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SPECTRAL AND PROJECTION PROPERTIES OF THE "TWO TIME" GREEN FUNCTIONS OF n PARTICLES IN THE NULL PLANE QUANTUM FIELD THEORY

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1. Introduction - The Statement of the Question

The system of interacting particles in quantum field theory can be described using the Bethe-Salpeter amplitudes or the quasipotential wave functions. Let us recall the definition of these quantities.

Let $|P, c>$ - be the state vector with the total 4-momentum P and quantum number of the particle c . The bound state of the particles a and b having the total 4-momentum P and quantum number of the particle c is described by the Bethe-Salpeter amplitude

$$\Psi_P(x,y) = <0|T\varphi_a(x)\varphi_b(y)|P,c>,$$

$\varphi_a(x)$ and $\varphi_b(y)$ are the Heisenberg operators and T is the operator of time ordering.

Due to the translational invariance, we have:

$$\Psi_P(x,y) = e^{-iP\frac{x+y}{2}} f_P(x-y) \ .$$

The amplitude $f_P(x-y)$ depends on the relative time $x_0 - y_0$ and it is impossible to interpret it as the wave function.

The quasipotential wave functions are determined as the values of the corresponding Bethe-Salpeter amplitudes on the given space-like surface. In the early paper/1/ as the space-like surface, we chose the surface of equal times for all particles.

In particular, the quasipotential wave functions of two particles had the form:

$$\chi_P(\vec{x}-\vec{y}) = f_P(x-y)\Big/_{x_0-y_0=0}$$

Note, that though the wave functions thus determined are formally non-covariant, nevertheless, all physical quantities (the spectrum of bound states, the scattering matrix and so on) obtained using these functions have a relativistically invariant meaning.

At present various ways of constructing relativistically covariant quasipotential wave functions [2] are known.

The present paper gives a systematic description of the properties of multiparticle quasipotential wave functions and the corresponding Green functions in quantum field theory on the null plane for arbitrary spin. The starting point of the unified approach is the spectral representation of the Green function. It makes it possible to study the structure of a quasipotential as a result of which one can formulate the correct relativistic equations in the many-body problem [3]. The projection and transformation properties of the Green function and quasipotential wave functions are specially investigated.

The projection properties can just be explained by the example of the two-particle quasipotential wave function. For the sake of simplicity, let us consider the scalar particles of the

type a, b and c to which there correspond second - quantized fields $\varphi_i(x)$ $(i=a,b,c)$ and the interaction Lagrangian of the form:

$$\mathcal{L}_I(x) = g : \varphi_a(x)\,\varphi_b(y)\,\varphi_c(x) .$$

In the lowest order of perturbation theory in the coupling constant g for the Bethe-Salpeter amplitude, we have:

$$f_P(k) \sim \frac{g}{[(\frac{1}{2}P+k)^2-m^2+i\varepsilon\,][(\frac{1}{2}P-k)^2-m^2+i\varepsilon\,]} ,$$

where

$$f_P(x-y) = \int d^4k\, e^{-ik(x-y)} f_P(k) .$$

The quasipotential wave function on the null plane (see § 2) is determined by the expression

$$\varphi_P(\underline{x}-\underline{y}) = f_P(x-y)\Big/_{x^+=y^+} .$$

In the considered approximation, we have

$$\varphi_P(\underline{x}-\underline{y}) = \int d^3\underline{k}\, e^{-i\underline{k}(\underline{x}-\underline{y})}\, \varphi_P(\underline{k}) ,$$

where

$$\varphi_P(\underline{k}) \sim \frac{g\,\theta(\eta)\,\theta(1-\eta)\, i\pi}{|P^+|\,\eta(1-\eta)\Big[P^2+P_\perp^2-\frac{(\frac{1}{2}P_\perp+k_\perp)^2+m^2}{\eta}-\frac{(\frac{1}{2}P_\perp-k_\perp)^2+m^2}{1-\eta}\Big]}$$

$$\eta = \frac{1}{2}+\frac{k^+}{P^+} .$$

One can see that in the given approximation the Fourier transform of the quasipotential wave function has the following projection properties[4]

$$\Phi_p(k^+, k_\perp) = 0, \qquad \text{if} \quad \eta < 0, \text{ or } \eta > 1.$$

It will be shown below that this important property is conserved for all quasipotential wave functions independent of perturbation theory.

