ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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1. Introductory Remarks*.

Without taking into account the spin effects in the c.m.s., the elastic scattering amplitude may be put as

$$A(\theta) = \frac{i\lambda}{2} \sum_{\ell=0}^{\infty} (2\ell+1)\, \xi_\ell \cdot \mathcal{P}_\ell\,(\cos\theta) \tag{1}$$

where $\xi_\ell = 1 - e^{2i\eta_\ell}$, η_ℓ is the complex phase shift.

If the wavelength is short enough as compared with the dimensions of the interacting particles, then the quantity ξ_ℓ can be considered as a smooth function of the quantum number ℓ. Under this assumption we get for the forward scattering

$$A(0) = \frac{i\lambda}{2} \sum_{\ell=0}^{\infty} (2\ell+1)\, \xi_\ell \cdot \Delta\ell \to \frac{i\lambda}{2} \int_0^{\infty} (2\ell+1)\, \xi_\ell \, d\ell \tag{2}$$

where $\Delta\ell = 1$. If L is the upper limit of the orbital number ℓ for which the quantity ξ_ℓ is already very small, $\bar{\xi}_\ell$ is the average value ξ_ℓ in the interval $0 < \ell < L$, then the differential forward scattering cross section will be:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\lambda^2}{4} \,|\, \xi_\ell \,|^2 L^4 = \tfrac{1}{4} \,|\, \bar{\xi}_\ell \,|^2 R^2 \left(\frac{R}{\lambda}\right)^2 \tag{3}$$

where $R = \lambda \cdot L$ is the radius of the sphere where the particles interact. Generally speaking, this radius is a function of the energy of the particle k. In the special case when the phase shifts of the scattered waves $\eta_\ell\,(k)$ depend on orbital momentum ℓ only through the ratio ℓ / k: $\eta_\ell\,(k) = \eta\,(\ell / k)$ the interaction radius R does not depend on particle energy (in this simplest case the phase shifts are the functions of collision parameters $\rho = \lambda \cdot \ell = \ell / k$ only).

2. Backward Scattering.

Let us consider now the backward scattering.

From (1) we get for $\theta = \pi$:

$$A(\pi) = \frac{i\lambda}{2} \sum_{\ell=0}^{\infty} (2\ell+1)\, \xi_\ell\, (-)^\ell =$$

$$= \frac{i\lambda}{2} \left[\sum_{s=0}^{\infty} (4s+1)\, \xi_{2s} - \sum_{s=0}^{\infty} (4s+3)\, \xi_{2s+1} \right] = \tag{4}$$

$$= \frac{i\lambda}{2} \left[\sum_{s=0}^{\infty} (4s+1)\, (\xi_{2s} - \xi_{2s+1}) - 2 \sum_{s=0}^{\infty} \xi_{2s+1} \right]$$

Further $\xi_{2s+1} - \xi_{2s} = \frac{d\xi_\ell}{d\ell} + \dots$ $(\Delta\ell = 1)$ and assuming that the second derivative is no longer essential (the condition of smoothness) we find from (4)

$$A(\pi) \to \frac{i\lambda}{2} \left[-\tfrac{1}{2} \int_0^{\infty} (2\ell+1) \frac{d\xi_\ell}{d\ell}\, d\ell \;-\; \int_0^{\infty} \xi_\ell \, d\ell - \int_0^{\infty} \frac{d\xi_\ell}{d\ell}\, d\ell \right] = \tag{5}$$

$$= \frac{i\lambda}{2} \, \frac{3}{2} \, \xi_0 \; .$$

Hence:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{9}{16} \lambda^2 \,|\, \xi_0 \,|^2 \tag{6}$$

(Here ξ_0 is ξ_ℓ for $\ell = 0$).

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* This work develops the one published earlier /1/.

If ξ_0 does not depend on energy, what will take place when absorption is large ($| \bar{e}^{2i\eta_\ell} | \ll 1$) then the energy dependence of the backward scattering cross-section will be $\sim 1/E^2$ (in the c.m.s.) or $\sim 1/E_0$ (in the lab . syst.). A stronger decrease of this cross section with the increasing energy would mean the growing of the transparency of the particle core.

