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ON THE ACCURACY OF CALCULATIONS OF THE STRONGLY DEFORMED ELEMENT PROPERTIES USING THE SUPERFLUID NUCLEAR MODEL

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Abstract

The accuracy of calculations according to the superfluid nuclear model has been investigated on the basis of a model with five levels which permits to obtain the exact solution on the electronic computer. Comparision has been made with the real case of a system consisting of 102 neutrons.

The investigation has shown that the accuracy of calculations is in the main restricted to the uncertainties in the details of the average field and the fluctuation in these single-particle levels but not to the inaccuracy of the mathematical method used.

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О ТОЧНОСТИ РАСЧЕТОВ СВОЙСТВ СИЛЬНОДЕФОРМИРОВАННЫХ ЭЛЕМЕНТОВ НА ОСНОВЕ СВЕРХТЕКУЧЕЙ МОДЕЛИ ЯДРА

Аннотация

Проведено исследование точности расчетов согласно сверхтекучей модели ядра на основе модели с 5 уровнями, допускающей получение точного решения с помощью электронной вычислительной машины, и для реального случая системы из 102 нейтронов.

Проведенное исследование показало, что точность расчетов ограничена, в основном, плохим знанием поведения уровней среднего поля и их флюктуацией, а не точностью используемого математического метода.

Работа издаётся только на английском языке.

On the basis of the superfluid nuclear model the behaviour of the single-quasi-particle levels of odd nuclei was investigated, the energies of the two-quasi-particle exicted states of even-even nuclei were calculated and the effect of paring correlations on the probabilities of alpha- , beta- and gamma transitions in strongly deformed nuclei was studied in the region $154 \leq A \leq 188$ and $225 \leq A \leq 255$ /1-3/. A satisfactory agreement is obtained between theoretical and experimental data.

As long as the calculations performed are approximate then the problem of their accuracy is of interest. As is known, the errors in the calculations made on the basis of the superfluid nuclear model are, first, due to the uncertainties in the details of the average field and, to the fluctuations in these single-particle levels in passing from one nucleus to another, and, second, they are due to an inaccuracy of the mathematical method used /4/.

The errors which are due to the behaviour of the average field levels were essentially decreased by locating them so that to obtain the best agreement between energies of the calculated single-quasi-particle levels of odd nuclei and the corresponding experimental values. The use of the modified Nilsson's scheme as the average field levels improves noticeably the agreement between calculated and experimental values of the even-even nucleus excited state energies as compared with calculations based on the Nilsson's scheme levels given in /5/.

In these calculations, however, use was made of a single scheme of the average field levels for a large group of nuclei and the alterations in this scheme were not taken into account in passing from nucleus to nucleus.

In the present paper we investigate the accuracy of the mathematical method based on the Bogolubov's canonical transformation and used for calculating the following characteristics of strongly deformed nuclei:

a) the energies of the single-quasi-particle excited states of system consisting of an odd number of nucleons,

b) the energies of the two-quasi-particle excited states of systems consisting of an even number of nucleons,

c) corrections which are due to the superfluidity of the ground- and excited nuclear states to the probabilities of the alpha-, beta-, and gamma transitions which enable one to calculate the relative values of log ft for beta transitions, the hindrance factors F in alpha decays and so on.

1. Exact and Approximate Solutions

We shall investigate the accuracy of the approximate method using the simplified model which was studied earlier by the two of us /6/. Consider the n-particle interaction described by the Hamiltonian

$$H = \sum_{s,\sigma} E(s)\, a^{+}_{s\sigma}\, a_{s\sigma} - G \sum_{s,s'} a^{+}_{s+}\, a^{+}_{s-}\, a_{s'-}\, a_{s'+} \tag{1}$$

Here $a^{+}_{s\sigma}$, $a_{s\sigma}$ are the nucleon creation and annihilation operators in states with quantum numbers s, σ ($s = 1, ..., \Omega$; $\sigma = \pm 1$), $E(s)$ are the energies of the twofold degenerated average field levels, G is the paring interaction constant. This problem is solved in /6/ exactly on the electronic computer for the case of the equidistant location of the average field levels with $\Omega = 5$, $n = 6$ for G equal to $0{,}5\,\Delta E$, $0{,}8\,\Delta E$, $1\,\Delta E$ and $1{,}25\,\Delta E$. The average field levels were denoted by 1,2,3,4,5, the distance between them $\Delta E = E(i+1) - E(i) = 1$ was taken to be a conventional unit of the energy. The method of the exact solution of this problem with basic formulas and notations and the comparisons of the exact solutions with the approximate ones are stated in Ref. /6/.

We will use further this exactly soluble simplified model to investigate the problem about the accuracy of the approximate calculations. We find the excitation spectrum of the even system for $n = 6$ and of the odd system for $n = 5$, the ground-state energy for $n = 4$ and the density of the number of pairs on the average field levels. Calculations are made for the case of equidistant location of the field levels E (s) (denote equid) and for the cases of the following change in the equidistant location of the levels; 1) level 3 is raised by 0,5 (denote (3+)), 2) level 3 is lowered by 0,5 (denote (3–)), 3) levels 2 and 3 are each raised by 0,5 (denote (2+, 3+)), 4) level 2 is lowered by 0,5, level 4 is raised by 0,5 (denote (2–, 4+)). The locations of the average field levels are chosen so as to correspond as well as possible to the behaviour of the average field levels in the region of strongly deformed nuclei.

We compare the exact solution of the problem (denote by m) with the following approximate ones: not taking into account the blocking effect as in the original method for treating pairing correlations (denote by a) and taking into account the blocking effect as in the superfluid nuclear model (denote by δ). As an example in Fig. 1 we give the energies of the ground- and two-quasi-particle excited states for $n=6$ calculated with the exact and approximate methods when $G = 1,25$ for case (2+, 3+). Similar figures in the case of equidistant location of the average field levels are given in Ref. /6/. The figures on the left denote there the average field levels which are occupied by the quasi-particles. We design by K the last filled single-particle level of the average field at $G = 0$, by K–1, K–2 the hole states, by K+1, K+2 the particle states.

