ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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REGGE POLES IN QUANTUM ELECTRODYNAMICS

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Abstract

On the basis of Regge's ideas and the perturbation theory the energy levels of positronium have been obtained.

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На основе синтеза идей Редже и теории возмущений получены энергетические уровни позитрония.

It was shown [1–3] that by combining Regge's ideas and the perturbation theory it is possible to get, with the aid of the renormalization group method, the Regge-type behaviour of the scattering amplitude for scalar particles

$$M \to f(s)\, t^{\alpha(s)} .$$

If the coupling constant is small, the trajectory $\alpha(s)$ can be calculated by the perturbation theory. Then, by solving $\alpha(s)=\ell$ we obtain the energy levels of the bound states. From this standpoint it is of particular interest to investigate Regge-type behaviour of the scattering amplitudes in electrodynamics where the coupling constant is small and there is a regular method for calculating the energy levels of the bound states. Here we apply this method to study the electron-positron system in the singlet state. We will show that the summation of a certain class of diagrams which is improved by means of a renormalization group method, leads to Regge-type behaviour of the scattering amplitude in the crossed channel, the obtained Regge trajectory giving the known energy leves of parapositronium.

In order to find Regge trajectory for the electron-positron system by the renormalization group method, we have to calculate the contributions to the scattering amplitude from the diagrams of the second and fourth orders shown in Fig. 1 and 2.

We denote by p_1 and q_1 the 4-momenta of the electron and the positron in the initial state, by p_2 and q_2 – the same quantities in the final state, by m – the electron mass, and $q = p_1 - p_2$, $s = -(p_1+q_1)^2$, $t = -(p_1-p_2)^2$, $u = -(p_1-q_2)^2$. The matrix element of the second order diagram is

$$M_2 = \frac{e^2}{t}\, V , \tag{1}$$

$$V = \bar{u}(p_2)\gamma_\alpha u(p_1)\, \bar{v}(-q_1)\gamma_\alpha v(-q_2) , \tag{2}$$

whereas the matrix element of the fourth order diagram 2a may be written as

$$M_4 = \frac{ie^4}{(2\pi)^4}\int d^4k\; \Phi(p_1, q_1, q, k) \tag{3}$$

$$\bar{u}(p_2)\gamma_\alpha[\hat{k}\gamma_\beta + 2p_{1\beta}]u(p_1)\,\bar{v}(-q_1)[\gamma_\beta\hat{k} - 2q_{1\beta}]\gamma_\alpha v(-q_2) ,$$

where

$$\Phi(p_1, q_1, q, k) = \frac{1}{[(p_1+k)^2+m^2][(q_1-k)^2+m^2][k^2+\lambda^2][(k+q)^2+\lambda^2]} \tag{4}$$

We shall see below that the remaining diagrams 2b, 2c etc. give inessential contribution.

Introduce five invariant amplitudes

$$\begin{aligned}
T_1 &= \bar{u}(p_2)u(p_1)\,\bar{v}(-q_1)v(-q_2) ,\\
T_2 &= \bar{u}(p_2)\, i\hat{Q}\, u(p_1)\, v(-q_2)\, v(-q_2) - \bar{u}(p_2)u(p_1)\,\bar{v}(-q_1)\, i\hat{P}\, v(-q_2) ,\\
T_3 &= \bar{u}(p_2)\, i\hat{Q}\, u(p_1)\, v(-q_1)\, i\hat{P}\, v(-q_2) ,\\
T_4 &= \bar{u}(p_2)\, i\hat{Q}\gamma_5 u(p_1)\,\bar{v}(-q_1)\, i\hat{P}\gamma_5 v(-q_2) ,\\
T_5 &= \bar{u}(p_2)\gamma_5 u(p_1)\,\bar{v}(-q_1)\gamma_5 v(-q_2),
\end{aligned} \tag{5}$$

where

$$P = \frac{p_1 + p_2}{2} \quad , \qquad Q = \frac{q_1 + q_2}{2} \quad , \tag{6}$$

and write the whole matrix element in the form:

$$M = M_2 + M_4 + \ldots = \sum_{i=1}^{5} A_i \; T_i \; . \tag{7}$$

On making elementary calculations we get

$$V = \frac{1}{4\omega} \left[(u-s) m^2 T_1 + (t - 4m^2) m \; T_2 + (s-u) T_3 + t T_4 \right], \tag{8}$$

