ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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INFRARED SINGULARITIES AND REGGE TRAJECTORIES IN ELECTRODYNAMICS

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A b s t r a c t

The form of the Regge trajectory in the lowest order in e^2 for the relativistic charge particle interaction is obtained on the basis of the analysis of the infrared singularities in quantum electrodynamics. In the non-relativistic limit this trajectory corresponds to the bound states of the Coulomb potential.

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"Инфракрасные особенности и траектории Редже в электродинамике"

Из анализа инфракрасиых особенностей в квантовой электродинамике получен в низшем порядке по e^2 вид траектории Редже для взаимодействия релятивистских заряженных частиц. В нерелятивистском пределе эта траектория соответствует связанным состояниям кулоновского потенциала.

In the present note we consider the form of a Regge trajectory for electron-positron interaction which was obtained in [1] from dispersion relation for photon-electron scattering. The generalization of this trajectory to the case of particles with unequal masses is also traeted.

If the matrix element of photon-electron scattering, calculated with introduction of the photon 'mass' $\sqrt{\lambda}$ into the photon Green function, is represented in the form*

$$M_{\lambda} = \exp[F(t)]\, M \tag{1}$$

where

$$F((p'-p)^2) = \frac{i\alpha}{8\pi^3} \int \frac{dk}{k^2-\lambda} \left(\frac{2p'-k}{2p'k-k^2} - \frac{2p-k}{2pk-k^2} \right)^2 \tag{2}$$

then, as shown in [1], for M we can write the following representation (m is the electron mass)

$$M = \sum_{b=s,u} \frac{A_b}{b-m^2} \exp\left[\beta(t) \ln \frac{m^2-b}{m^2} + \gamma(t)\right] + M_a \tag{3}$$

where $A_b/(b-m^2)\ (b=s,u)$ are Born terms corresponding to the two diagrams of the second order in which the additional magnetic moment of electron is taken into account, and β and γ are the series in α. In the lowest order we have

$$\beta(t) = (\alpha/\pi)\, t \int_{4m^2}^{\infty} \frac{t'-2m^2}{\sqrt{t'(t'-4m^2)}} \frac{dt'}{t'(t'-t-i\epsilon)}, \tag{4}$$

the quantity M_a (more exacly the invariant coefficients of its spinor terms) is an analytic function of s, u, t, at least in the lowest (fourth) order perturbation theory, satisfying a Maudelstam representation with the cuts as singularities.

We see that the first term in (3) for large s is a Regge term [2] where the power of s is

$$a(t) = -1 + \beta(t), \tag{5}$$

It is reasonable to assume that the second term in (3) M_a can give only higher order corrections to this expression.

The behaviour of the quantity (5) (the Regge trajectory) is represented in the figure.

The Regge equation

$$a(t) = \ell, \qquad \ell = 0,1,2,\dots \tag{6}$$

determines the bound states in the t -channel, i.e. the bound states of electron-positron system. It has solutions only at $0<t<4m^2$; in this case

$$a(t) = -1 + (\alpha/\pi)\left[1 + \frac{2t-4m^2}{\sqrt{t(4m^2-t)}} \operatorname{arctg} \sqrt{\frac{t}{4m^2-t}}\right]. \tag{7}$$

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* s, u, t denote the Mandelstam variables for the direct, crossed and third channels of the reaction respectively; α is the fine structure constant; the system of units $\hbar = c = 1$, the vector product $ab = a^0 b^0 - \vec{a}\,\vec{b}$.

In the non-relativistic approximation $(m \to \infty)$. this expression turns into

$$\alpha((2m+E)^2) = -1 + \alpha\sqrt{\frac{m}{-4E}} \tag{8}$$

what corresponds to the Coulomb levels of electron-positron system with the radial quantum number equal to 0 .

The obtained formulas can be generalized to the cas e of particles with unequal masses by means of the dimensionality considerations.