It is known that in calculations according to the superfluid nuclear model the number of particles is conserved on the average and the wave functions Ψ of the n -particle system contain admixtures of states with $n - 2$, $n+2$ and so on numbers of particles. The wave function Ψ of the ground state of the even system reads

$$\Psi = \prod_s (u_s + u_s a^+_{s+} a^+_{s-}) \Psi_0 \qquad (2)$$

where $a_{s\sigma} \Psi_0 = 0$, $v_s^2 = \frac{1}{2}\left(1 - \frac{E(s) - \lambda}{\sqrt{C^2 + \{ E(s) - \lambda \}^2}} \right)$, $u_s^2 = 1 - v_s^2$, C is the correlation function, λ - is the chemical potential of the system. As was noted in Ref. /7/ the accuracy of calculation of the excitation energy will be improved if instead of the wave functions (2) we use the normalized projections of these wave functions on the subspace of n-particles. The exact calculations both for the excitation energies and pair distribution densities are also compared with the approximate ones by the methods a and δ with the projected wave functions (denoted by a_p and δ_p).

To describe more exactly the behaviour of the single-particle levels we have used in /2/ experimental data on the odd nucleus levels. The average field levels were altered so as to obtain agreement between calculated energy values and corresponding experimental data. In connection with describing more exactly the behaviour of the average field levels the problem arises how to calculate exactly the energies of the single-quasi-particle excited states in the superfluid nuclear model.

Consider a system consisting of five particles. The energies of the ground- and excited states of the system for the case (2+, 3+) at $G = 1$ are given in Fig. 2 (1,2,3,4,5 are levels occupied by a quasi-particle) . From Fig. 2 it is seen that the energies of the ground state in cases of the exact solution and δ_p are practically coincident and lie by almost one level below than in the solutions by the a and b methods. The calculations using the b method gives a correct description both of the sequence of the energy levels and of the distances between them. At the same time the calculations using the a method lead to a changed order of levels and to a considerable decrease of the distance between them as compared to the exact calculations. In order to demonstrate the accuracy of the approximate methods in Fig. 3 we plot the excitation energies of the system with $n = 5$ for the case (3+). Above the value $E(k) = 0$ corresponding to the ground state we draw positions of the particle excited states, and below positions of the hole states. The results of calculations for $G = 1$ and 1,25 for all above cases of behaviour of the average field levels show that the excitation energies of system consisting of an odd number of particles are calculated in this simplified model by a method with the accuracy (5-15)%. The accuracy of similar calculations in the case of strongly deformed nuclei is expected to be noticeably improved. From comparison of the spectra given in Figs. 3 and 2 it is seen that the raising of the level 2 by 0,5 leads to greater change in the behaviour of the energy levels in the case of the exact solution than the errors of the approximate b method in comparison with the corresponding exact solution. From Fig. 3 it is seen that in passing from $G=1$ to $G = 1,25$ the spectrum of the single-quasi-particle levels becomes narrow, what provides evidence of the correctness of conclusions/1/ about the influence of the superfluidity on the single-particle levels of odd-nuclei.

Thus, the investigation shows that the approximate mathematical method used in the superfluid nuclear model gives a fairly good accuracy in calculating the energies of the single-quasi-particle excited states of the strongly deformed odd nuclei.

2. Density of the Number of Pairs on the Average Field Levels.

As is known the pairing correlations of the superconductive type lead to a diffusion of the Fermi surface. Hence, the average values of the particle number operator in the state s , N_s in the case of the exact solution and v_s^2 in the approximate calculations, become different from 1 and zero. Corrections to the probabilities of alpha- beta- and gamma transitions which are due to the superfluidity of the ground- and excited states consist of the combinations of the quantities v_s and u_s. Therefore it is necessary to make clear to what accuracy the densities of number of pairs on the average field levels are calculated by the method of the superfluid nuclear model.

We compare the distribution of number of pairs in the ground- and excited states calculated by the exact and approximate methods. We consider, first, the distribution of the density in the ground state of the system consisting of an even number of particles. For this in Table 1 we give the values of $\overline{N}_s$, $<\Phi_o | N_s | \Phi_o>$ and v_s^2 for the case (3+) at $G=1$ and $G = 1,25$; here $<\Phi_o | N_s | \Phi_o>$ is the distribution of the density in states described with projected wave functions. The results of the analogous calculations for the case (equid) at $G = 1,25$, 1, 0,8 and 0,5 are given in Ref./6/. From Table 1 it is seen that v_s^2's describe well the den sity of pairs, the ratio $\frac{v_s^2}{N_s}$ changes between 0,95 and 1,2. Note the function v_s^2 yields a somewhat larger diffusion of the Fermi surface compared with the exact solution, while the function $<\Phi_o | N_s | \Phi_o>$, on the contrary, leads to a decrease of this diffusion. Calculations with projected wave functions do not lead to a noticeable improvement of the approximation v_s^2.

We consider the density of tbe number of pairs in the two-quasi-particle excited states of system consisting of even number of particles. In Table 2 we gave the distribution for the case (2+, 3+) at $G = 1,25$. From the Table it is seen that the calculations by the method b describe well the distribution of the pair density in the excited states. In the exact calculations the importance of the blocking effect is demostrated. The density of the number of pairs on the average field levels changes noticeably in the transition from the ground to the excited states. This is seen from the comparison of pair distributions in the case (3+) given in Tables 1 and 3. Note the exact calculations yield a somewhat larger depression of pair correlations in the excited states compared with the ground ones than the calculations by the b method . The calculations with projected wave functions lead to a noticeable weakening of the superfluidity as compared with the exact calculations. In the calculations by the a method the number of particles is not conserved even on the average and the density of pairs strongly differs from that in the case of the exact method.

It should be noted that for comparatively small values of G and in the case of a small density of the average field levels near the Fermi surface energy in the two-quasi-particle states (K, K + 1) and sometimes in other cases, the pairing correlations calculated by the b method are practically absent. The exact calculations in these case yield a noticeable depression of pair correlations, however the diffusion of the Fermi surface remains still essential enough. This occurs, e.g., in the case (3+) at G= 1, what is seen from Table 3. In raising G the usual picture is restored, e.g., at G = 1,25. At G = 0,5 in the case (equid.) the pairing correlations calculated by the b method are absent in states (K, K+1) , (K, K+2) and (K–1, K+1). In those cases when the pairing correlations calculated by the b method are absent, i.e. $C \approx 0$ we have a significant disagreement between the approximate calculations and the exact ones.

