ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ
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ON THE PRODUCTION OF "SOFT" π MESONS IN PION-NUCLEON AND NUCLEON-NUCLEON COLLISIONS

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Abstract

It is shown that in the limiting case of production of "soft" π mesons ($k \to 0$, k is the four-momenta of the π-meson being produced) in $\pi - N$ and $N - N$ collisions the invariance under the time reversal leads to some consequences for observables which are independent of the dynamics. So, for example, the recoil nucleon polarization in the production of "soft" π^0 mesons in $\pi - N$ collisions vanishes.

Аннотация

Показано, что в предельном случае рождения "медленных" π -мезонов ($k \to 0$, k - 4 - импульс рождающегося π -мезона) в $\pi - N$ и $N - N$ столкновениях инвариантность относительно обращения времени приводит к ряду не зависящих от динамики следствий для наблюдаемых величин. Так, например, поляризация нуклонов отдачи в процессе рождения "медленных" π^0 -мезонов в $\pi - N$ столкновении оказывается равной нулю.

Препринт издается только на английском языке.

1. The time reversal invariance imposes rather severe restrictions on the elastic scattering amplitude (see, e.g. [1]). In the general case of inelastic scattering this invariance leads only to the relation between the amplitudes of the direct and the inverse processes. We shall consider the meson production in $\pi - N$ and $N - N$ – collisions and show that in those cases when the produced π – meson energy may be neglected the T – invariance enables us to obtain some useful consequences for observables. The "soft" π – meson production processes have been considered by Nambu and Lurié [2] who have obtained, in the model with conserved axial current, " low-energy theorems" connecting the elastic scattering amplitude with that of production of a zero-energy massless π – meson. Our consideration is based on the invariance requirements only; however, the results obtained are also valid in the limiting case $k \to 0$ (k is the four-momentum of a produced meson). Since real π–mesons have a finite mass the relations considered below can not, strictly speaking, be satisfied in fact. Nevertheless, it should be expected that they are approximately true at rather high energies, when the energy of an additional meson may be neglected compared to the energies of other particles.

2. We first consider the π^0 meson production in the reaction

$$\pi + p \to \pi + p + \pi^0 . \tag{1}$$

The matrix element of this process can be written in the following form

$$\langle p'q'k | S | pq \rangle = -(2\pi)^4 i \left(\frac{m^2}{p_0' p_0 2q_0' 2q_0 2k_0} \right)^{1/2} \tag{2}$$

$$\bar{u}(p') M(p'q'k; pq) u(p) \delta(p' + q' + k - p - q).$$

Here p, p' and q, q' are the nucleon and meson momenta, k is the π^0 – meson momentum, m is the nucleon mass and $p_0 = -ip_4$ etc.

As is known (see, e.g. [3]) the emission of a particle with momentum k is equivalent to the absorption of an antiparticle with momentum k, i. e.

$$M(p'q' - k; pq) = M(p'q'; pqk). \tag{3}$$

This relation connects the amplitude of process (1) in the unphysical region with that describing the absorption of π^0 (π^0 is the purely neutral particle) in the reaction

$$\pi^0 + \pi + p \to \pi + p . \tag{4}$$

Now we may use the invariance of the S – matrix under the time reversal and connect the matrix element of process (4) with that of the inverse process (1). We get

$$\langle p'q' | S | pqk \rangle = - {}_t\langle pqk | S | p'q' \rangle_t \tag{5}$$

where e.g., the state $|p'q'\rangle_t$ describes nucleon and meson with momenta $-p'^*$ and $-q'^*$ and reversed spins. The minus sign in (5) is due to π^0 transformation properties under the time reversal. From (2), (3), (5) and the invariance under space inversion it follows that

$$M(p'q' - k; pq) = (U^+ M(pqk; p'q') U)^T . \tag{6}$$

Here U is the unitarity matrix satisfying the relations

$$U^{+} \gamma_{\mu} U = \gamma_{\mu}^{T}$$

and T indicates transposition. Eq. (6) is similar to the well – known crossing symmetry and connects the amplitude of inelastic process (1) with that of the same process in the unphysical region. In the limiting case of production of a zero energy massless π –meson we obtain from (6) the following restriction on the process amplitude in the physical region

$$M(p'q'o;\, pq) = (U^{+} M(pqo;\, p'q')\, U)^{T} . \tag{7}$$

In the real case of the finite rest mass of π –meson it may be expected that this relation will be satisfied only in those cases when the π^{0}–meson energy in c.m.s. is close to the rest mass μ and the total energy of the system and the momentum transfer are much larger than μ. In the invariant form these conditions can be written in the form

$$\frac{P \cdot k}{\sqrt{s}} \approx \mu , \quad s >> \mu , \quad |t| >> \mu \tag{8}$$

where $P = p + q$ is the total four–momentum of the system, $s = - P^{2}$ and $t = -(p' - p)^{2}$.