where

$$\omega = \frac{s(s+t-4m^2)}{4} \; . \tag{9}$$

and

$$\begin{aligned} \frac{16\pi^2}{e^4} M_4 &= 2(2m^2 - s) [J_0 + 2I_1 + 2I_2] V \\ &+ \left[-4 I_1 + \frac{(u-s)(s-2m^2)}{\omega t} J_{aa} + \frac{(u-s)(u-2m^2)}{\omega t} K_0 + 2(K_2 - K_1) \right] m^2 T_1 \\ &+ \left[2I_1 + \frac{(t-4m^2)(s-2m^2)}{\omega t} J_{aa} + \frac{(t-4m^2)(u-2m^2)}{\omega t} K_0 + 2(K_1 + K_2) \right] m \; T_2 \\ &+ \left[8I_2 + \frac{u-s}{2\omega} (J_{aa} + K_0) + \frac{(t-4m^2)^2}{2\omega t} (J_{aa} - K_0) + 2(K_1 - K_2) \right] T_3 \\ &+ \left[\frac{s-2m^2}{\omega} J_{aa} + \frac{u-2m^2}{\omega} K_0 + 2(K_1 - K_2) \right] T_4 \\ &+ 8\left[\frac{K_0 - J_{aa}}{t} - K_2 - 2K_3 - K_4 \right] T_5 \; , \end{aligned} \tag{10}$$

where

$$J_0 = \frac{1}{\pi^2 i} \int d^4k \; \Phi(p_1, q_1, q, k) \, , \tag{11}$$

$$J_a = \frac{1}{\pi^2 i} \int d^4k \; \Phi(p_1, q_1, q, k) \, k_a = I_1 (p_1 - q_1)_a + I_2 q_a \, , \tag{12}$$

$$\begin{aligned} J_{a\beta} = \frac{1}{\pi^2 i} \int d^4k \; \Phi(p_1, q_1, q, k) \, k_a k_\beta &= K_0 \delta_{a\beta} + K_1 (p_1 + q_1)_a (p_1 + q_1)_\beta \\ &+ K_2 (p_1 - q_1)_a (p_1 - q_1)_\beta + K_3 [q_a (p_1 - q_1)_\beta + q_\beta (p_1 - q_1)] + K_4 q_a q_\beta \, . \end{aligned} \tag{13}$$

In the asymptotic region of the variable $t \; (t \to \infty)$ we have

$$J_0 = -2 \frac{\ln t/m^2}{t} F(s) \, , \tag{14}$$

where

$$f(s) = \int_{4m^2}^{\infty} \frac{ds'}{(s'-s) \sqrt{s'(s'-4m^2)}} \; . \tag{15}$$

For $s < 4m^2$, $4m^2 - s \ll m^2$

$$f(s) = \frac{\pi}{\sqrt{s(4m^2 - s)}} \tag{16}$$

Near the threshold $|s-4m^2| << m^2$, $sf(s) >> 1$. In this region among the terms in (10) giving the contributions of the type $\frac{1}{t}\ln\frac{t}{m^2}$ the terms proportional to J_0 make the main contribution. (s is always in the above-mentioned region when the energy levels are calculated.) In this region

$$
\begin{aligned}
I_1 &\to \frac{t}{2u} J_0 ,\\
I_2 &\to \frac{s-4m^2}{2u} J_0 ,\\
K_0 &\to -\frac{(s-4m^2)t}{4u} J_0\\
K_1 &\to -\frac{(s-4m^2)t}{4us} J_0 ,\\
K_2 &\to \frac{(t-s+4m^2)t}{4u^2} J_0,\\
K_3 &\to \frac{(s-4m^2)t}{2u^2} J_0 ,\\
K_4 &\to \frac{(s-4m^2)^2}{2u^2} J_2 .
\end{aligned}
\tag{17}
$$

The matrix elements of the diagrams $2b$, $2c$ etc give the contributions independent of s and, therefore, they are not essential in the above region.