In investigating the properties of the strongly deformed nuclei on the basis of the superfluid model of a nucleus in /2/ there were cases when in states (K, K+1) and rarely in states (K, K+2) and (K–1, K+1) the pairing correlations were absent. The present investigations show that in these cases the accuracy of calculations by the b method becomes worse. Therefore it is desirable that in case where $C \approx 0$ the method should be somewhat modified to obtain better agreement with the corresponding exact solutions. It should be noted that in this approach some interesting results have been obtained by Mikhailov I. /8/

Consider the density of pairs in the ground- and excited states of system consisting of odd number of particles. Table 4 contains a part of the results obtained. We first consider regularities obtained in the exact solution. In all cases the pairing correlations in the ground states are somewhat more strongly depressed as compared to the hole K–1, K–2 and to the particle K+ 1, K+2 ones. From the comparison of the pair density in the ground states of even and odd systems given in Tables 1 and 4 it is seen that the pairing correlations in the system consisting of odd number of particles are noticeably depressed compared with that consisting of even number of particles. Thus, the calculations made provide evidence of the correctnees of conclusions drawn on the basis of the superfluid nuclear model that the correlation function C(K) of the ground state of system of 2N – 1 particles is somewhat less than the C correlation function of the ground state of system of 2N particles but the values of $C(K \pm i)$ for the excited states turned out to be somewhat larger than those of C (K) .

From Table 4 it is seen that the calculations by the b method describe sufficiently well the density of pairs on the average field levels. The quantities v_s^2 well reproduce all regularities in the behaviour of the corresponding quantities $\overline{N}_s$.

Corrections to the probabilities of alpha-, beta- and gamma transitions which are due to the superfluidity of the ground- and excited states of atomic nuclei are of the following form:

$$R \approx \zeta \prod_s (u_s u'_s + v_s v'_s) \tag{3}$$

If we restrict ourselves to the transitions between states with 0,1,2 quasi-particles we obtain ζ consisting of combinations of u_s and v_s which are referred to the ground and single-particle states. The product can then contain u'_s and v'_s which are referred to the two-quasi-particle states.

The calculations of v_s and u_s for the ground states of systems consisting of even number of particles and for the single-quasi-particle states by the *b* method are in satisfactory agreement with the exact ones. The ratio $\frac{v_s^2}{\overline{N}_s}$ changes in the limits

$$0{,}9 < \frac{v_s^2}{\overline{N}_s} < 1{,}5$$

It should be noticed that comparatively large deflections of $v_s^2/\overline{N}_s$ from unity are due to the fact that in the simplified model under consideration Ω and n are taken to be small.

The products in (3) are calculated by the b method with a sufficiently good accuracy, except when the correlation function of one of the states is close to zero. In these cases the calculations by the *b* method gives understated values.

In calculating corrections to the probabilities of the alpha-, beta- and gamma transitions in strongly deformed nuclei there were some cases where in the final state the correlation function C was zero. From the analysis it follows that in such cases the understated values for R's were obtained. So, e.g., in calculating corrections to the probabilities of beta transitions in[2] it was found that at C=0 the products $\prod_s (u_s u'_s + v_s v'_s)$ assumed the values 0,5 – 0,7, while at $C \neq 0$ they were 0,6 – 0,99. This fact points out the necessity of modification of the b method where C's are very close to zero.

The expressions for ζ in the case of gamma transitions are of a rather complicated form, so, e.g., for transitions with the conservation of the number of particles

$$\zeta = (u_{s_2}(s_1)\, u_{s_1}(s_2) - \eta \; v_{s_1}(s_2)\, v_{s_2}(s_1)) \qquad (4)$$

where $\eta = 1$ for electric and $\eta = -1$ for magnetic transitions.

Since in cases with $\eta = 1$ large errors can appear, then corrections R_γ should be calculated by the exact method and compared with the results of calculation by the *b* method. The results are given in Table 5. The corrections R_γ^m to the magnetic transitions caclulated by the b method are in a satisfactory agreement with the exact calculations. The lower values down to 0,4 compared to the exact ones are obtained by the b method for corrections R_γ^e . In calculating corrections R_γ for electromagnetic transitions from the two-quasi-particle states to the ground ones use should be made of simpler calculations by the a method. The accuracy of calculation of corrections R_γ may be considered sufficient as far as small fluctuations in the average field levels lead to a large change of R_γ's and the experimental data on the probabilities of gamma transitions are insufficiently exact.

The investigation carried out showed that the densities of pairs in the ground- and excited states calculated by the *b* method were in satisfactory agreement with the results of the exact calculations. Small changes in the behaviour of the average field levels lead to approximately the same changes of the exact solutions as the differences between exact and approximate calculations. Since in increasing Ω, n and decreasing G the accuracy of calculations by the b method is expected to be improved, hence it follows that the accuracy of calculations of corrections R is sufficiently good.

From the above analysis it follows that in calculating on the basis of the superfluid nuclear model corrections R to the probabilities of the alpha-, beta- and gamma transitions there is no need in a further improvement of the mathematical method (except states with $C \approx 0$) if the behaviour of the average field levels will not be described more exactly.

3. Energies of the Two-Quasi-Particle Excited States

We investigate the problem of the accuracy of calculation of the energies of the two-quasi-particle states of system consisting of even number of nucleons. We make use of the simplified model and compare the exact solution of the problem (m) with the approximate ones by the a method without the account of the blocking effect, by the b method with the account of the blocking effect and by the a_p and b_p ones with the projected wave functions. The results of calculations of the energies of the ground and two-quasi-particle excited states for the case of the equidistant location of the average field levels at G = 0,5; 0,8, 1 and 1,25 are given in Ref. /6/ and for the case (2+, 3+) at G = 1,25 in Fig. 1. The energy of the ground state of the system in the case of the exact solution for all values of G is less than that obtained by the approximate a or b methods. From Fig. 1 it is seen that the calculations by the b method well describe the sequence of the excited states and well reproduce the character of their behaviour. For example, in exact solution in the case (2+, 3+) small distances between the levels (3,5) and (2,4) and also (1,3) and (1,5) are obtained. In the b approximation these particularities are conserved. The absolute energy values calculated by the b method differ noticeably from those obtained by the exact method, the first are lower especially for the (K, K+1) level.

