Coutogouris, Frautschi and Wong there is a similar proposal, only they assumed the residues of the Regge poles in the inelastic interaction to be factorisable.

They obtained the following inelastic amplitude splitting up both the upper and lower vertex functions:

$$A \approx g(s_{n_1})\, s_n^{\alpha(t)}\; g(s_{n-n_1}) \approx g(s_{n_1-n_2})\, s_{n_3}^{\alpha(t_1)}\; g(s_{n_2})\, s_n^{\alpha(t)} \times \tag{3}$$
$$\times\; g(s_{n-n_1-n_3})\, s_{n-n_1}^{\alpha(t_2)}\; g(s_{n_3})$$

(here s_i is the total four momentum squared of the i appropriate particle).

This procedure may be continued until we obtain a picture (fig. 2), in which each particle is emitted from different vertices. This picture is very similar to the multiperipheral model of Amati et al.[4], only instead of the exchange of the one pion propagator the exchanged particle is a Regge pole.

Nevertheless one can raise two objections against this picture. The first one is that the amplitude obtained by Contogouris et al is unsymmetric. The $s^{\alpha(t)}$ propagator (the centre of the chain) corresponds to a section of the chain, which is arbitrarily chosen.

A more serious objection is that this model gives $f(s_1, s_2) \approx s_1 . s_2$ in formula 1, which would give a cross section proportional to s^2 , as is easily seen from formula 3.

It was proved by Ter-Martirosyan [3] for the case of 3 particle production that one has to write the amplitude as $A \approx s_{12}^{\alpha(t_1)}\; s_{23}^{\alpha(t_2)}$ if both s_{12} and s_{23} are large $(s_{ik} = (q_i + q_k)^2$, q_i are the four momenta of the emitted particles). If we generalise this picture to n particle production we must add a factor $s_{i,i+1}^{\alpha(t_i)}$ for the i th propagator of the chain $(t_i = (p_1 - \sum_{k=1}^{i} q_k)^2$, p_1 and p_2 are the four momenta of the particles in the initial state). At the vertices there appear the usual residues continued in the external masses.

Our picture is wholly symmetric in the sections of the chain, and as we shall see it removes the difficulties connected with the behaviour of the total cross sections.

In the chain each second propagator must be a pion- Regge pile due to the G parity conservation.

We have for the n particle exchange term the following contribution to the imaginary part of the elastic scattering amplitude:

$$Im\, A_n(s,t) = \int A_n(1)\; A_n(2)\; \delta(\Sigma\; q_i - p_1 - p_2) \prod_{i=1}^{n} d^4 q_i\, \delta(q_i^2 - \mu^2)\,, \tag{4}$$

where $A_n(1)$ and $A_n(2)$ are the n particle production amplitudes.

We adopt the following notations:

$$s_i = (\sum_{k=i}^{n} q_k)^2, \qquad s_{i,i+1} = (q_i + q_{i+1})^2,$$

$$t_i = (p_1 - \sum_{k=1}^{i} q_k)^2 = (p_2 - \sum_{k=i+1}^{n} q_k)^2,$$

$$t'_i = (p_3 - \sum_{k=1}^{i} q_k)^2 = (p_4 - \sum_{k=i+1}^{n} q_k)^2,$$

$$s_1 = s\,, \quad s_n = q_n^2 = 1,$$

$$t'_0 = t_0 = 1\,.$$

Here p_3 and p_4 are the momenta of the outgoing particles.

Using the delta function of the momentum conservation we integrate over q_n. We introduce new integration variables instead of $d^4 q_i$ in the following manner:

$$q_i \to s_{i+1}\,,\; t_i\,,\; t'_i\,,\; q_i^2\,.$$

For

$$t = (p_1 - p_3)^2 = 0$$

$$d^4 q_i = \frac{\pi}{2^2}\; \frac{\delta(t_i - t'_i)\, dt_i\, dt'_i\, ds_{i+1} dq_i^2}{\sqrt{\Delta(s_i, t_{i-1}, 1)}}\,,$$

where

$$\Delta(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$$

With the help of delta functions we can integrate over t'_i and q_i^2. If we expand the Regge exponents in series around $t_i = 0$, and we keep only the linear part we can integrate over t_i as well.