2. Two-Time Green Functions

The Green function of n interacting particles in quantum field theory on the null plane is defined as a vacuum expectation value of the time ordered products of the corresponding Heisenberg field operators, and has the form[5]

$$G^{(n)}(x_1, \dots x_n; y_1, \dots y_n) = \langle 0 | T_+ \Psi_1(x_1) \dots \Psi_n(x_n) \bar{\Psi}_n(y_n) \dots \bar{\Psi}_1(y_1) | 0 \rangle . \qquad (1)$$

For the parametrization of the components of four vectors x_i, y_i and others, the following variables of the "light cone" are used:

$$x = (x^-, x^+, x_\perp), \quad x^\pm = \frac{1}{2}(x^0 \pm x^3), \quad x_\perp = (x^1, x^2)$$

$$\underline{x} = (x^-, x_\perp), \quad x = (x^+, \underline{x}).$$

In the momentum space it is convenient to introduce the following notations:

$$p = (p^-, p^+, p_\perp), \quad p^\pm = p^0 \pm p^3, \quad p_\perp = (p^1, p^2), \quad \underline{p} = (p^+, p_\perp)$$

$$px = p^- x^+ + p^+ x^- - p_\perp x_\perp = p^- x^+ + \underline{p}\underline{x} .$$

Ψ and $\bar{\Psi}$ are conjugate to each other Heisenberg field operators of i-th particle. T_+ is the operator of time ordering. In the simplest case of two fields

$$T_+ \Psi_i(x) \Psi_j(y) = \theta(x^+ - y^+) \Psi_i(x) \Psi_j(y) \pm \theta(y^+ - x^+) \Psi_j(y) \Psi_i(x)$$

(minus corresponds to fermions).

Note, that in quantum field theory on the null plane, the canonical commutation relations are given at fixed "time" variable $x^+ = x^0 + x^3$. Correspondingly, in perturbation theory there are considered the x^+-ordered products of field free operators.

This results in a certain difference in the language of diagrams as compared to the covariant theory for particles with spin.

Let us define the "two-time" Green function of n-particles

$$\tilde{G}^{(n)}(x^+; \underline{x}_1, \dots \underline{x}_n; y^+; \underline{y}_1, \dots \underline{y}_n) = G^{(n)}(x_1, \dots x_n; y_1 \dots y_n)\Big|_{\substack{x_1^+ = \dots = x_n^+ = x^+ \\ y_1^+ = \dots = y_n^+ = y^+}} . \qquad (2)$$

It is convenient to introduce the following operators

$$A(x) = \Psi_1(x_1) \dots \Psi_n(x_n)\Big|_{x_1^+ = \dots = x_n^+ = x^+}$$

$$\bar{A}(y) = \bar{\Psi}_n(y_n) \dots \bar{\Psi}_1(y_1)\Big|_{y_1^+ = \dots = y_n^+ = y^+}$$

and to write down the "two-time" Green function (2) in terms of these operators

$$G^{(n)}(x,y) = \langle 0 | T_+ A(x) \bar{A}(y) | 0 \rangle = \tag{3}$$

$$= \theta(x^+ - y^+) \langle 0 | A(x) \bar{A}(y) | 0 \rangle \pm \theta(y^+ - x^+) \langle 0 | \bar{A}(y) A(x) | 0 \rangle .$$

The signs ($\pm$) are chosen depending on the number of fermion operators in $A(x)$.