Now we consider what consequences for the observables * Eq. (7) leads to. From the usual requirements of invariance we get the following expression for the "soft" π^{0} – meson production amplitude

$$M(p'q'o;\, pq) = a\gamma_{5} + b\gamma_{5}\gamma\cdot(q + q') \tag{9}$$

where a and b are functions of the invariants s and t. By using (7) we have

$$b = 0$$

and the amplitude reduces to one term

$$M(p'q'o;\, pq) = a\gamma_{5} . \tag{10}$$

Such a simple form of the amplitude enables us to draw conclusions independent of dynamics about the polarization phenomena in the "soft" π^{0}–meson production. The recoil proton polarization is found to be

$$\xi_{\mu} = 2\kappa \cdot \xi^{0} \kappa_{\mu} - \xi^{0}_{\mu} \tag{11}$$

where ξ^{0}_{μ} is the proton polarization in the initial state and κ_{μ} is the unit vector in the direction of the momentum transfer $p' - p$ (we use the covariant description of the polarization [4]). The polarization vanishes at all angles if initial protons are unpolarized. It is interesting to note that for scalar π^{0}– meson the condition (7) will be satisfied automatically and the polarization will not vanish identically. Therefore the experimental confirmation of the vanishing of the polarization would be an additional argument in favour of the π^{0} – meson pseudoscalarity [5].

It is obvious that the abovementioned consideration can be directly applied to the case of the π^{0}– meson production in the reaction of the type: $a + b \to a + b + \pi^{0}$ where a and b are any particles.

So, for example, for the π^{0} – meson production in $p - p$ – collisions (7) should be replaced by the following relation

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* Neglecting the dependence on the momentum k in the amplitude we keep, of course, k in the phase – space volume.

$$M(p_1'p_2'o;p_1p_2) = (U_1^+U_2^+M(p_1p_2o;p_1'p_2')U_2U_1)^T \tag{12}$$

where p_1, p_2 and p_1', p_2' are the nucleon momenta before and after collision and U_1 and U_2 are matrices affecting the spin variables of the first and the second nucleons $/U_1^+\gamma_\mu^{(1)}U_1 = \gamma_\mu^{(1)T}/$. From the invariance requirements together with (12) and Pauli principle we get

$$M(p_1'p_2'o;p_1p_2) = A[\gamma_5^{(1)}+\gamma_5^{(2)}] + B[\gamma_5^{(1)}\gamma^{(2)}\cdot(p_1+p_1')+\gamma_5^{(2)}\gamma^{(1)}(p_2+p_2')]. \tag{13}$$

The polarization arising in collisions of un-polarized particles is non-vanishing in this case. However, the polarization correlation C_{nn} in the direction of the normal to the scattering plane vanishes. For polarized incident protons Eq. (12) leads, as in the case of the elastic scattering [1,6], to the following expression for the reaction cross section

$$\sigma = \sigma_0(1+\xi_0\cdot\xi). \tag{14}$$

Here ξ_0 is the incident proton polarization, ξ is the polarization arising in unpolarized particle collisions. We notice that (12) is competely analogous to the relation for the scattering amplitude, following from the T-invariance in the case of elastic collisions. Due to this fact we obtain for the cross section of "soft" π-meson production by polarized protons the expression (14). Respectively, (14) holds in more general case of the "soft" π^0-meson production in collisions of polarized protons with unpolarized particles with an arbitrary spin (cf. (1)).

3. We go over to the consideration of the charged meson production. In this case by transferring the π-meson from the initial state to the final one, we pass to the oppositely charged meson. Therefore under the time reversal we pass in fact to another process and obtain in this way relations between amplitudes of different processes. We consider, e.g., the reaction

$$\pi^+ + p \to \pi^+ + \pi^+ + n, \tag{15}$$

when one of π^+-mesons in the final state is slow. Acting in the same way as in the case of the neutral meson production we get the relation

$$M_+(p'q'o;pq) = (U^+M_-(pqo;p'q')U)^T \tag{16}$$

where M_+ is the amplitude of the reaction (15) and M_- is the amplitude of the "soft" π^- production in the reaction

$$\pi^+ + n \to \pi^- + \pi^+ + p, \tag{17}$$

or, due to the charge symmetry, the amplitude of the "soft" π^+ production in the reaction

$$\pi^- + p \to \pi^+ + \pi^- + n. \tag{18}$$

On the basis of the invariance requirements we have

$$\begin{aligned} M_+(p'q'o;pq) &= a_+\gamma_5 + b_+\gamma_5\gamma\cdot(q+q') \\ M_-(pqo;p'q') &= a_-\gamma_5 + b_-\gamma_5\gamma\cdot(q+q'). \end{aligned} \tag{19}$$

From (16) it follows:

$$a_+ = a_- \qquad b_+ = -b_- . \tag{20}$$

This means that the differential cross sections of the reaction(15) and (17) or (18) are equal and the recoil nucleon polarizations are opposite.

In a similar way it can be shown that the differential cross sections of "soft" π meson production in the reactions

$$p + p \to n + p + \pi^+ \qquad n + p \to p + p + \pi^- \tag{21}$$

are equal. In some cases, due to the charge symmetry, it is possible to obtain restrictions of the type (7) on the charged meson production amplitude too. So, for example, the "soft" π^- – meson production amplitude of the reaction

$$\pi^- + p \to \pi^- + \pi^+ + n \tag{22}$$

obeys this requirement, and, consequently, the recoil neutron polarization vanishes.

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