For the cases (equid.), (3–), (3+), (2+, 3+), (2–, 4+) and other of the average field behaviour at G=0,8, 1 and 1,25 it is shown that the energy of the (K, K+1) level calculated by the exact method lies below the corresponding values of the gap 2C. This fact proves the importance of the blocking effect for the same location of the average field levels as in the case of the strongly deformed nuclei.

In calculations by the a method the correct sequence of the two-quasi-particle levels is not conserved and the particularities of the spectrum are not transferred. The differences of energies between the ground- and excited states do not strongly differ from the exact values, however the density of the two-quasi-particle states obtained in the case of the a method is about twice as large as that in the case of the exact solution. The distances between the two-quasi-particle excited states calculated by the b method are approximately the same as in the case of the exact solution. The densities of the levels of the two-quasi-particle state energies calculated by the m and b methods are approximately identical

It should be noticed that in the considered simplified model the accuracy of solutions by the a and b methods is worse than that in the case of the strongly deformed nuclei because of very small values of Ω and n and also relatively large values of G. At the same time the role of the blocking effect is somewhat overestimated.

The energies of the ground- and excited states calculated by the a_p and b_p methods with the projected wave functions are in a fairly good agreement with those calculated by the exact method, especially for large G's. In this case the simplified model gives better agreement with the exact solutions than the calculations in the region of strongly deformed nuclei.

Owing to the fact that the calculations according to the superfluid nuclear model are based on the experimental data on pairing energies it is necessary to compare the energies of the ground- and excited states calculated by the m and b methods for the same values of the pairing energies P. In Fig. 4 we give the energies of the ground- and excited states calculated exactly (m) for $P=P_0$ and G=1, by the b nethod at G=1, and by the b method at $P=P_0$. If in the exact method G= 1, $P = P_0$,then in the b method the pairing energy takes the value $P=P_0$ at G = 1,09. From Fig. 4 it is seen that the errors in the differences of energies in the b method are twice as small compared with the exact one if calculations are made for one and the same value of the pairing energy $P = P_0$ as compared to those in calculations made for one and the same value of the pairing interaction constant G = 1. As long as the calculations made in accordance with the superfluid nuclear model yield the correct sequence of the levels of the even-even nucleus energy, the accuracy of calculations becomes higher due to the fact that they are based on the experimental pairing energies.

We investigate the influence of the fluctuation in the average field levels on the change of the excited state spectrum. In Fig. 4 we gave the exact solutions for cases (3–) and (3+) at G = 1 . From Fig. 4 it is seen that the change in the position of the only one average field level leads to a marked change of energies both of the ground- and excited states and in a number of cases to the change of the excites states sequence. Thus, an insignificant change in the behaviour of the average field levels leads to a large change of the energy spectrum as compared to that which follow from the approximate solution of the problem.

As was already noted the energies of the ground- and two-quasi-particle states calculated by the a_p and b_p methods are fairly close to those obtained by the exact method. Use this fact in order to estimate the accuracy of calculations of the even-even nucleus energy. Calculate the energies of the ground- and excited states by the b, a_p and b_p methods for systems consisting of 102 neutrons. We use the scheme of the neutron single-particle levels of the average field in the region 98< N <110. Calculations are made with the same parameters and in just the same way as in /2/.

The projection wave function and the energy of the ground state of the n particle system ($\nu = n/2$) are of the form:

$$\phi = \{ \sum_{s_1<s_2<\dots<s_\nu} \frac{v_{s_1}^2 v_{s_2}^2 \dots v_{s_\nu}^2}{u_{s_1}^2 u_{s_2}^2 \dots u_{s_\nu}^2} \}^{-1/2} \sum_{s_1<s_2<\dots<s_\nu} \frac{v_{s_1} v_{s_2} \dots v_{s_\nu}}{u_{s_1} u_{s_2} \dots u_{s_\nu}} \beta^+_{s_1} \beta^+_{s_2} \dots \beta^+_{s_\nu} \Psi_0 \qquad (5)$$

$$\epsilon_p = \{ \sum_{s_1<s_2<\dots<s_\nu} \frac{v_{s_1}^2 v_{s_2}^2 \dots v_{s_\nu}^2}{u_{s_1}^2 u_{s_2}^2 \dots u_{s_\nu}^2} \}^{-1} \{ 2 \sum_{s_1<s_2<\dots<s_\nu} ([E(s_1) + E(s_2) + \dots + E(s_\nu)] \frac{v_{s_1}^2 v_{s_2}^2 \dots v_{s_\nu}^2}{u_{s_1}^2 u_{s_2}^2 \dots u_{s_\nu}^2} -$$

$$- G \sum_{f_1 f_2} \frac{v_{f_1} v_{f_2}}{u_{f_1} u_{f_2}} \sum_{s_2<s_3<\dots<s_\nu \neq f_1 f_2} \frac{v_{s_2}^2 v_{s_3}^2 \dots v_{s_\nu}^2}{u_{s_2}^2 u_{s_3}^2 \dots u_{s_\nu}^2} \qquad (6)$$

where $\beta_s = a_{s-} a_{s+}$. These and similar formulas will be used in calculating by the a_p and b_p methods.

The results of calculations of the energy differences (expressed in MeV) between two-quasi-particle (with $f_1 \neq f_2$) and ground states are represented in Fig. 5. The calculations by the b method are made at $G = 0{,}022\, h\omega^\circ$, which corresponds to the pairing energy P = 1,26 MeV and at $G = 0{,}023\, h\omega^\circ$, P=1. 51 MeV. Calculations by the a_p and b_p methods are performed at $G = 0{,}022\, h\omega^\circ$, in this case the pairing energy P=1,57 MeV. The pairing energies obtained by the b method at $G = 0{,}023\ h\omega^{\circ}$ are very close to those obtained by the a_p and b_p methods at $G = 0{,}022\ h\omega^{\circ}$ and to experimental data. The comparison of energies calculated by the b method at $G = 0{,}023\ h\omega^{\circ}$ with those by the a_p method shows that the order of the levels is one and the same in both cases, and the difference of energies does not exceed 100-200 KeV. In calculations by the b_p method the (K, K+1) and (K−1, K+1) state energies are not given since they are not calculated with a sufficient accuracy because C's are very small in those states. From the comparison of calculations by the b and b_p methods it follows that the excitation energies in the case of the b method are noticeably lower than those calculated by the b_p method. Notice that the method b_p as well as a_p yield somewhat larger values of the excitation energies. At the same time the calculations by the b method reproduce the special features of the spec trum revealed in calculations by the b_p method.

