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E - 1100

ON THE RADIATIVE CORRECTIONS IN THE NEUTRON DECAY

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Abstract

It is shown that if the terms of the order $\alpha\, m/M$ are neglected then the radiative corrections to the neutron decay change only the values of the decay constants, but do not change the correlation or polarization decay properties. The cause lies in the γ_5 –invariance of the theory in the considered approximation which forbids the variants S, T and P.

Аннотация

Показано, что если пренебречь членами порядка $\alpha \frac{m}{M}$, то радиационные поправки к β –распаду нейтрона изменяют лишь значения постоянных распада, но не меняют корреляционных или поляризационных свойств распада. Причина этого лежит в γ_5 –инвариантности теории в рассматриваемом приближении, которая запрещает варианты S, T и P.

Препринт издается только на английском языке.

I. Introduction

The neutron decay represents the simplest process which, in principle, allows one to check the correctness of the universal weak interaction theory and the Gell–Mann and Feynman hypothesis on the conservation of the vector current. In the paper [1] the influence of the weak magnetism on the neutron decay has been investigated. It has been noted there that the weak magnetism (together with the kinematical corrections for the recoil) is the only source of corrections, if we restrict ourselves to the effects whose contribution does not exceed the fraction m/M of the main effect (m and M are the electron and nucleon masses). In this connection it is important to check whether radiative corrections can contribute essentially in the order under consideration

The raditive corrections to the neutron decay have been considered in a number of papers [2–4]. However, in all these calculations the averaging over the particle spins was performed and only the formulas for the radiative corrections to the decay probability were given. For our purpose it would be necessary to have formulas in which the spin would be written explicitly. With the aid of such formulas we could obtain corrections directly to the effective interaction Hamiltonian.

From the former papers [2–3] (see also [5]) it is known that first of all the radiative corrections to the decay constants are divergent. Therefore, we can speak about the magnitude of the constants with the accuracy up to α. However, it is not obligatory that all the observable effects depend on the divergent integrals. Let us consider, for example, the electron polarization. If the electron mass is neglected then the electron longitudinal polarization both in vector and pseudovector variant would exactly be equal to -1. The change in the polarization can occur in this case only due to the appearance of the admixture of other three variants S, T, P for which the polarization sign would be opposite. The electron non-zero mass leads to the polarization value being equal to $-v/c$ (velocity) instead of 1 but it remains equal in both variants V and A. The electron polarization will therefore change only owing to the admixture of the variants S, T and P. If the quantities g_V and g_A are believed to be experimental constants, then, obviously, the question we are interested in is the following: whether the interactions of the type S, T and P appear in the terms of the order α or they will be of the order $\alpha\, m/M$. The results described in the following show that other types of interactions do not appear in the terms of the order α. The renormalization of the decay constants remains the only effect of the radiative corrections with the accuracy considered. Owing to this fact the check of the vector current conservation can be performed, in principle, with the error of the order $\alpha \; m/M \approx 5.10^{-6}$.

2. The effective Hamiltonian with the radiative corrections

As it has been noted we consider only the radiative correction effects of the order α and neglect the effects of the order $\alpha\, m/M$ Therefore in calculating the radiative corrections use can be made of the following beta-decay Hamiltonian without the radiative corrections [1)].

$$H_0 = \frac{G}{\sqrt{2}} \langle e | \gamma_\mu (1+\gamma_5) | \nu \rangle \langle p \mid \gamma_\mu (1 + a\gamma_5) \mid n \rangle . \qquad (1)$$

The diagrams corresponding to the lowest-order radiation corrections are represented in fig. I. The matrix elements of the diagrams a and b corresponding to the renormalization of the external lines are proportional to Hamiltonian (1). We have therefore to consider only the matrix element of the diagram c [2)]

1) The effect of the weak magnetism gives a contribution to radiative terms of the order $\alpha\; m/M$.

2) We notice that all the terms in (2) are invariant with respect to the transformation $(u_e,\, u_\nu) \to \gamma_5 (u_e,\, u_\nu)$.

$$M_s = \frac{G}{\sqrt{2}} \frac{e^2}{16\pi^2} \{ 4pP\, I^0\, \bar{u}_e \gamma_\mu (1+\gamma_5)\, v_\nu\, \bar{u}_p \gamma_\mu (1 + a\gamma_5)\, u_n$$

$$+ 2 I_\alpha [\, \bar{u}_e \hat{P}\, \gamma_\alpha \gamma_\mu (1+\gamma_5)\, v_\nu \bar{u}_p \gamma_\mu (1 + a\gamma_5)\, u_n - \bar{u}_e \gamma_\mu (1+\gamma_5)\, v_\nu\, \bar{u}_p\, \hat{p}\, \gamma_\alpha \gamma_\mu (1 + a\gamma_5)\, u_n] \tag{2}$$

$$- I_{\alpha\beta}\, \bar{u}_e \gamma_\sigma \gamma_\alpha \gamma_\mu (1+\gamma_5)\, v_\nu\, \bar{u}_p \gamma_\sigma\, \gamma_\beta \gamma_\mu (1 + a\gamma_5)\, u_n \} ,$$

