ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ
Лаборатория ядерных проблем

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E - 1012

DETERMINATION OF THE ROOT-MEAN-SQUARE RADIUS OF TRANSITION He^3 — H^3

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The experimental investigation of μ -meson capture in He^3 is of great importance for checking the universal weak interaction theory. However, the muon-nucleon interaction constant can be determined exactly only in the case when the mean-square radius of transition $He^3 \to H^3$, $< H^3 | r^2 | He^3 >$ is known. The theoretical calculations of this value depend on the choice of the nucleon-nucleon interaction potential and of the nuclear model. Calculations carried out by Werntz[1] and Fujii and Primakoff[2] yield different results. Following Werntz $r = 1.56 \cdot 10^{-13}$cm, Fujii and Primakoff have obtained $r = 1.78 \cdot 10^{-13}$cm. The aim of the present note is to show that r can be claulated if we know the ratio of Panofsky for He., P_{He}, i.e. the ratio of the probabilities of the processes

$$\pi^- + He^3 \to H^3 + \pi^0 \quad \text{and} \quad \pi^- + He^3 \to H^3 + \gamma .$$

The calculation is based on the following assumptions: a) the π -meson capture is considered in terms of the impulse approximation; b) in the nuclear wave function only the S-wave is taken into account.

Following I. Pomeranchuk[3] the π -meson capture in hydrogen will be described by phenomenological potentials: $a\delta(\bar{r}_p - \bar{r}_n)$ for process $\pi^- + p \to n + \pi^0$, $\frac{b}{\sqrt{\omega}}(\bar{\sigma}\bar{e})\delta(\bar{r}_p - \bar{r}_n)$ for process $\pi^- + p \to n + \gamma$, where $\bar{r}_n$ is the π -meson coordinate, $\bar{r}_p$ is the proton coordinate, ω is the γ -quantum energy, $\bar{e}$ is the unit vector of the γ -quantum polarization, a and b are constants. The ratio $|a|^2/|b|^2$ is easily expressed in terms of the Panofsky ratio for hydrogen, which is measured with great accuracy. The Panofsky ratio for hydrogen according to the V.T Cocconi data[4] is $P_H = 1,53 \pm 0.02$.

The nuclear wave functions He^3, H^3 are

$$\Psi = \frac{1}{\sqrt{2}}(\chi'\xi'' - \chi''\xi')\psi(r_1, r_2, r_3)$$

where

$$\chi' = \frac{1}{\sqrt{6}}(2a_1a_2b_3 - a_1b_2a_3 - b_1a_2a_3)$$
$$\chi'' = \frac{1}{\sqrt{2}}(a_1b_2a_3 - b_1a_2a_3)$$

a_i is the state of i nucleon with the spin projection + 1/2,

b_i is the state with the spin projection - 1/2, ξ', ξ'' are constructed in a similar way in the isotopic spin space[6],

$\psi(r_1, r_2, r_3)$ is symmetrical in all three coordinates. After having calculated the matrix elements of

$$H_{\pi_0} = a\sum_{i=1}^{3}\delta(r - r_i)\tau_i^-$$

and

$$H_\gamma = \frac{b}{\sqrt{\omega}} \sum_{i=1}^{3} \delta(r - r_i)(\vec{\sigma}_i \vec{e})\, \tau_i^-$$

it is not difficult to get

$$P_{He} = P_H \sqrt{\frac{E(\mu + m)^3 M}{E_H(\mu + M)^3 m}} \quad \frac{\omega_H}{\omega} \quad \frac{M + \omega}{m + \omega_H} \quad \frac{1}{/\langle e^{i\vec{k}\vec{r}}\rangle/^2}$$

where m is the neutron mass, M is the tritium mass, μ is the π^0 -meson mass, $E = 4{,}06$ MeV is the kinetic energy of H^3 and π^0 , $E_H = 3{,}3$ MeV is the kinetic energy of n and π^0 , $\omega = 135{,}80$ MeV is the γ quantum energy in process $\pi^- + He^3 \to H^3 + \gamma$, $\omega_H = 129{,}46$ MeV is the γ quantum energy is process $\pi^- + p \to n + \gamma$.

The exponent in the matrix element can be expanded in power series

$$\langle e^{i\vec{k}\vec{r}} \rangle = 1 - \frac{1}{3!} k^2 \langle r^2 \rangle + \frac{1}{5!} k^4 \langle r^4 \rangle - \frac{1}{7!} k^6 \langle r^6 \rangle$$

We restrict ourselves to the three first terms of the expansion; $\langle r^4 \rangle$ and $\langle r^6 \rangle$ may be approximately expressed in terms of $\langle r^2 \rangle$. We take $\psi(r_1, r_2, r_3)$ as a product of some single-particle functions

$$\psi(r_1, r_2, r_3) = \phi(r_1)\,\phi(r_2)\,\phi(r_3)$$

The estimates $\langle r^4 \rangle$ and $\langle r^6 \rangle$ depend mainly on the behaviour of $\phi(r)$ for large r . For $\phi = e^{-ar}$ we obtain $\langle r^4 \rangle = \frac{5}{2} \langle r^2 \rangle^2$, $\langle r^6 \rangle = \frac{35}{3} \langle r^2 \rangle^3$; for $\phi = e^{-ar^2}$ we obtain $\langle r^4 \rangle = \frac{5}{3} \langle r^2 \rangle^2$, $\langle r^6 \rangle = \frac{35}{9} \langle r^2 \rangle^3$ for $\phi = \theta(r - R)$ we get $\langle r^4 \rangle = \frac{25}{21} \langle r^2 \rangle^2$, $\langle r^6 \rangle = \frac{125}{81} \langle r^2 \rangle^3$. Calculations show that $/\langle e^{i\vec{k}\vec{r}} \rangle/^2$ changes about by 3% due to the choice of $\phi(r)$ and about 3% due to term $k^6 r^6$. It should be noted that these estimations are carried out for $r = 1{,}5.10^{-13}$cm, for smaller radius the error decreases down to 1%. The Panofsky ratio for helium at different r is: $r = 1{,}46.10^{-13}$cm, $P_{He} = 2{,}37$; $r = 1{,}56.10^{-13}$cm, $P_{He} = 2{,}47$; $r = 1{,}78.10^{-13}$cm $P_{He} = 2{,}71$.

Preliminary experiment results on the π -capture in He^3 |5| yield $P_{He} = 2{,}16 \pm 0{,}28$. This value of the Panofsky ratio for helium corresponds to the radius of transition

$$r \left(= 1{,}24 \begin{smallmatrix} +0{,}30 \\ -0{,}46 \end{smallmatrix} \right) \cdot 10^{-13} \text{ cm.}$$

We note once again that calculations are carried out in terms of the impulse approximation. An independent determination of the root-mean-square radius in electron scattering experiments on He is of great interest both for the theory of μ -capture in He^3 and the check of the applicability of the impulse approximation in the theory of π -capture in He^3.

In conclusion the author expresses his gratitude to R.M. Suliaev and S.S. Gershtein for stimulating this work , I thank also Yu.A. Shcherbakov and I.V. Falomkin for the discussions and valuable remarks.

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Reveived by publishing department
on June 16, 1962.