ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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E - 1095

REGGE-POLES AND PERTURBATION THEORY II

Phys. Lett, 1963, v 4, N 5, p 308-310.

Дубна 1962 год

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In recent years the problem as to how to find the singularities of partial wave amplitudes in the l-plane has been widely discussed. In the papers |1,2| a method is suggested to calculate the Regge trajectories by means of perturbation theory. As is shown in |1,2| perturbation theory contains information about the singularities in the l-plane. However, the extraction of this information is far from being trivial.

In the present note the structure of the perturbation series terms in the presence of branch cuts in the l-plane is analysed by the example of the electron scattering in the Coulomb field. It is shown how the expansions in the charge for the Regge trajectories can be obtained in the case under consideration.

The amplitude of the electron scattering can be represented in the form:

$$T(z,p^2) = f_1(z,p^2) + \frac{i\vec{\sigma}[\vec{p}_2\vec{p}_1]}{p^2} f_2(z,p^2) \tag{1}$$

where z is the cosine of the scattering angle, $\vec{p}_1$ and $\vec{p}_2$ are the initial and final electron momenta, and $p^2 = \vec{p}_1^{\,2} = \vec{p}_2^{\,2}$.

We expand in a usual manner the amplitudes f_1 and f_2 in the partial waves:

$$\begin{aligned} f_1(z,p^2) &= \sum_{n=0}^{\infty} [(n+1) f_n^+(p^2) + n f_n^-(p^2)] P_n(z) \\ f_2(z,p^2) &= \sum_{n=0}^{\infty} [f_n^-(p^2) - f_n^+(p^2)] P_n'(z). \end{aligned} \tag{2}$$

In the given case the analytical continuations of the partial waves throughout the l-plane together with the poles have the branch points. The amplitudes f_ℓ^- and f_ℓ^+ have the cuts from $-e^2$ to e^2 and from $-1-e^2$ to $-1+e^2$ respectively (e^2 is the fine structure constant)[4].

We make calculations starting from perturbation theory up to the terms of the second order in e^2 for large s and obtain $(s = -2p^2(1-z))$:

$$f_1 = -e^2\frac{E+m}{2E}\left[\frac{1}{2(E+m)^2} + \frac{2E}{s(E+m)} + e^2\frac{E-m}{4\sqrt{-p^2}(E+m)^2}\ln\left(-\frac{s}{\lambda^2}\right) + \frac{e^2}{s}\frac{2E^2}{\sqrt{-p^2}(E+m)}\ln\left(-\frac{s}{\lambda^2}\right)\right] + 0\left(\frac{1}{s^2}\right) \tag{3}$$

$$f_1 - zf_2 = -e^2\frac{E+m}{2E}\left[\frac{1}{s} + \frac{e^2}{s}\cdot\frac{E+m}{2\sqrt{-p^2}}\ln\left(-\frac{s}{\lambda^2}\right)\right] + 0\left(\frac{1}{s^2}\right) \tag{4}$$

where $E = \sqrt{p^2 + m^2}$, λ is the photon "mass" introduced to exclude the infrared divergence. On the other hand, by applying to the formulae (2) the Sommerfeld-Watson transformation modified following Mandelstam [5] we get for large z :

$$f_1 = \frac{a_+ + 1}{2a_+ + 1}\beta_+(p^2)(-z)^{a_+(p^2)} + \beta_-(p^2)(-z)^{a_-(p^2)} +$$

$$+ \frac{1}{2i\sqrt{\pi}}\int_{c^+}\frac{(\ell+1)\Gamma(\ell+\frac{1}{2})}{\Gamma(\ell+1)\sin\pi\ell} f_\ell^+(p^2)(-2z)^\ell d\ell \tag{5}$$

$$+ \frac{1}{2i\sqrt{\pi}}\int_{c^-}\frac{\ell\Gamma(\ell+\frac{1}{2})}{\Gamma(\ell+1)\sin\pi\ell} f_\ell^-(p^2)(-2z)^\ell d\ell + \ldots$$

$$f_1 - zf_2 = \beta_+(p^2)(-z)^{a_+(p^2)} + \frac{1}{2i\sqrt{\pi}}\int_{c^+}\frac{(2\ell+1)\Gamma(\ell+\frac{1}{2})}{\Gamma(\ell+1)\sin\pi\ell} f_\ell^+(p^2)(-2z)^\ell d\ell + \ldots \tag{6}$$

where $a_\pm$ are the main poles of the amplitudes $f_\bullet^\pm$ in the l-plane, $C^\pm$ are the contours round the cuts $L^\pm$. Comparing the formulae (4) and (6) we note that the expansion in e^2 of the trajectory $a_+(p^2)$ may be of the form: $-1 + O(e^2)$. However, to obtain this expansion from the formula (4) by the method used in paper [2] it is necessary to subtract preliminarily the contribution from the cut. To find this contribution we turn to formulae (3) and (5).