We finally obtain the following expression:

$$Im\, A_n(s,0) \approx \frac{\pi^{n-1}}{\sqrt{s^2-4s}\; 2^{3n-1}\; \alpha'_+(0)^{\frac{n}{2}-1}\, \alpha'_-(0)^{\frac{n}{2}-1}} \times$$

$$\times \int \delta(s_n - 1)\, f^2(t_{n-1,\,max}) \prod_{i=1}^{n-1} (ds_{i+1}\; \frac{(\frac{s_{i,i+1}}{s_0})^{2\alpha_\pm(t_{i,max})}(1 + ctg^2 \frac{\pi\alpha_\pm}{2})}{\sqrt{\Delta(s_{i+1}, t_{i,max}, 1)}}\; f^2(t_{i\,max}, t_{i-1\,max}))\,, \tag{5}$$

where f is the Regge residue, $\alpha_+(t)$ and $\alpha_-(t)$ are the Pomeranchuk and pion trajectories respectively. (For i even we have + for i odd $-$).

$s_{i,i+1}$ and $t_{i\,max}$, the maximum value of the momentum transfer (forward scattering), may be given as a function of s_i. The region of integration is given by the following condition : $\sqrt{s_{i-1}} > 1 + \sqrt{s_i}$

In formula 5) we must integrate essentially over the possible values of n two dimensional q_i vectors in their c.m.s., with the subsidiary conditions: $\Sigma q_{io} = \sqrt{s}$, $\Sigma q_{i1} = 0$, $q_{io}^2 - q_{i1}^2 = 1$, where we denote the one dimensional space like component by q_{i1}.

In any case we have $q_{i0} \le \frac{\sqrt{s}}{2}$. We should like to estimate the behaviour of the factor $\prod_{i}^{n-1} \left(\frac{s_{i,i+1}}{s_0} \right)^{2\alpha_{\pm}(t_{i\,max})}$. We choose q_{io} to be in the region of $O(s^{a_i})$, $0 \le a_i \le ½$ for any s, and we calculate the contribution to $Im\, A_n\ (s,0)$. The value of a_i is not too essential, the main point is whether $s_{i,\,i+1}$ depends on s or not. However in any case we must have at least two vectors with $a_{i_1} = a_{i_2} = ½$.

It is easy to see the behaviour of $s_{i,\,i+1}$

$$s_{i,i+1} = O(s^{|a_i - a_{i+1}|}) \quad \text{if} \quad q_{i1} \cdot q_{i+1,1} > 0,$$

$$s_{i,i+1} = O(s^{a_i + a_{i+1}}) \quad \text{if} \quad q_{i1} \cdot q_{i+1,1} < 0.$$

Let's define formally a_i for vectors pointing in the direction of $\vec{p}_2$ to be negative so $q_{io} \approx O(s^{|a_i|})$. Then we always have

$$s_{i,\,i+1} = O(s^{|a_i - a_{i+1}|}).$$

For $a_{i+1} \le a_i$ we obtain the condition $\sum_{i=1}^{n-1} |a_i - a_{i+1}| = 1$ because $a_i = -a_n = ½$.
The behaviour of $t_{i\,max}$ is very simple in this case, $t_{i\,max} \to 0$ for $s \to \infty$ if $|a_i - a_{i+1}| \neq 0$ and $t_{i\,max} = O(1)$ if $|a_i - a_{i+1}| = 0$.

Using the fact that $\alpha_+(0) = 1$, $\alpha_-(0) < 0$, the maximum contribution is obtained if $a_i = a_{i+1}$ for i even.