If absorption is large, the conclusion about the dependence $\sim 1/E^2$ will remain valid, only the numerical coefficient in (6) may vary.

Therefore, in case of large absorption it is more accurate to write

$$\left(\frac{d\sigma}{d\Omega} \right)_\pi = a \lambda^2 \tag{6'}$$

where the magnitude of a must be equal to $\frac{9}{16} | \xi_0 |^2$.

3 . Angular Distribution in the Region of 180°.

Let us consider now the form of the angular distribution in the 180° region. For this purpose we note that

$$\left(\frac{d\, P_\ell(z)}{d z} \right)_{z=-1} = (-)^{\ell+1} \tfrac{1}{2} \ell (\ell + 1) \qquad (\text{see } /2/),$$

further $\Delta z = \Delta \cos (\pi - \theta) = \tfrac{1}{2} \theta^2 + \dots$.

Therefore

$$A (\pi - \theta) = A(\pi) + \frac{i\lambda}{2} \tfrac{1}{2} \sum_{\ell=0}^{\infty} (2\ell + 1) \ell (\ell + 1) \xi_\ell (-)^{\ell+1} \frac{\theta^2}{2} + \dots \tag{7}$$

The sum over ℓ entering this formula can be transformed as follows:

$$\begin{aligned} S &= \tfrac{1}{2} \sum_{\ell=0}^{\infty} (2\ell + 1) \ell (\ell + 1) \xi_\ell (-)^{\ell+1} = -\tfrac{1}{2} \sum_{s=0}^{\infty} (4 s + 1) 2s (2s + 1) \xi_{2s} + \\ &+ \tfrac{1}{2} \sum_{s=0}^{\infty} (4s + 3) (2s + 1) (2s + 2) \xi_{2s+1} = \\ &= - \tfrac{1}{2} \sum_{s=0}^{\infty} (4s + 1) (2s) (2s + 1) (\xi_{2s} - \xi_{2s+1}) + 3 \sum_{s=0}^{\infty} (2s + 1)^2 \xi_{2s+1} \to \\ &\to \tfrac{1}{4} \int_0^{\infty} [(2\ell + 1) \ell (\ell + 1)] \frac{d\xi_\ell}{d\ell} d\ell + 3/2 \int_0^{\infty} (\ell + 1)^2 (\xi_\ell + \frac{d \xi_\ell}{d \ell}) d\ell = \\ &= - \tfrac{3}{4} \int_0^{\infty} (2\ell + 1) \xi_\ell \, d\ell - \int_0^{\infty} \xi_\ell \, d\ell - 3/2 \, \xi_0 \end{aligned} \tag{8}$$

Note that the first term is equal to -3/4 $A(0)$, where $A(0)$ is a forward scattering amplitude.

The substitution of expression (8) into (7) gives:

$$A (\pi - \theta) = A(\pi) - [\tfrac{3}{4} A (0) + \frac{i\lambda}{2} \int_0^{\infty} \xi_\ell \, d \ell + \tfrac{3}{4} i\lambda \xi_0] \frac{\theta^2}{2} + \dots \tag{9}$$

If an interaction radius R is far larger than λ the terms in square brackets, except 3/4 $A(0)$, are not essential. Therefore, the cross section $\left(\frac{d\sigma}{d\Omega} \right)_\pi$ can be written in the form :

$$\left(\frac{d\sigma}{d\Omega} \right)_{\pi-\theta} = \left(\frac{d\sigma}{d\Omega} \right)_\pi - 3/8 \, [A^*(\pi) A(0) + A^*(\pi) A(0)] \, \theta^2 + \dots \tag{10}$$

or for the real amplitudes:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\pi-\theta} = \left(\frac{d\sigma}{d\Omega}\right)_{\pi}\left(1-\frac{\theta^2}{\theta_0^2}+\ldots\right) \tag{11}$$

where

$$\theta_0^2 = \tfrac{3}{4}\left[\left(\frac{d\sigma}{d\Omega}\right)_{\pi} / \left(\frac{d\sigma}{d\Omega}\right)_{0}\right]^{1/2} \tag{12}$$

It is supposed in formulae (10) and (11) that $\theta \ll \theta_0$. So, the assumption about the smooth behaviour of phase shifts leads to the existence of backward scattering maximum whose curvature is defined by the angle θ_0.