where p , P are the four-momenta of electron and proton,

$$I^0 = \frac{1}{\pi^2 i} \int d^4 k \cdot \frac{1}{[k^2 + 2kp][k^2 - 2kP][k^2 + \lambda^2]} ,$$

$$I_\alpha = \frac{1}{\pi^2 i} \int d^4 k \cdot \frac{k_\alpha}{[k^2 + 2kp][k^2 - 2kP][k^2 + \lambda^2]} = I^{(1)} P_\alpha + I^{(2)} p_\alpha , \tag{3}$$

$$I_{\alpha\beta} = \frac{1}{\pi^2 i} \int d^4 k \cdot \frac{k_\alpha k_\beta}{[k^2 + 2kp][k^2 - 2kP][k^2 + \lambda^2]} =$$

$$= I^{(3)} \delta_{\alpha\beta} + I^{(4)} P_\alpha P_\beta + I^{(5)} p_\alpha p_\beta + I^{(6)} (p_\alpha P_\beta + p_\beta P_\alpha) .$$

The first term in (2) is proportional to the Hamiltonian (1) and the second and the third ones are equal

$$- 2 I^{(1)} M^2\, \bar{u}_e \gamma_\mu (1 + \gamma_5)\, v_\nu \cdot \bar{u}_p \gamma_\mu (1 + a\gamma_5) u_n$$

$$- 2 I^{(1)} \bar{u}_e \gamma_\mu (1 + \gamma_5)\, v_\nu \cdot \bar{u}_p \cdot \hat{p} \hat{P} \gamma_\mu (1 + a\gamma_5) u_n \tag{4}$$

$$+ 2 I^{(2)} \bar{u}_e \hat{P} \hat{p}\, \gamma_\mu (1 + \gamma_5)\, v_\nu \cdot \bar{u}_p \gamma_\mu (1 + a\gamma_5) u_n$$

$$+ 2 I^{(2)} m^2\, \bar{u}_e \gamma_\mu (1 + \gamma_5)\, v_\nu \cdot \bar{u}_p \gamma_\mu (1 + a\gamma_5) u_n$$

and

$$-2(1 + a)\, I_{\alpha\alpha}\, \bar{u}_e \gamma_\mu (1 + \gamma_5)\, v_\nu \cdot \bar{u}_p \gamma_\mu (1 + \gamma_5)\, u_n$$

$$-2(1 - a)\, I^{(3)} \bar{u}_e \gamma_\mu (1 + \gamma_5)\, v_\nu \cdot \bar{u}_p \gamma_\mu (1 - \gamma_5)\, u_n \tag{5}$$

$$-2(1 - a)\, I^{(4)} \bar{u}_e \hat{P} (1 + \gamma_5)\, v_\nu\, \bar{u}_p \hat{P} (1 - \gamma_5)\, u_n$$

$$-2(1 - a)\, I^{(5)} \bar{u}_e \hat{p} (1 + \gamma_5)\, v_\nu \cdot \bar{u}_p \hat{p} (1 - \gamma_5)\, u_n$$

$$- 2(1-a) I^{(6)} u_e \hat{p} (1+\gamma_5) v_\nu \; u_p \hat{P} (1-\gamma_5) u_n$$

$$- 2(1-a) I^{(6)} u_e \hat{P} (1+\gamma_5) v_\nu u_p \hat{p} (1-\gamma_5) u_n \tag{5}$$

respectively. The first term in (4) is proportional to Hamiltonian (1), the three first terms in (5) give a contribution of the order α only to the variants V and A in the non-relativistic approximation. The other terms are of the order $\alpha m/M$. Thus, in the order α radiative corrections lead only to the renormalization of the constants g_A, g_V but not to the appearance of the other variants, i.e. in the considered approximation the effective Hamiltonian is also of the form (1) taking into account the radiative corrections. Therefore in the order α the radiative corrections do not change the correlations or polarization decay properties and the latter are completely defined by the weak magnetism.

3. The connection with the γ_5 invariance

The physical meaning of the obtained results becomes clear if we consider what occurs with the effective Hamiltonian when we multiply the lepton spinors u_e and u_ν by γ_5 i.e. make the following substitution

$$u_e \to \gamma_5 u_e, \qquad u_\nu \to \gamma_5 u \tag{6}$$

$$\bar{u}_e \to - \bar{u}_e \gamma_5, \qquad \bar{u}_\nu \to - \bar{u}_\nu \gamma_5 .$$

It is known that if the electron mass is assumed to be equal to zero then the weak interaction Lagrangian is invariant under the transformation (1) (in the general case we have to change the sign of the mass $m \to -m$). The electromagnetic interaction is invariant under the transformation (1) as well. On the other hand in such a substitution the variants S , T and P change their signs. Hence, it follows that for $m \to o$ only the terms corresponding to the vector and pseudoscalar variants can remain in the effective Hamiltonian. The other terms must be proportional to the electron mass, i.e. they occur only in the order $\alpha \; m/M$.

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