ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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REGGE POLES AND PERTURBATION THEORY

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Abstract

The method to evaluate the Regge trajectories starting from the perturbation theory is proposed.

In the previous paper[1] a method has been proposed to investigate the asymptotic behaviour of the scattering amplitude using information from perturbation theory. This approach makes it possible to calculate the Regge trajectories. In the present note the Regge trajectories for potential scattering are evaluated starting from perturbation theory. As we have earlier seen[1] it is necessary to sum up the terms proportional to $\frac{1}{s}(\ln s)^n$ in order to find the asymptotic behaviour of the scattering amplitudes from perturbation theory. In this way we obtain the following asymptotic expression for the scattering amplitude for large s

$$T(s,t)=\beta(t)\, s^{\alpha(t)} \tag{1}$$

In order to calculate $\alpha(t)$ from perturbation theory we have to find the expansion in powers of g of the following expression for large s:

$$\alpha(t)=\frac{\partial \ln T(s,t)}{\partial \ln s} \tag{2}$$

In the case of the Yukawa potential $V(r)=-g\frac{e^{-\mu r}}{r}$ we obtain the expression for the first Regge trajectory, substituting the Born expansion for large s up to the terms of g^3 order

$$\alpha_1(k^2)=-1+\frac{g}{2\sqrt{-k^2}}+\frac{g^2}{2k^2}\ln\left(1+\frac{\mu}{2\sqrt{-k^2}}\right) \tag{3}$$

where k^2 is the energy and $s=-2k^2(1-\cos\theta)$. For $\mu=0$ we get from (3) the exact expression of the first Regge trajectory for the Coulomb potential.

In order to calculate the second Regge trajectory one needs to sum up the terms proportional to $\frac{1}{s^2}(\ln s)^n$. We ought however to keep in mind here that some of these terms have to be preliminarily referred to the part of the amplitude emerged from the first pole, because it has the form[2]

$$T_1(s,k^2)=f(k^2)\, z^{\alpha(k^2)} F\left(-\frac{\alpha}{2},\ \frac{1-\alpha}{2};\ \frac{1-2\alpha}{2};\ \frac{1}{z^2}\right) \tag{4}$$

where $z=1+\frac{s}{2k^2}$ and F is the hypergeometric function.

Subtracting from the Born expansion the contribution from the first pole we obtain up to the terms of g^2 order

$$\tilde{T}=T-T_1=-\frac{g}{s^2}(\mu^2+2k^2)\left[1+\frac{g\ln\frac{s}{\mu^2}}{2\sqrt{-k^2}}\left(1+\frac{\mu^2}{2k^2}\right)\right] \tag{5}$$

Now, with the aid of (2) we get the expression of the second Regge trajectory

$$\alpha_2(k^2)=-2+\frac{g}{2\sqrt{-k^2}}\left(1+\frac{\mu^2}{2k^2}\right) \tag{6}$$

For $\mu=0$ formula (6) leads to the exact expression of the second Regge trajectory for Coulomb potential. Formulae (3) and (6) for the integer values of $\alpha=\ell$ give the energy spectrum of the system with the radial quantum numbers $n_r=0,1$.

In a similar manner one can calculate the other Regge trajectories up to any order in g. Above we have shown that information contained in perturbation theory enables one to obtain the knowledge of both the asymptotic behaviour of the scattering amplitude and bound states of the system. In our case we can compare the results we have got with information obtained directly from the Schrödinger equation

$$\frac{d^2u}{dr^2}+\left[k^2-\frac{\alpha(\alpha+1)}{r^2}+g\frac{e^{-\mu r}}{r}\right]u(r)=0$$

Representing u in the form $u = r^{a+1}\phi$, expanding a and ϕ in powers of g

$$a_n = -n + g a_n^1 + g^2 a_n^2 + \dots$$
$$\phi = \phi_0 + g\,\phi_1 + g^2 \phi_2 + \dots$$

and solving successively the equations in each order in g we get for a_1 and a_2 the expressions coinciding with formulae (3) and (6).

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