Further we want to obtain the spectral representation for the Green function (3).

Using the expansion over the total set of physical states $|m\rangle$, the properties of the translational invariance

$$\langle 0 | A(x) | m \rangle = e^{-iP_m^- x^+} \langle 0 | A(\underline{x}) | m \rangle$$

$$A(\underline{x}) = A(x)\big|_{x^+ = 0}$$

and the Fourier representation for the θ-function , one can write (3) in the following spectral form:

$$\tilde{G}^{(n)}(x,y) = \tilde{G}^{(n)}(x^+ - y^+; \underline{x}; \underline{y}) =$$

$$= \int_{-\infty}^{\infty} dP^- e^{-iP^-(x^+ - y^+)} \int_0^{\infty} dz \left[\frac{G_1(z; \underline{x}, \underline{y})}{P^- - z + i\varepsilon} \mp \frac{G_2(z; \underline{x}, \underline{y})}{P^- + z - i\varepsilon} \right], \tag{4}$$

where for the spectral functions $G_{1,2}$ we have the expression in terms of the n-particle quasipotential wave functions

$$G_1(z; \underline{x}, \underline{y}) = \frac{i}{2\pi} \sum_m \delta(z - P_m^-) \chi_{om}(\underline{x}) \bar{\chi}_{om}(\underline{y})$$

$$G_2(z; \underline{x}, \underline{y}) = \frac{i}{2\pi} \sum_m \delta(z - P_m^-) \chi_{mo}(\underline{x}) \bar{\chi}_{mo}(\underline{y}) \tag{5}$$

$$\chi_{om}(\underline{x}) = \chi_{om}(\underline{x}_1, \ldots \underline{x}_n) = \langle 0 | \psi_1(0, \underline{x}_1) \ldots \psi_n(0, \underline{x}_n) | m \rangle$$

$$\bar{\chi}_{om}(\underline{y}) = \langle m | \bar{A}(\underline{y}) | 0 \rangle$$

$$\chi_{mo}(\underline{x}) = \langle m | A(\underline{x}) | 0 \rangle \qquad \bar{\chi}_{mo}(\underline{y}) = \langle 0 | \bar{A}(\underline{y}) | m \rangle .$$

The summation over m in (1.8) means the integration over 4-momentum $P_m (P_m^- > 0, P_m^+ > 0, P_\perp)$ $P_m^2 > 0$ and the summation over the rest of quantum numbers on which the given physical state $|m\rangle$ may depend.

Let us define the Fourier transforms of the spectral functions

$$G_{1,2}(z; \underline{x}_1 \ldots \underline{x}_n; \underline{y}_1 \ldots \underline{y}_n) = \frac{1}{(2\pi)^n} \int e^{-i\sum_{j=1}^n (\underline{p}_j \underline{x}_j - \underline{q}_j \underline{y}_j)} G_{1,2}(z; \underline{p}; \underline{q}) \prod_{j=1}^n d\underline{p}_j^3 d\underline{q}_j^3 \tag{6}$$

From (6) using (5), we have

$$G_1(z; \underline{p}_1 \ldots \underline{p}_n; \underline{q}_1 \ldots \underline{q}_n) = \frac{i}{(2\pi)^{1-n}} \sum_m \delta(z - P_m^-) \chi_{om}(\underline{p}_1 \ldots \underline{p}_n) \bar{\chi}_{om}(\underline{q}_1 \ldots \underline{q}_n)$$

$$G_2(z; \underline{p}_1 \ldots \underline{p}_n; \underline{q}_1 \ldots \underline{q}_n) = \frac{i}{(2\pi)^{1-n}} \sum_m \delta(z - P_m^-) \chi_{mo}(\underline{p}_1 \ldots \underline{p}_n) \bar{\chi}_{mo}(\underline{q}_1 \ldots \underline{q}_n), \tag{7}$$

where

$$\chi_{om}(\underline{x}_1 \ldots \underline{x}_n) = \int e^{-i\sum \underline{p}_j \underline{x}_j} \chi_{om}(\underline{p}_1 \ldots \underline{p}_n)\, d\underline{p}_1^3 \ldots d\underline{p}_n^3$$

$$\chi_{mo}(\underline{x}_1 \ldots \underline{x}_n) = \int e^{-i\sum \underline{p}_j \underline{x}_j} \chi_{mo}(\underline{p}_1 \ldots \underline{p}_n)\, d\underline{p}_1^3 \ldots d\underline{p}_n^3 .$$