Then we have

$$\prod_{i} \left(\frac{s_{i,\,i+1}}{s_0} \right)^{2\alpha_{\pm}(t_{i\,max})} = s^2 \cdot c \tag{6}$$

where c is independent of s.

It is not difficult to show that the exponent of s on the right hand side of formula 6 decreases if we exchange two particles i and $i+1$ (with $a_i > a_{i+1}$). For this exchange we have some $t_{i\,max} \to -\infty$, the corresponding $\alpha(t_{i\,max})$ going to -1, which always decreases the power.

We then see that the maximal behaviour of the above product s^2 is obtained if we have k groups of particles in which all the exponents a are the same, so $(\Sigma\, q_i)^2$ for the particles of a group is independent of s. In the first and in the last group we have always an odd number of particles. In these groups $|a_i| = ½$.

The most interesting case is when the number of groups is equal to two.

Then $q_{io} = c_i\, s^{½}$, $\Sigma\, c_i = 1$, $c_i < ½$. Particles with $i \le k$ are in the first group, for $i \ge k+1$ in the second group.

$$\left(\sum_{i=1}^{k} q_i\right)^2 = O(1),\ \left(\sum_{i=k+1}^{n} q_i\right)^2 = O(1).$$

The above equations must be understood in the sense that these quantities are independent of s. We have still

$$s_{k,\,k+1} = O(s), \qquad s_{i,\,i+1} = O(1) \quad \text{for} \quad i \neq k,$$

$$s_i = O(s), \qquad \text{for} \quad i \le k,$$

$$s_i = O(1), \qquad \text{for} \quad i \ge k+1.$$

So to obtain the contribution of the above region of variables q_i we must perform the following substitution of integral variables in integral 5 :

$$s_i = d_i s \quad \text{for} \quad i \le k \qquad d_1 > d_2 > \dots > d_k , \quad d_1 = 1 .$$

$$s_i = s_i \quad \text{for} \quad i \ge k+1 , \quad \sqrt{s_i} < 1 + \sqrt{s_{i-1}}$$

Asymptotically $\sqrt{\Delta (s_i , t_{i-1} , 1} = d_i s$, for $i \le k$, $\log \frac{s_{k,k+1}}{s_0} k \approx \log \frac{s}{s_0}$ so after the substitution we obtain for the contribution to $Im\, A_n(s,0)$

$$Im\, A'_n(s,0) \approx \frac{s}{\log (\frac{s}{s_0})} R$$

where R is an integral, which is convergent and independent of s .

In a similar way we obtain a contribution

$$Im\, A''_n (s,0) \approx \frac{s}{\log^{k+1} (\frac{s}{s_0})} R_k$$

if we have k groups of particles.

Using the optical theorem we obtain the asymptotic behaviour of the cross section of the creation of n particles:

$$\sigma_n (s) \approx c_n \frac{1}{\log \frac{s}{s_0}} \tag{7}$$

So the model gives picture which is identical with the picture of the multi-fire ball model, but asymptotically the main contribution comes from a two centre emission of particles.

Exactly speaking the main contribution to the cross section in our model satisfies the following definition of the two centre model:

At a given multiplicity n , there are two groups of particles (with k and n-k particles) in the c.m.s. of these groups the distributions of particles and the distribution of the total energy of the fire ball are independent of s, the primary energy. The distribution of k does not depend on s as well.

In our model just like in the model of the mentioned authors/1,2/ there is a section of the chain, where the energy $s_{k,\,k+1} = O(s)$, but it is not equal to s . So in formula (1) there appears a factor $(\frac{s_{k,\,k+1}}{s_0})^{2\alpha(t_{max})-1}$ instead of $(\frac{s}{s_0})^{2\alpha(t_{max})-1}$. If s_1 and s_2 are great in the c.m.s. system of the fire balls $q_{k+1,0} = O(s_2^{1/2})$, $q_{k,0} \approx O(s_1^{1/2})$. Transforming back into the total c.m.s. we obtain that $s_{k,\,k+1} \approx c \frac{s}{s_1 s_2}$ so together with the factor $f(s_1 , s_2) \approx s_1 \; s_2$ we obtain the appropriate behaviour for the cross section.