4. The more precise definition of the formulae.

All the formulae listed above can be made more accurate if one keeps the sum up to some value $\ell = L$ and only for $\ell > L$ the sum is replaced by an integral.

In other words, one consideres the necessary smoothness to come only for $\ell > L$.

Formulae (1.2.3) will be then rewritten as follows:

$$A(0) = \frac{i\lambda}{2}\left[\sum_{\ell=0}^{L}(2\ell+1)\xi_\ell + \int_{L+1}^{\infty}(2\ell+1)\xi_\ell\, d\ell\right] \tag{2'}$$

If absorption is large ($\xi_\ell \cong 1$), then

$$A(0) = \frac{i\lambda}{2}\left[(L+1)^2 + \int_{L+1}^{\infty}(2\ell+1)\xi_\ell\, d\ell\right] \tag{2''}$$

Further

$$A(\pi) = \frac{i\lambda}{2}\left[\sum_{\ell=0}^{\infty}(2\ell+1)\xi_\ell(-)^\ell + \left(L+\frac{5}{2}\right)\xi_{L+1}\right] \tag{5'}$$

and for $\xi_\ell \cong 1$ for $\ell \le L+1$, we find:

$$A(\pi) = \frac{i\lambda}{2}\,\frac{3}{2}\,\xi_{L+1} \tag{5''}$$

This formula is formally valid for $L=-1$ too ; in this case it coincides with (5).

Formula (9) assumes the form:

$$A(\pi-\theta) = A(\pi) - \frac{i\lambda}{2}\Big\{\tfrac{3}{4}\int_{L+1}^{\infty}(2\ell+1)\xi_\ell\, d\ell + \int_{L+1}^{\infty}\xi_\ell\, d\ell + \tag{9'}$$

$$+\tfrac{1}{4}\left[(2L+1)\;L(L+1)+6(L+1)^2\right]\xi_{L+1} - \tfrac{1}{2}\sum_{\ell=0}^{L}(2\ell+1)\ell(\ell+1)\xi_\ell\,(-)^{\ell+1}\Big\}\frac{\theta^2}{2}+\ldots$$

and particularly for $\xi_\ell \cong 1$ at $\ell \le L+1$

$$A(\pi-\theta) = A(\pi) - \frac{i\lambda}{2}\left[\tfrac{3}{4}\int_{L+1}^{\infty}(2\ell+1)\xi_\ell\, d\ell + \tfrac{3}{4}(L^2+3L+2) + \int_{L+1}^{\infty}\xi_\ell\, d\ell\right]\frac{\theta^2}{2}+\ldots$$

5. Comparison with the Experiment.

The comparison with the experimental data available indicates qualitative agreement with formula (6'). Particularly, for the pion energy of 2,5 GeV $\left(\frac{d\sigma}{d\Omega}\right)_\pi = 0.10^{3}$ mb and for 7÷ 8 GeV, $\left(\frac{d\sigma}{d\Omega}\right)_\pi = 0{,}02$ mb /4/. For $|\xi_0| = 0{,}7$ the

the corresponding theoretical values, in accordance with (6), will be 0, 10 mb and 0,03 mb.

However, we have no measurements yet which would be sufficiently full and reliable and which might allow to make more comprehensive comparison with the experiment.

It seems to be especially interesting for us to measure the energy dependence of $\left(\frac{d\sigma}{d\Omega}\right)_{\pi}$. This measurement could show whether nucleon transparency varies in its cental part- in the nucleon core.

References

1. Д.Блохинцев. ЖЭТФ 42, 880 (1962) и то же Nuovo Cimento Ser. X, 23, 1061 (1962).
2. Е.Гобсон. Теор. сферическ. функций, стр. 37, ИЛ (1952).
3. W.Kwan, W.I.Lawrence and U.P.Martin. Phys. Rev. Lett., 7, 125 (1961).
4. Р.А.Арипов, В.Г.Гришин, Л.В.Сильвестров, В.Н.Стрельцов. Препринт Р-765, Дубна(1962

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