From the comparison made we may conclude that the energies of the two-quasi-particle excited states of even-even strongly deformed nuclei are calculated by the b method with the accuracy up to 10%.

Thus, the accuracy of calculations of the two-quasi-particle excited states energies on the basis of the superfluid nuclear model is restricted to the uncertainties in the details of the average field and the neglect of the fluctuations in the transition from nucleus to nucleus, but not to the inaccuracy of the mathematical method.

4 Conclusion

Here we give the approximations which have been used in calculating some properties of strongly deformed elements on the basis of the superfluid nuclear model.

The exact equations for the nuclear many-body problem were obtained in /9/ by means of the Bogolubov variational principle. In further investigations based on these equations the following approximations were made:

1. The density function $F(f, f') = \langle a_f^{+} a_{f'} \rangle$ was assumed to be diagonal. i.e.

$$F(f, f') = F(f)\,\delta(f - f') \qquad (7)$$

In the approximation (7) the average field and the residual interactions of the superconducting type /4/ were singled out explicitly from the most general type of the interaction between nucleons. The neglect of the non-diagonal part of F (f,f') leads to errors of the order (1–2)% in the energies of the single- and two-quasi-particle excited states and to errors less than 1% in the corrections R to the probabilities of the alpha- beta- and gamma transitions to the non-collective levels.

2. Interactions leading to the superconducting type pairing correlations described by the function G(s+ ,s-; s'-,s'+) are considered to be independent of the quantum numbers s and s'. i.e.

$$G = \mathrm{Const} \qquad (8)$$

The pairing interaction constants G were calculated in /2/ using the experimental data on pairing energies. It was shown that G changes as A^{-1} in the transition from one nucleus to another and form the region $154 < A < 188$ to $225 < A < 255$. Hence, it follows that the approximation (8) is satisfied with high accuracy.

On the basis of the approximations (7) and (8) the superfluid nucleus model was formulated which takes into account the residual interactions between nucleons described by the Hamiltonian(1). In accordance with this model the accuracy of calculations depends on:

3. the accuracy of the determination of the average field levels,

4. the accuracy of the mathematical method used in solving this problem.

The investigations carried out have shown that the accuracy of calculations according to the superfluid nuclear model is restricted, in general, to the uncertainties in details of the average field and the fluctuation of these levels but not to the inaccuracy of the mathematical method .

Taking into consideration the errors of calculations which come from the abovementioned sources we draw the following conclusions.

a) the accuracy of calculations of the energies of the two-quasi-particle levels of even-even nuclei amounts to (10 – 20) %,

b) the accuracy of calculations of the corrections R to the probabilities of the alpha- , beta- and gamma transitions varies between 10% and 100%.

The calculations made in /2/ and the comparisons with the experimental data agree with these conclusions. The calculations showed that on the basis of the superfluid nuclear model one may investigate not only the general regularities in the behaviour of nuclei but specific features of each of them.

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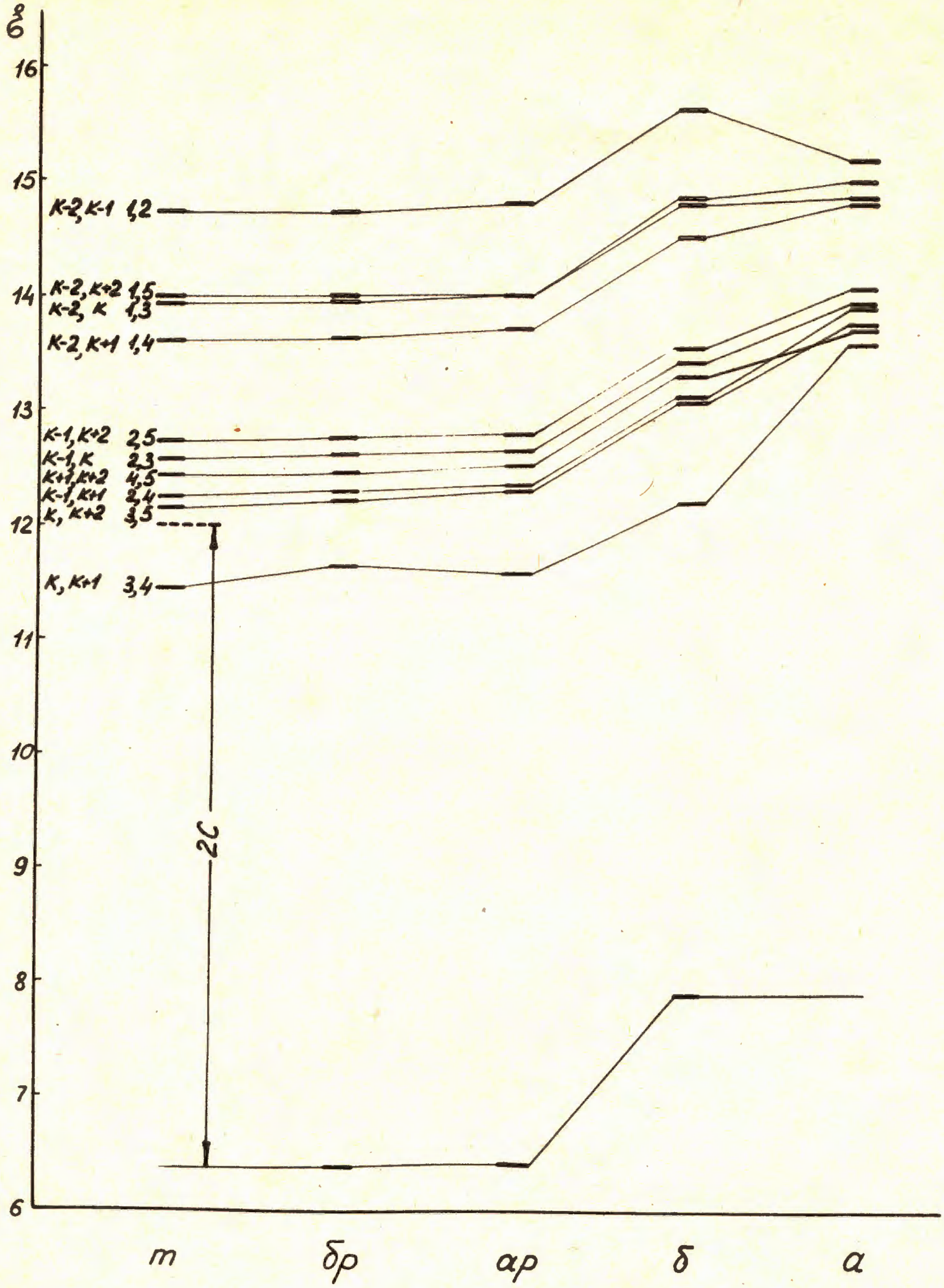