We also show that the spectral functions $G_{1,2}$ in the momentum space have the following important properties

$$G_1(z; \underline{p}_1, \ldots \underline{p}_n; \underline{q}_1 \ldots \underline{q}_n) = 0 , \tag{8}$$

if at least one of the variables p_i^+ or q_j^+ is smaller than zero, and

$$G_2(z; p_1 \dots p_n; q_1 \dots q_n) = 0 \tag{9}$$

if at least one of the variables p_i^+ or q_j^+ is larger than zero.

First, let us show the validity of (8).

To this end, consider the Fourier transform of the n-particle quasipotential wave function

$$\chi_{0m}(p_1 \dots p_n) = \frac{1}{(2\pi)^{3n}} \int e^{i\sum_{j=1}^{n} p_j x_j} \langle 0|\psi_1(x_1) \dots \psi_n(x_n)|m\rangle \prod_{j=1}^{n} dx_j^3 = \tag{10}$$

$$= \frac{1}{(2\pi)^{3n-3}} \sum_{m_1} \delta(p_1 - p_{m_1}) \int e^{i\sum_{j=2}^{n} p_j x_j} \langle 0|\psi_1(0)|m_1\rangle\langle m_1|\psi_2(x_2)\dots|m\rangle \prod_{j=2}^{n} dx_j^3$$

Taking into account that $p_{m_1}^+ > 0$, we have that $\chi_{0m}(p_1 \dots p_n)$ is equal to zero if $p_1^+ < 0$. To prove the validity of this statement for any p_j^+, we use the properties of field commutation on the null plane and placing an arbitrary operator $\psi_j(x_j)$ to the first place in the left-hand side, we rewrite eq.(10) as follows:

$$\chi_{0m}(p_1 \dots p_n) = \pm \frac{1}{(2\pi)^{3n-3}} \sum_{m_1} \delta(p_j - p_{m_1}) \int e^{i\sum_{\ell \neq j} p_\ell x_\ell} \langle 0|\psi_j(0)|m_1\rangle$$
$$\langle m_1|\psi_1(x_1) \dots \psi_{j-1}(x_{j-1})\, \psi_{j+1}(x_{j+1}) \dots \psi_n(x_n)|m\rangle \prod_{\ell \neq j} dx_\ell^3$$

Thus, considering that for the physical states $p_{m_1}^+ \geq 0$ we are convinced that

$$\chi_{0m}(p_1 \dots p_n) = 0 \qquad \text{(if at least one } p_i^+ < 0\text{)} \tag{11}$$

Analogously, one can make certain that

$$\chi_{m0}(p_1 \dots p_n) = 0 \qquad \text{(if at least one } p_i^+ > 0\text{)} \tag{12}$$

Taking into account (11), (12) and (7) we can verify the validity of properties (8) and (9).