Since the expansion of $a_-(p^2)$ should start with -1 but not with zero*, then comparing (3) and (5) we find up to the terms of e^4 order

$$\frac{1}{2i\sqrt{\pi}}\int_c\frac{\ell\Gamma(\ell+\frac{1}{2})}{\Gamma(\ell+1)\sin\pi\ell} f_\ell^-(p^2)(-2z)^\ell d\ell = \tag{7}$$

$$= -e^2\frac{E+m}{2E}\left[\frac{1}{2(E+m)^2} + e^2\frac{E-m}{4\sqrt{-p^2}(E+m)^2}\ln\left(-\frac{s}{\lambda^2}\right)\right] + \ldots$$

Using the fact that the amplitudes f_ℓ^+ and f_ℓ^- can be represented on the corresponding cuts in the form (see for example [4])

$$f_\ell^+ = [\phi_1(\ell+1) + \phi_2(\ell+1)]\exp\{i\pi(\ell+1)\} \tag{8}$$

$$f_\ell^- = [\phi_1(\ell) - \phi_2(\ell)]\exp\{i\pi\ell\}$$

where $\phi_1(\ell)$ is an even function and $\phi_2(\ell)$ is an odd one of ℓ, we find the connection between the integrals along contours C^+ and C^-. After simple transformations have been made we get

$$\frac{1}{2i\sqrt{\pi}}\int_{c^+\gamma}\frac{(2\ell+1)\Gamma(\ell+\frac{1}{2})}{\Gamma(\ell+1)\sin\pi\ell} f_\bullet^+(p^2)(-2z)^\ell d\ell =$$

* This fact is connected with the character of the level structure in the central field and, by the example of the Coulomb field, is easily seen from the Sommerfeld fine structure formula (see e.g. [6]).

$$= - e^2 \frac{2p^2(E+m)}{2Es} \left[- \frac{1}{2(E+m)^2} + e^2 \frac{E-m}{4\sqrt{-p^2}(E+m)^2} \ln\left(-\frac{s}{\lambda^2}\right)\right] + \dots \qquad (9)$$

Subtracting (9) from (4) we single out the contribution from the pole term

$$(f_1 - zf_2)_R = - \frac{e^2}{s}\left(1 + e^2 \frac{m}{\sqrt{-p^2}} \ln\left[-\frac{s}{\lambda^2}\right]\right) + 0(e^4) \; . \qquad (10)$$

Now following[2], we get:

$$a_+(p^2) = -1 + \frac{e^2 m}{\sqrt{-p^2}} \; . \qquad (11)$$

The other trajectories can be calculated in a similar way. For example, from (3) and (5) we have:

$$a_-(p^2) = -1 + e^2 \frac{E}{\sqrt{-p^2}} + 0(e^4) \, . \qquad (12)$$

These results can be compared with the conclusions obtained from the Dirac equation in the central field:

$$\begin{aligned} &\frac{dF}{dr} + (1-\kappa)\frac{F}{r} + (E - m - V)G = 0 \\ &\frac{dG}{dr} + (1+\kappa)\frac{G}{r} - (E + m - V)F = 0 \end{aligned} \qquad (13)$$

where

$$\kappa = \begin{cases} -(\ell+1) & j = \ell + 1/2 \\ \ell & j = \ell - 1/2 \end{cases} \qquad (14)$$

and

$$V(r) = - \frac{e^2}{r} + \sum_{n=0}^{\infty} V_n r^n$$

By inserting F and G in the form

$$F = r^{\gamma} f \, , \quad G = r^{\gamma} g \, , \quad \gamma = -1 + \sqrt{\kappa^2 - e^4}$$

where f and g have no longer a cut in the l-plane and expanding f, g and $a_{\pm}$ in powers of e^2, with the aid of the successive solution of the equations in each order of expansion we obtain, e.g., for $V(r) = -\frac{e^2}{r}$ expressions (11) and (12). We emphasize that in the case of the potentials of the form (14) the cuts coincide with L^+ and L^- as before. The formula (8) remains valid as well. Thus, the developed method of calculating the cut in perturbation theory can be applied to a more general case too.

It is seen from the aforesaid that the contribution from the cut to the terms of the perturbation series is of the same type as the contributions from the poles.

However this is not always so, e.g. in the case of the relativistic scalar particle in the external field of the type (14) the contribution from the cut in perturbation theory differs from the pole one by the fact that the first one contains the non-integer powers of and is therefore easily singled out. In general, from the Klein Gordon equation it is easily seen that the term quadratic in the potential is responsible for the cut. In the light of the aforesaid it is very interesting to consider similar questions in the quantum field theory.

In conclusion the authors would express their gratitude to N.N. Bogolubov, D.I.Blokhintzev, S.S. Gershtein, Ya.A. Smorodinsky, O.A. Khrustalev for the interesting discussions and remarks.

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Received by Publishing Department
on October 3, 1962.