Fig. 1 Comparison of the exact (m) energy levels with the approximate ones calculated by: the (a) method, with projection (ap), the (b) method, with projection (b$_p$), b and a methods for the case (2+, 3+) at G= 1,25

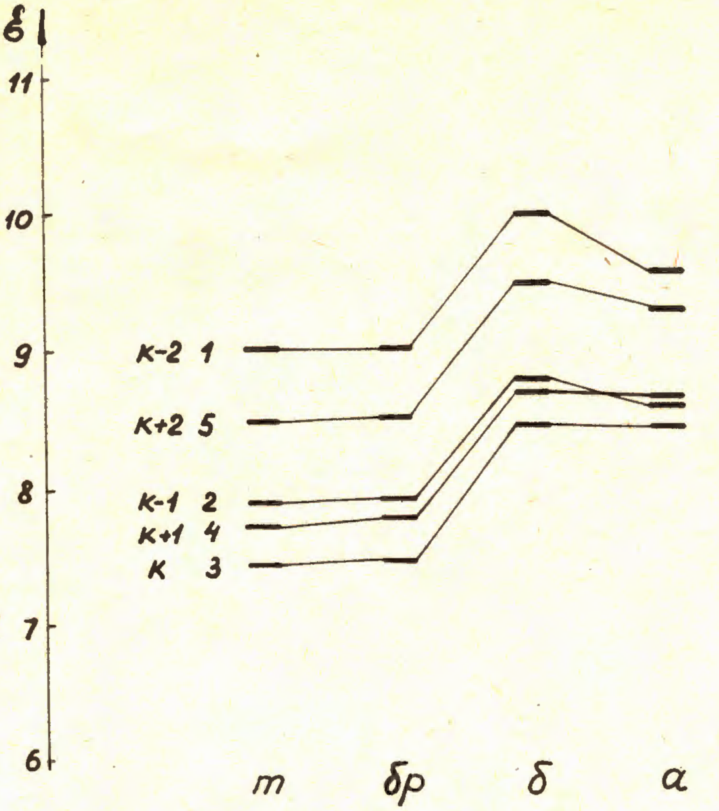

Fig. 2 Ground- and excited state energies with n=5 in the case (2+,3+) at G=1.

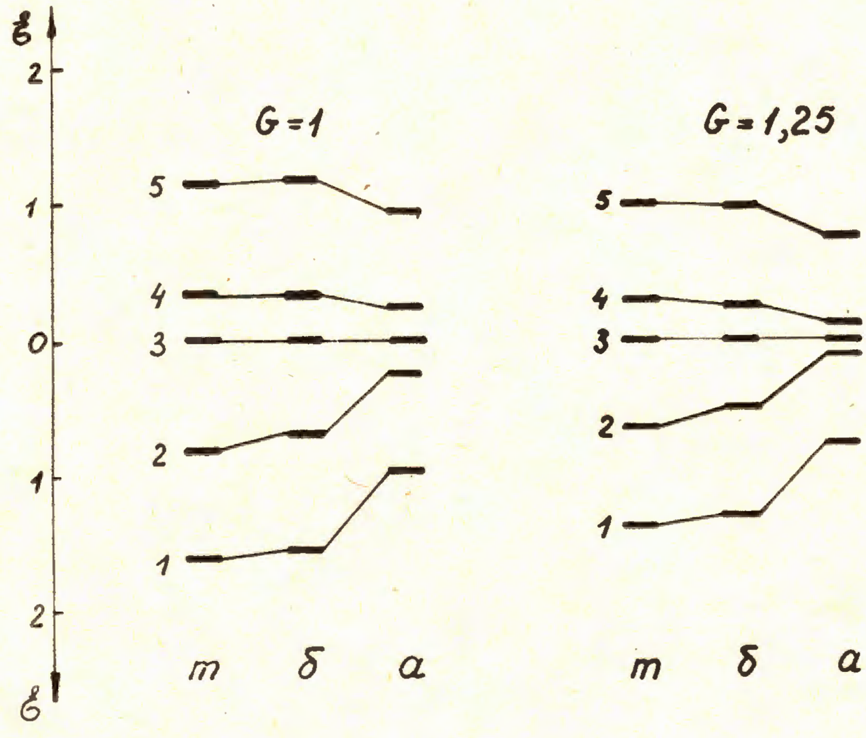

Fig. 3 Spectrum of the odd system levels in the case (3+).

