ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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REGGE POLES AND BETHE-SALPETER EQUATION

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Abstract

An approximate method is suggested to solve the equation of Bethe-Salpeter type and to find with its aid the trajectories of the Regge poles.

Bethe-Salpeter equation for scalar particles is taken as an example for the consideration.

Аннотация

Предлагается приближенный метод решения уравнения типа Бете-Солпитера и нахождения с его помощью траекторий полюсов Редже.

Рассмотрение проведено на примере уравнения Бете-Солпитера для скалярных частиц.

Introduction

In our previous papers[1,2] the properties of the Regge poles were studied on the basis of the perturbation theory. As was shown earlier[3], in order to get the information from the perturbation theory we are in need of an additional analysis which is, generally speaking, not trivial. When the Regge trajectories are sought for by using the perturbation theory a certain difficulty arises. There appear the contributions from the cuts which are mixed up with those from the poles. In this connection it seems interesting to investigate the structure of the Regge properties with the help of equations of Bethe-Salpeter type.

A similar approach was discussed in papers[4]. Before we start the analysis of Beth-Salpeter equation, let us be concerned with the Schrödinger equation with the potential of the Yukawa type.

1. Schrödinger Equation for Partial Amplitudes

As is well known, the Schrödinger equation has the form

$$(k^2+E)\psi(\vec{k})+\int d\vec{k}'\, V(\vec{k}-\vec{k}')\,\psi(\vec{k}')=0\,. \tag{1.1}$$

In the case under consideration $V(\vec{k}-\vec{k}')$ is as follows

$$V(\vec{k}-\vec{k}') = \int_{\mu^2}^{\infty} \frac{\tilde{V}(\nu)}{(\vec{k}-\vec{k}')^2+\nu}\, d\nu\,. \tag{1.2}$$

We write Eq. (1.1) for the partial amplitudes. With this aim we represent $\psi(k)$ as

$$\psi(\vec{k}) = \Sigma\,(-i)^{\ell}\ \frac{1}{k}\,\psi_\ell(k)\,P_\ell\,(\cos\theta_{\vec{k}})\,. \tag{1.3}$$

By using the representation

$$\frac{1}{(\vec{k}-\vec{k}')^2+\mu^2} = \sum_{\ell=0}^{\infty}\ (2\ell+1)\ Q_\ell\Big(\frac{k^2+k'^2+\mu^2}{2kk'}\Big)\,P_\ell\,(\cos\theta_{\vec{k}\vec{k}'}) \tag{1.4}$$

and substituting (1.3) and (1.4) into (1.1) we obtain

$$(k^2+E)\psi_\ell(k)+2\pi\int_{\mu^2}^{\infty} d\nu\int_0^{\infty} dk'\ \tilde{V}(\nu)\ Q_\ell\Big(\frac{k^2+k'^2+\nu}{2kk'}\Big)\,\psi_\ell(k')=0\,. \tag{1.5}$$

The partial amplitudes in the r - space

$$\tilde{\psi}(\vec{r}) = \sum_{\ell=0}^{\infty}\ \frac{1}{r}\,\tilde{\psi}_\ell(r)\,P_\ell\,(\cos\theta_{\vec{r}}) \tag{1.6}$$

are related to the partial amplitudes $\psi_\ell(k)$ in the following manner

$$\psi_\ell(k) = \int_0^{\infty} dr\sqrt{kr}\ J_{\ell+\frac{1}{2}}\,(kr)\,\tilde{\psi}_\ell(r) \tag{1.7}$$

$$\tilde{\psi}_\ell(r) = \int_0^\infty dk \sqrt{kr}\; J_{\ell+1/2}(kr)\, \psi_\ell(k). \tag{1.7}$$

In calculating the Regge trajectory one should have in mind that the Regge exponent is a function of the parameter $e^2/\sqrt{E}$. When investigating the bound states this parameter is not small and its magnitude is of the order of unity. Therefore, in order to find $a(E)$ we ought to be able to sum all the terms of the type $(e^2/\sqrt{E})^n$. Let us take an example, when the potential in Eq. (1.5) is equal to the sum of the Coulomb and Yukawa ones.

$$\tilde{V}(\nu) = -\frac{1}{2\pi^2}[\, e^2\,\delta(\nu) + g^2\delta(\nu-\mu^2)\,]. \tag{1.8}$$

Eq. (1.5) becomes

$$(k^2+E)\,\psi_\ell(k) - \frac{e^2}{\pi}\int_0^\infty dk'\, Q_\ell\!\left(\frac{k^2+k'^2}{2kk'}\right)\psi_\ell(k') =$$

$$= \frac{g^2}{\pi}\int_0^\infty dk'\, Q_\ell\!\left(\frac{k^2+k'^2+\mu^2}{2kk'}\right)\psi_\ell(k'). \tag{1.9}$$

To sum up all the terms of the order $(e^2/\sqrt{E})^n$ it is necessary to take in (1.9) the Coulomb partial amplitudes as a zero approximation. In this approximation the main Regge trajectory is

$$a_0 = -1 + \frac{e^2}{2\sqrt{E}}. \tag{1.10}$$

If we treat the right-hand side as a perturbation, for the Regge trajectory of Eq. (1.9) it is possible in this approximation to get

$$a = -1 + \frac{e^2}{2\sqrt{E}} + \frac{g^2}{2\sqrt{E}}\left(1+\frac{\mu}{2\sqrt{E}}\right)^{-\frac{e^2}{\sqrt{E}}}. \tag{1.11}$$

The choice of the zero approximation made above allowed to obtain the correct expansion in a small parameter g^2.