Now, determine the Fourier transform of the "two-time" Green function (4)

$$\tilde G^{(n)}(x^+ y^+; x_1 \dots x_n; y_1 \dots y_n) = \frac{1}{(2\pi)^{4n}} \int e^{-iP^-(x^+ - y^+) - \sum_{j=1}^{n}(p_j x_j - q_j y_j)}$$
$$\tilde G^{(n)}(P^-; p_1 \dots p_n; q_1 \dots q_n)\, dP^- \prod_{j=1}^{n} dp_j^3\, dq_j^3 . \tag{13}$$

Substituting (6) and (13) in (4) one obtains

$$\tilde G^{(n)}(P^-; p; q) = \int dz \left[\frac{G_1(z; p, q)}{P^- - z + i\varepsilon} \mp \frac{G_2(z; p; q)}{P^- + z - i\varepsilon} \right] \tag{14}$$

$$G_1(z; p; q) = 0 \qquad \text{(if at least one } p_i^+, q_j^+ < 0\text{)}$$

$$G_2(z; p; q) = 0 \qquad \text{(if at least one } p_i^+, q_j^+ > 0\text{)}$$

The spectral representation (14) is the analog of the spectral representation in the total energy of the "two-time" Green function in quantum field theory. However, there appears an essential separation between the upper and lower parts of the light cone characteristic of quantum field theory on the null plane. Namely, the "retarded" part of the Green function (14)(the first component) completely determines the behaviour of the Green function at positive $p_i^+ q_j^+$ and the advanced part (the second component) - the Green function at negative p_i^+, q_j^+.

If one takes into account that in quantum field theory on the null plane the "dressed" vacuum $|0\rangle$ (the eigenfunction of a total Hamiltonian with the minimal energy $\hat{P}^-|0\rangle = 0$ coincides with the "bare" one, i.e. with the analogous solution of a free Hamiltonian, it is easy to show essential simplifications in the spin structure of the "two- time" Green function.

In fact, the quasipotential wave functions can be presented as follows:

$$\chi_{om}(p_1 \dots p_n)=\sum_{i_1\dots i_n} U_1^{i_1}(p_1)\dots U_n^{i_n}(p_n)\,\varphi_{om}^{i_1\dots i_n}(p_1\dots p_n)$$

$$\chi_{mo}(p_1\dots p_n)=\sum_{i_1\dots i_n} \mathcal{V}_1^{i_1}(p_1)\dots \mathcal{V}_n^{i_n}(p_n)\,\varphi_{mo}^{i_1\dots i_n}(p_1\dots p_n),$$

where

$$\varphi_{om}^{i_1\dots i_n}(p_1\dots p_n)=(2\pi)^{-3n}(p_1^+\dots p_n^+)^{-1}\langle p_1 i_1\dots p_n i_n|m\rangle$$

is the projection of the state vector $|m\rangle$ on the state of n-free particle with the momenta $p_1\dots p_n$ and the spin projections $i_1,\dots i_n$

$$U_{ex}^i(p)=\langle 0|\Psi_{ex}^+(0)|ip\rangle;\quad \mathcal{V}_{ex}^i(p)=\langle pi|\Psi_{ex}(0)|0\rangle$$

are the spinors describing the one-particle states *). The spectral densities can be rewritten in the form of the projection operators:

$$\sigma_1(z;p_1\dots p_n;q_1\dots q_n)=\sum_{\substack{i_1\dots i_n\\ j_1\dots j_n}} U_1^{i_1}(p_1)\dots U_n^{i_n}(p_n)\,\sigma_{1\,i_1\dots i_n j_1\dots j_n}(z;p_1\dots p_n;q_1\dots q_n)\,\bar{U}_1^{j_1}(q_1)\dots\bar{U}_n^{j_n}(q_n)$$

$$\sigma_2(z;p_1\dots p_n;q_1\dots q_n)\sum_{\substack{i_1\dots i_n\\ j_1\dots j_n}} \mathcal{V}_1^{i_1}(p_1)\dots \mathcal{V}_n^{i_n}(p_n)\,\sigma_{2\,i_1\dots i_n j_1\dots j_n}(z;p_1\dots p_n;q_1\dots q_n)\,\bar{\mathcal{V}}_1^{j_1}(q_1)\dots\bar{\mathcal{V}}_n^{j_n}(q_n).$$