The main defect of our calculations is that we did not take into account the final state interactions of the emitted particles. These interactions may play an essential role for small energies, so the effect of them is essentially a change of the constant in formula 7, but they do not alter the type of the asymptotic behaviour.

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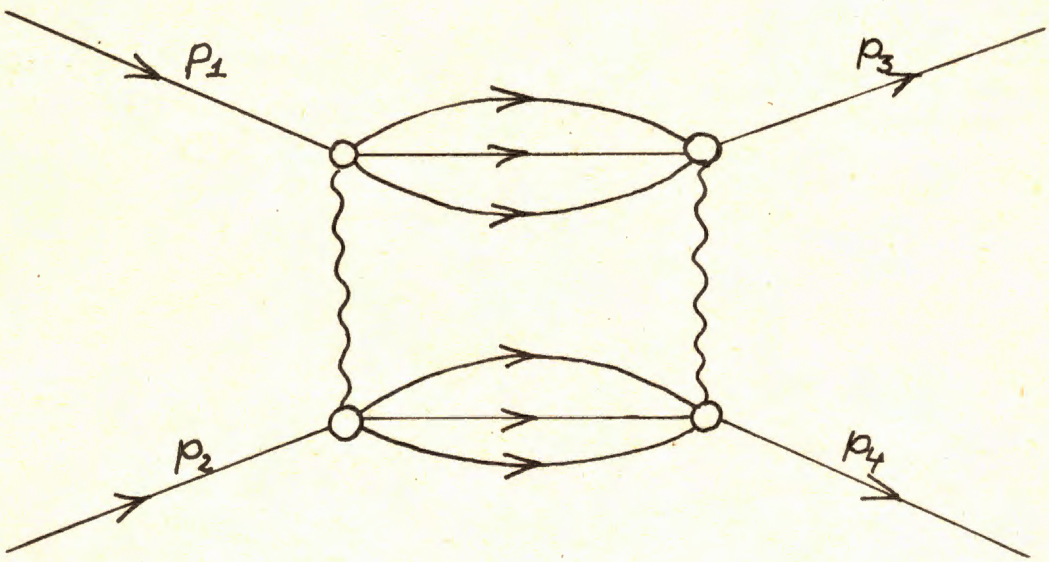

Fig. 1. Diagram for the contribution of many particle intermediate states to the imaginary part of the amplitude in the model of Miyazawa and Suzuki/1/. We denote ordinary particles by straight lines, Regge poles by wavy ones.

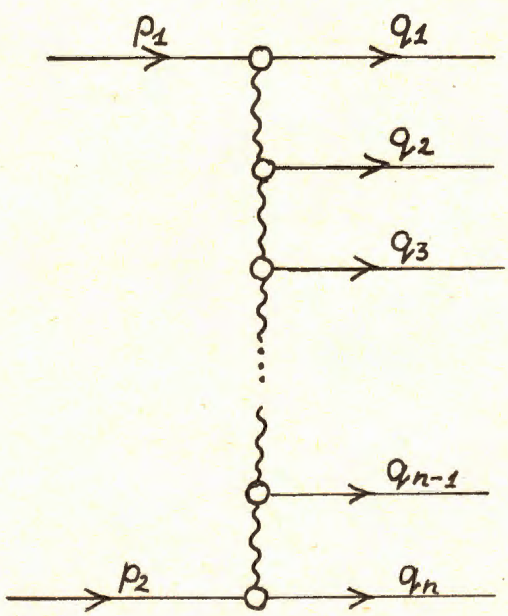

Fig. 2 Diagram for the n particle production amplitude in the model of Contogouris, Frautschi and Wong/2/ and in our model.