The application of the renormalization group method to the invariant amplitudes Ai yields no information on the bound states of the electron-positron system. Different amplitudes behave differently, for the choice of these amplitudes is quite arbitrary and they are not connected with the physical bound states. It is necessary, therefore, to apply the renormalization group method to the consideration of the behaviour not of the invariant amplitudes, but of the physical ones: the scattering amplitudes in the singlet and triplet states

$$M^s_{mm'} = \sum_{\mu\mu'} C^{sm}_{1/2\mu\, 1/2\, m-\mu}\; C^{sm'}_{1/2\mu'\, 1/2\, m'-\mu'} \;\langle \mu, m-\mu \,|\, M \,|\, \mu', m'-\mu' \rangle . \tag{18}$$

Substituting (7) into (18), we get for the singlet amplitudes

$$M^\circ = \frac{1}{s}\,[\,(t-4m^2)A_1 + 2m(s-u)A_2 + \{(s-m^2)t+(s-2m^2)^2\}A_3 + usA_4 + tA_5\,] \tag{19}$$

Eqs. (2), (8) and (18) yield the singlet amplitude in the second order. In this order each of the terms in (19) is tending to the constant value at $t\to\infty$. However, in the expression for M° they are cancelled, and we have

$$M^\circ_2 = e^2\,\frac{2(s-2m^2)}{s}\,\frac{1}{t} . \tag{20}$$

A similar equation for M° in the fourth order is of the form

$$M^\circ_4 \to -\frac{e^4}{4\pi^2}\,\frac{(s-2m^2)^2}{s}\,J_0 . \tag{21}$$

It should be noted just now that each term in (19) in the fourth order has at $t\to\infty$ an asymptotic behaviour like $\ln t/m^2$. In the expression for M°, however, these terms are mutually calcelled.

Thus, for the singlet amplitude up to the terms of the fourth order we have

$$M^0 = e^2 \frac{2(s-2m^2)}{s} \frac{1}{t} \left[1 + \frac{e^2}{4\pi^2} (s - 2m^2) f(s) \ln t/m^2 \right]. \tag{22}$$

Hence, as is easily seen, the renormalization group method gives for Regge trajectory in the region $0 < 4m^2 - s << m^2$ *)

$$\ell(s) = -1 + a \frac{s - 2m^2}{\sqrt{s(4m^2 - s)}} . \tag{23}$$

The contributions from the diagrams $2b$, $2c$ etc in the region $0 < 4m^2 - s << m^2$ are not essential.

Thus, the summation of a certain class of diagrams, improved by the renormalization group method, gives Regge trajectory which determines the energy levels of positronium. It should be noted that Regge trajectory corresponding to positronium is not the main one for the asymptotic behaviour of the scattering amplitude.

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*) See [4], where a discussion is given how this formula is related to the infrared singularity.

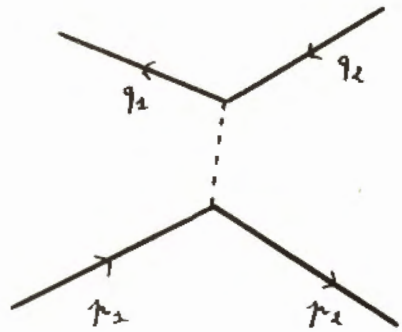

Fig. 1.

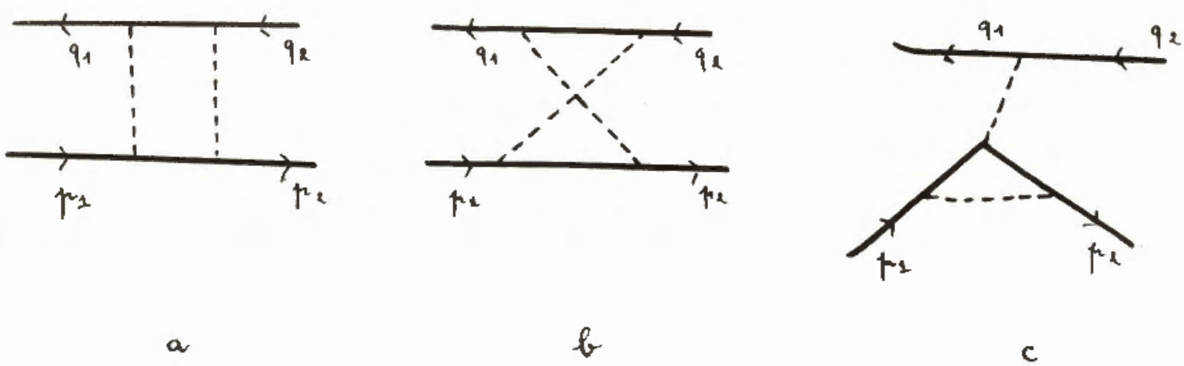

Fig. 2.