*) Here and below we use the invariant normalization of the one-particle state vectors:

$$\langle \vec{p}, i|\vec{p}', i'\rangle=\langle p, i|p', i'\rangle=(2\pi)^3 2p^0\delta_{ii'}\delta^{(3)}(\vec{p}-\vec{p}\,')=(2\pi)^3 p^+\delta_{ii'}\delta^{(3)}(p-p')$$

Let us return to the spectral representation (14). From the property of the translational invariance it follows: $\sum_{j=1}^{n}(p_j-q_j)=0$. We introduce the vector $P=\sum_{j=1}^{n} p_j=(P^+,P_\perp)$ and determine 4-vector $P=(P^-,P^+,P_\perp)$ $P^-=\frac{1}{P^+}(P^2+P_\perp^2)$. Using the definition of spectral densities (7) and the property of the translational invariance of wave functions, one may get, using (14), the spectral representation of the "two-time" Green function in the total 4-momentum P squared.

$$\tilde{G}^{(n)}(P^2;p_1\dots p_n;q_1\dots q_n)=\int_0^\infty ds\,\frac{\sigma(s;p_1\dots p_n;q_1\dots q_n)}{P^2-s+i\varepsilon}, \quad (15)$$

where

$$\sigma(s;p;q)=\sigma_1(s;p;q)\prod_{j=1}^{n}\theta(p_j^+)\theta(q_j^+)\mp\sigma_2(s;p;q)\prod_{j=1}^{n}\theta(-p_j^+)\theta(-q_j^+)$$

$$\sigma_1(s;p;q)=i(2\pi)^{4n-1}\sum_m P^+\delta(s-P_m^2)\,\chi_{om}(p)\,\bar{\chi}_{om}(q)$$

$$\sigma_2(s;p;q)=i(2\pi)^{4n-1}\sum_m P^+\delta(s-P_m^2)\,\chi_{mo}(p)\,\bar{\chi}_{mo}(q).$$

Taking into account the law of conservation of three-dimensional momentum

$$p_1+p_2+\dots+p_n=q_1+q_2+\dots+q_n$$

we write down the Green function in the form

$$\tilde{G}^{(n)}(P^2;p_1\dots p_n;q_1\dots q_n)=\delta^{(3)}\Big(\sum_{j=1}^{n}p_j-\sum_{j=1}^{n}q_j\Big)\,\tilde{G}^{(n)}(P;p_1\dots p_{n-1};q_1\dots q_{n-1}). \quad (16)$$

We, also, show that the Green function (16) depends especially on the variables P^+ and $P_\perp$

$$(P^+)^{2n-2} G^{(n)}(P; P_i^+ P_{i\perp}; q_j^+ q_{j\perp}) = S_P \tilde{G}^{(n)}(P^2; \eta_i, P_{i\perp} - \eta_i P_\perp; \xi_j, q_{j\perp} - \xi_j P_\perp) S_P^{-1}, \tag{17}$$

where due to (8) and (9)

$$\eta_i = \frac{P_i^+}{P^+}, \quad \xi_j = \frac{q_j^+}{P^+} \quad (i,j = 1 \dots n-1)$$
$$0 < \eta_i, \xi_j < 1.$$

S_P and S_P^{-1} the known transformation matrices acting on the spin indices. For the scalar particles S_P is equal to unity.

The dependence of the Green function on the scale variables η_i and ξ_j only, is a consequence of the invariance of the n-particle 4-dimensional Green function (1) with respect to the Lorentz rotations in the plane (x^0, x^3):

$$G^{(n)}(x_1^+, x_1^-, x_{1\perp}, \dots; y_1^+, y_1^-, y_{1\perp} \dots) =$$
$$S_\lambda G^{(n)}(\lambda x_1^+, \lambda^{-1} x_1^-, x_{1\perp} \dots; \lambda y_1^+, \lambda^{-1} y_1^-, y_{1\perp} \dots) S_\lambda^{-1}. \tag{18}$$