2. Bethe-Salpeter Equation for Partial Amplitudes

As is well-known, the Bethe-Salpeter equation for the scalar two-particle wave function is

$$\psi(\epsilon,\vec{k}) = -\lambda F(\epsilon,\vec{k}) \int \frac{\psi(\epsilon',\vec{k}')\, d\epsilon'\, d^3k'}{(\vec{k}-\vec{k}')^2-(\epsilon-\epsilon')^2+\mu^2} \tag{2.1}$$

where

$$F(\epsilon,\vec{k}) = G(\epsilon+E,\vec{k})\,G(\epsilon-E,\vec{k})$$

$$G^{-1}(\epsilon+E,\vec{k}) = (\epsilon+E)^2 - k^2 - \mu^2 + i\delta$$

$$\lambda = \frac{4i}{(2\pi)^4}\, g^2. \tag{2.2}$$

As a zero approximation it is reasonable in Eq.(2.1) to single-out explicitly the potential-like part. For this purpose we rewrite the equation in the form

$$\psi(\epsilon,\vec{k}) = -\lambda F(\epsilon,\vec{k}) \int Q(\vec{k},\vec{k}')\,\psi(\epsilon',\vec{k}')\,d\epsilon'\,d^3k' -$$

$$-\lambda F(\epsilon,\vec{k}) \int \left[\frac{1}{(\vec{k}-\vec{k}')^2-(\epsilon-\epsilon')^2+\mu^2} - Q(\vec{k},\vec{k}') \right] \psi(\epsilon',\vec{k}')\,d\epsilon'\,d^3k'. \tag{2.3}$$

Integrating Eq.(2.3) over ϵ, we have

$$\psi(\vec{k}) = -\lambda F(\vec{k}) \int Q(\vec{k},\vec{k}')\,\psi(\vec{k}')\,d^3k' -$$

$$-\lambda \int d\epsilon\,d\epsilon'\,d^3k'\,F(\epsilon,\vec{k}) \left[\frac{1}{(\vec{k}-\vec{k}')^2-(\epsilon-\epsilon')^2+\mu^2} - Q(\vec{k},\vec{k}') \right] \psi(\epsilon',\vec{k}') \tag{2.4}$$

where

$$\psi(\vec{k}) = \int \psi(\epsilon,\vec{k})\,d\epsilon$$

$$F(\vec{k}) = \int F(\epsilon,\vec{k})\,d\epsilon = \frac{i\pi}{2\sqrt{k^2+m^2}\,(k^2+m^2-E^2)}. \tag{2.5}$$

The kernel $Q(\vec{k},\vec{k}')$ is determined from the condition that the second term in the right-hand side of Eq.(2.4) vanishes. In the lowest approximation this condition yields for the kernel $Q(\vec{k},\vec{k}')$ the following expression

$$Q(\vec{k},\vec{k}') = \frac{1}{F(\vec{k})\,F(\vec{k}')} \int \int d\epsilon\,d\epsilon' \frac{F(\epsilon,\vec{k})\,F(\epsilon',\vec{k}')}{(\vec{k}-\vec{k}') - (\epsilon-\epsilon')^2+\mu^2} \tag{2.6}$$

In this approximation Eq. (2,4) is written down as

$$(\vec{k}^2+m^2-E^2)\,\psi(\vec{k}) = -i\pi\lambda \frac{1}{2\sqrt{\vec{k}^2+m^2}} \int Q(\vec{k},\vec{k}')\,\psi(\vec{k}')\,d^3\vec{k}'. \tag{2.7}$$

Passing in Eq. (2,7) to the partial amplitudes, we get for α the following expansion

$$\alpha = \alpha_0 + \left(\frac{g^2}{4\pi m^2}\right) \chi \left(\frac{g^2}{\sqrt{m^2-E^2}}\right) + \ldots \text{ where}$$

$$\alpha_0 = -1 + \frac{g^2}{4\pi m^2} \cdot \frac{m}{2\sqrt{m^2-E^2}}.$$

A detailed investigation of Eq. (2,7) is given in /5/.

It is seen from here, that in the field theory as well, the Regge factor is a function both of the coupling constant g^2 and the parameter $g^2/\sqrt{m^2-E^2}$ which is not small, indeed. The method we have developed above permits to sum up the terms of the type $(g^2/\sqrt{m^2-E^2})^n$ effectively, allowing thereby to construct the correct expansion of the Regge factor in the coupling constant g^2.

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