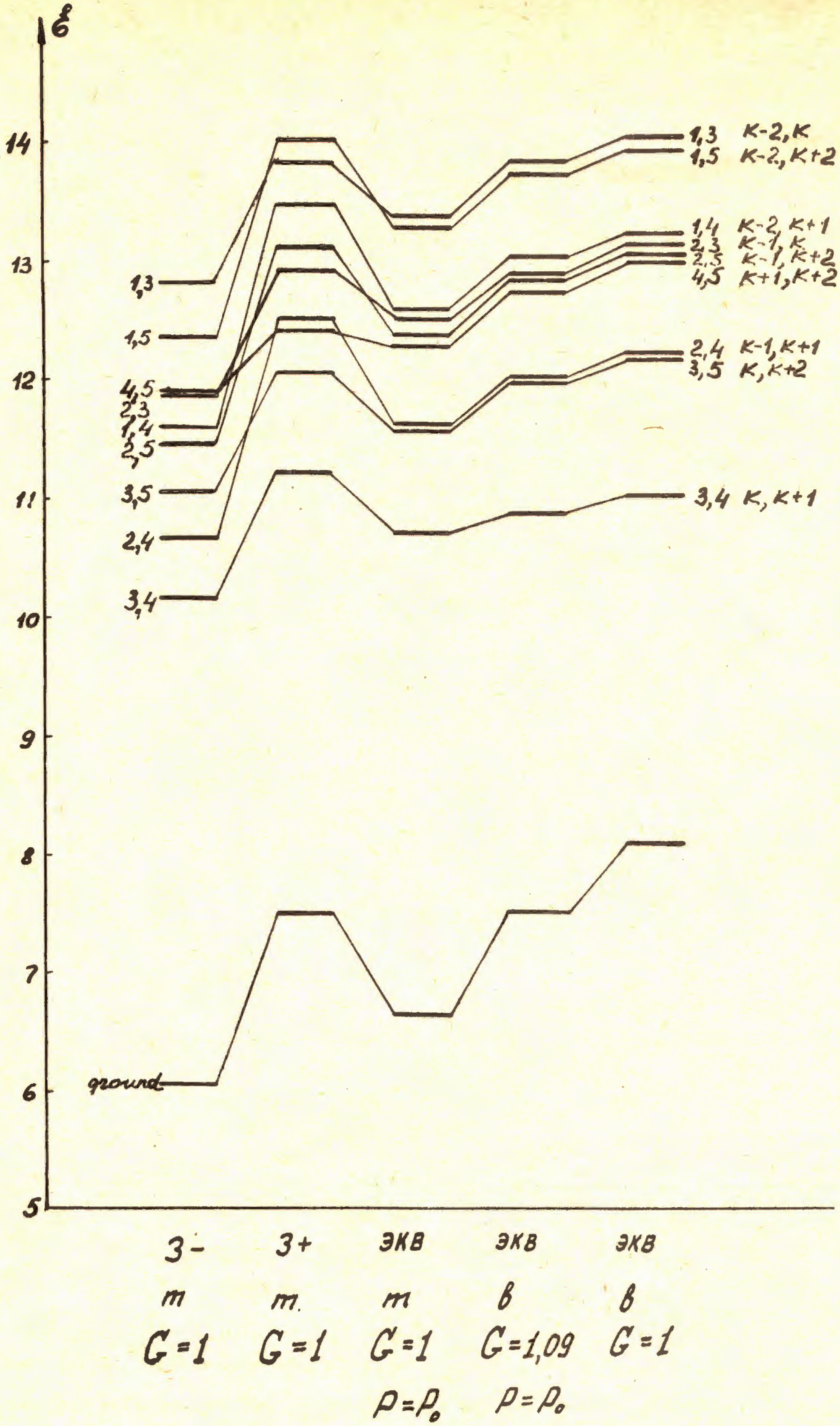

Fig. 4 Energies of the ground- and two-quasi-particle excited states of system consisting of even number of particles.

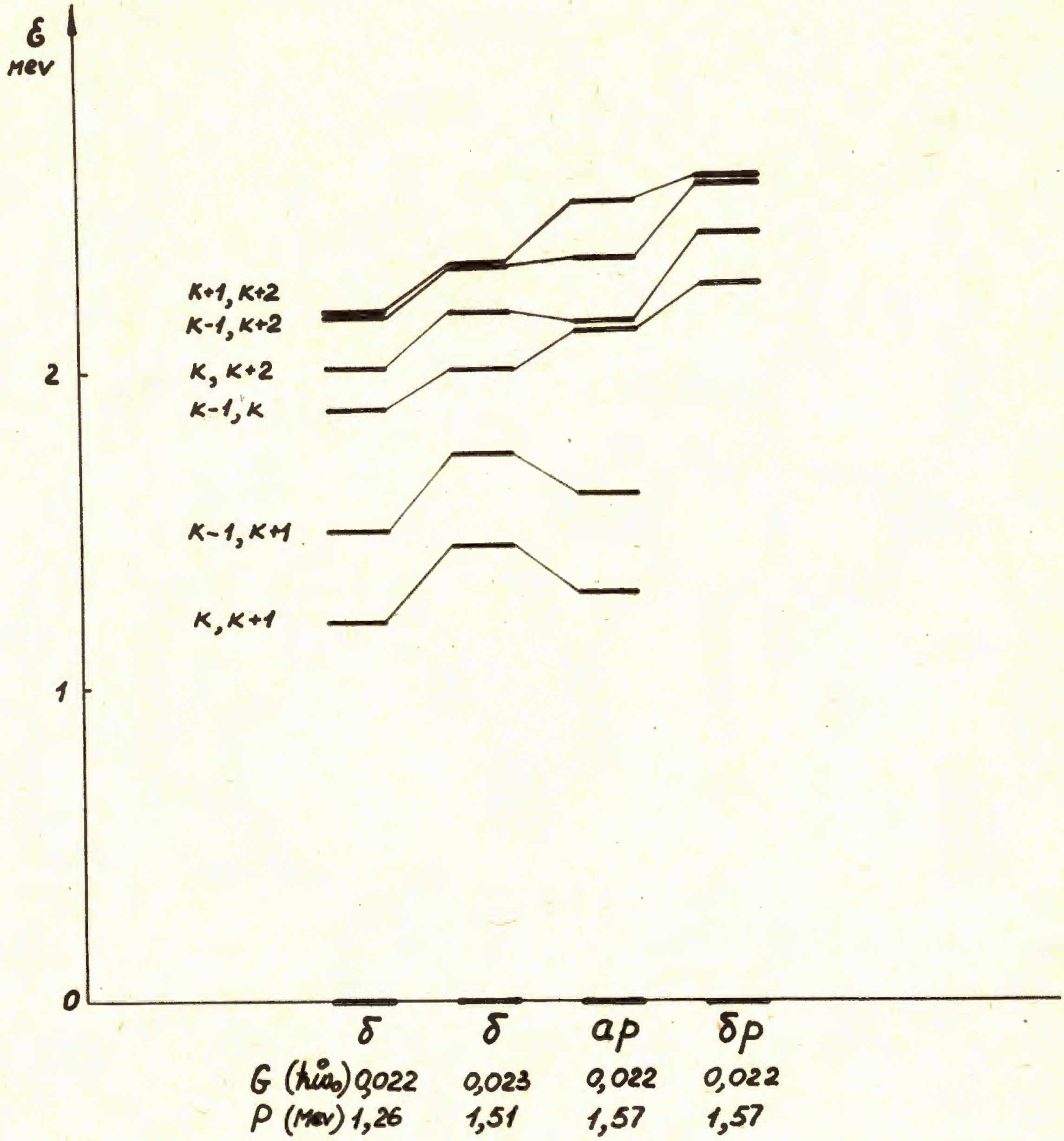

Fig. 5. Energies of the two-quasi-particle excited states of the neutron system with N=102 calculated on the basis of the superfluid nuclear model by the b method, with projected wave functions (ap, b_p).