We consider a particle with charge ze, mass m and momenta p and p' at the beggining and at the aend of the reaction, colliding with a particle with these parameters to be equal to Ze, M, P and P' respectively. To find, following Regge, the bound states of these particles we have to find the asymptotic behaviour of the matrix element M_λ of the scattering of these particles at $t \to \infty$. From the dimensionality considerations it must contain a term of the form

$$\frac{1}{t}\left(\frac{t}{\lambda}\right)^{\beta(s)}. \tag{9}$$

Therefore to find $\beta(s)$ it is sufficient to consider the infrared singularities of M_λ which are given by the formula (1), where instead of F we need to take the expression[3]

$$-2zZ[F((p+P)^2) - F((p'-P)^2)] + z^2F((p-p')^2) + Z^2F((P-P')^2). \tag{10}$$

The power β is the coefficient of $\ell n\ 1/\lambda$ in this expression at $t \to \infty$:

$$\beta = -zZ(\alpha/\pi)[\psi(s)+1] + (\alpha/2\pi)[(z+Z)^2(\ell n\frac{m^2}{t} + 1) + 2Z(z+Z)\ell n\frac{M}{m}] \tag{11}$$

$$\psi(s) = (s-m^2-M^2)\int_{(M+m)^2}^{\infty}\frac{ds'}{\sqrt{k(s')}\,(s'-s-i\epsilon)} \tag{12}$$

$$k(s) = [s-(m-M)^2][s-(m+M)^2]\ . \tag{13}$$

We see that it gives the asymptotic behaviour of the form (9) provided the particles have opposite charges $z = -Z$.
In this case (for $z = \pm 1$) the Regge power is

$$\alpha(s) = -1 + \beta(s) \tag{14}$$

$$\beta(s) = (\alpha/\pi)[\psi(s)+1]\ , \tag{15}$$

At $(m-M)^2 < s < (m+M)^2$ it is equal to

$$\alpha(s) = -1 + (\alpha/\pi)[1 + 2\frac{s-m^2-M^2}{\sqrt{-k(s)}}\ arctg\ \frac{s-(m-M)^2}{\sqrt{-k(s)}}]\ . \tag{16}$$

If one of the particles is at rest $(M \to \infty)$, the energy of the other particle is E and $p^2 = m^2 - E^2$, then for

$|E| < m$ this formula gives

$$\alpha((M+E)^2) = -1+(\alpha/\pi)[1+\frac{2E}{p}\,\mathrm{arctg}\,\frac{m+E}{p}], \tag{17}$$

For small p this expression turnes into the Sommerfeld formula up to the terms of the order $\left(\frac{p}{m+E}\right)^2$.

The method considered here allows one to obtain the main Regge trajectory up to α what enables one to describe the Coulomb interaction. This conclusion coincides with the conclusion of papers /4/ that Regge trajectories can be obtained in quantum field theory in lowest orders in coupling constant. The description of more fine effects requires more elaborate methods as it has been analysed in /5/.

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References

1. Л.Д.Соловьев. ЖЭТФ (in print), preprint ОИЯИ (in print).
2. T.Regge. Nuovo Cimento 14, 951 (1959); 18, 957 (1960).
3. D.R.Yennie, S.C.Frautschi, H.Suura. Ann. of Phys. 13, 379 (1961).
4. Б.А.Арбузов, А.А.Логунов, А.Н.Тавхелидзе, Р.Н.Фаустов, Phys. Lett. 2, 150 (1962).
   Б.А.Арбузов, А.А.Логунов, А.Н.Тавхелидзе, Р.Н.Фаустов, А.Т.Филиппов, Preprint ОИЯИ E-1061.
5. Б.А.Арбузов, Б.М.Барбашов, А.А.Логунов, Нгуен Ван Хьеу, А.Н.Тавхелидзе, Р.Н.Фаустов, А.Т.Филиппов Preprint ОИЯИ E-1095 Dubna 1962.

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