Table I

Distribution of density in the ground state of the even system in case (3+1)

s		$G=1$			$G=1,25$		
		$\bar{N}_s$ m	$\langle \Phi_0 \vert N \vert \Phi_0 \rangle$ b_p	v_s^2 b	$\bar{N}_s$ m	$\langle \Phi_0 \vert N \vert \Phi_0 \rangle$ b_p	v_s^2 b
1	K-2	0,91	0,93	0,90	0,88	0,90	0,86
2	K-1	0,86	0,87	0,83	0,82	0,82	0,78
3	K	0,61	0,59	0,57	0,59	0,58	0,57
4	K+1	0,41	0,42	0,45	0,45	0,45	0,48
5	K+2	0,21	0,19	0,25	0,26	0,25	0,31

Table 2

Distribution of the number of pairs in the two-quasi-particle excited states of the even system in case (2+, 3+) at G=1,25

K_1, K_2 s_1, s_2	s_i	m	b_p	b	a
K,K+1	1	0,969	0,996	0,974	0,863
3,4	2	0,936	0,987	0,908	0,732
	5	0,095	0,017	0,119	0,328
K,K+2	1	0,946	0,972	0,930	0,864
3,5	2	0,864	0,885	0,766	0,732
	4	0,190	0,143	0,304	0,495
K-1,K+1	1	0,967	0,988	0,965	0,863
2,4	3	0,867	0,899	0,765	0,582
	5	0,166	0,113	0,270	0,328
K+1,K+2	1	0,928	0,950	0,907	0,863
4,5	2	0,792	0,791	0,698	0,732
	3	0,280	0,259	0,395	0,582
K-1, K	1	0,965	0,985	0,965	0,863
2,3	4	0,785	0,804	0,685	0,495
	5	0,240	0,211	0,350	0,328
K-2,K-1	3	0,863	0,874	0,816	0,532
1,2	4	0,726	0,792	0,728	0,495
	5	0,341	0,335	0,456	0,328

Table 3

Density of the number of pairs in the two-quasi-particle exited states of the even system in case (3+)

K_1, K_2 / $\mathfrak{z}_1, \mathfrak{z}_2$	$\mathfrak{z}_i$	G = I		G = 1,25	
		m	*b*	*m*	*b*
K,K+1	1	0,981	1,000	0,970	0,986
3,4	2	0,969	1,000	0,953	0,966
	5	0,050	0,000	0,077	0,048
K,K+2	1	0,965	0,967	0,948	0,928
3,5	2	0,935	0,903	0,907	0,834
	4	0,100	0,130	0,145	0,238
K-1,K+2	1	0,978	0,982	0,967	0,965
2,4	3	0,903	0,829	0,867	0,765
	5	0,119	0,189	0,166	0,270
K+1,K+2	1	0,951	0,935	0,929	0,898
4,5	2	0,898	0,818	0,862	0,771
	3	0,151	0,247	0,209	0,331
K-1,K	1	0,977	0,979	0,965	0,965
2,3	4	0,837	0,726	0,795	0,685
	5	0,186	0,295	0,240	0,350

Table 4

Distribution of the number of pairs in the ground and excited states of system with odd number of particles

$K_1 s_1$	s_i	G=I case (2-,4+)		G=I case (3+)		G=1,25 case (3+)	
		m	b	m	b	m	b
K,3	1	0,945	0,947	0,931	0,912	0,895	0,853
	2	0,929	0,915	0,878	0,794	0,824	0,723
ground	4	0,071	0,085	0,122	0,206	0,176	0,277
	5	0,055	0,053	0,069	0,088	0,105	0,147
K+1,4	I	0,891	0,844	0,914	0,886	0,875	0,830
	2	0,846	0,763	0,836	0,738	0,779	0,685
	3	0,201	0,324	0,178	0,290	0,240	0,346
	5	0,062	0,069	0,072	0,088	0,106	0,139
K+2,5	1	0,882	0,832	0,893	0,855	0,867	0,799
	2	0,834	0,747	0,794	0,686	0,721	0,641
	3	0,204	0,321	0,183	0,274	0,244	0,320
	4	0,080	0,100	0,130	0,185	0,168	0,240
K-1,2	1	0,939	0,931	0,927	0,923	0,896	0,880
	3	0,799	0,676	0,604	0,528	0,564	0,507
	4	0,153	0,237	0,328	0,377	0,359	0,394
	5	0,109	0,156	0,141	0,172	0,181	0,219
K-2,1	2	0,920	0,900	0,873	0,846	0,831	0,793
	3	0,796	0,679	0,610	0,550	0,576	0,534
	4	0,166	0,253	0,357	0,408	0,389	0,426
	5	0,118	0,168	0,160	0,196	0,204	0,247

Table 5

Corrections R_γ^e to electric, R_γ^m to magnetic transitions in system consisting of odd number of particles at G=1

Transition $J_i \to J_k$		Case (3+)				Case (2-,4+)			
		m		b		m		b	
		R_γ^e	R_γ^m	R_γ^e	R_γ^m	R_γ^e	R_γ^m	R_γ^e	R_γ^m
1	3	0,35	0,84	0,24	0,76	0,58	0,94	0,41	0,79
2	3	0,26	0,90	0,11	0,87	0,55	0,96	0,35	0,84
4	3	0,49	0,99	0,26	0,98	0,55	0,96	0,35	0,84
5	3	0,58	0,97	0,42	0,92	0,58	0,94	0,41	0,79