The matrix S_λ acts on the spin indices, and realizes the transformation of the field operators ψ_i, $\bar{\psi}_j$. Note, that under the rotation in the plane (x^0, x^3) the arbitrary 4-vector $A = (A^+, A^-, A_\perp)$ is transformed by the law:

$$A^+ \to \lambda A^+, \quad A^- \to \lambda^{-1} A^- \quad A_\perp \to A_\perp.$$

For the "two-time" Green function the property (18) is conserved. As a result, the Fourier transform (16) is the "homogeneous" function of the variables P^+, P_i^+, q_j^+

$$\tilde{G}^{(n)}(P^2, P^+, P_\perp; \dots q_i^+, q_{i\perp} \dots; \dots p_j^+ p_{j\perp} \dots) =$$
$$= \lambda^{2n-2} S_\lambda \tilde{G}^{(n)}(P^2, \lambda P^+, P_\perp; \dots \lambda q_i^+, q_{i\perp} \dots; \dots \lambda p_j^+, p_{j\perp} \dots) S_\lambda^{-1}. \tag{19}$$

It follows from (19) that $\tilde{G}^{(n)}$ depends in a nontrivial way only on the scale variables η_i and ξ_j (17).

Consider the Lorentz transformation given by the two-dimensional vector $v_\perp$

$$A^+ \to A^+; \quad A^- \to A^- + v_\perp A_\perp + \frac{1}{2} A^+ v^2, \quad A_\perp \to A_\perp + v_\perp A^+.$$

The Green function (1) is invariant with respect to these transformations. For the Fourier transform of the "two-time" Green function it follows

$$\tilde{G}^{(n)}(P^2, P_\perp; \eta_i, P_{i\perp}; \xi_j, q_{j\perp}) =$$
$$= S_{v_\perp} G^{(n)}(P^2, P_\perp + P^+ v_\perp; \eta_i, P_{i\perp} + P_i^+ v_\perp, \xi_j, q_{j\perp} + q_j^+ v_\perp) S_{v_\perp}^{-1}. \tag{20}$$

By choosing $v_\perp \sim -\frac{P_\perp}{P^+}$, we come to eq. (17).

3. Quasipotential Green Functions of Free Particles

Let us consider the Green function of one particle with an arbitrary spin, making use of the representation (15). Separating explicitly in the spectral densities $\sigma_{1,2}$ the contributions of single-particle states , we have

$$\sigma_1(s,p,q)=i\,\delta^{(3)}(p-q)\,\theta(p^+)\,\delta(s-m^2)\sum_i\langle 0|\Psi(0)|ip\rangle\langle ip|\bar\Psi(0)|0\rangle$$

$$\sigma_2(s,p,q)=-i\,\delta^{(3)}(p-q)\,\theta(-p^+)\,\delta(s-m^2)\sum_i\langle 0|\bar\Psi(0)|-pi\rangle\langle -pi|\Psi(0)|0\rangle .$$

Denoting the projection operators on the states with positive and negative energies in terms of $\Lambda^{(+)}$ and $\Lambda^{(-)}$ respectively,

$$\Lambda^{(+)}(p)=\sum_i\langle 0|\Psi(0)|pi\rangle\langle ip|\bar\Psi(0)|0\rangle$$

$$\Lambda^{(-)}(p)=\sum_i\langle 0|\bar\Psi(0)|-pi\rangle\langle -pi|\Psi(0)|0\rangle$$

the single-particle propagator in the pole approximation can be rewritten as follows:

$$S(p)=\tilde G_0^{(1)}(p)=i\,\frac{\theta(p^+)\Lambda^{(+)}(p)\pm\theta(-p^+)\Lambda^{(-)}(p)}{p^2-m^2+i\varepsilon}\,. \qquad (21)$$

Analogously one can calculate the quasipotential propagator $\tilde G_0^{(2)}$ of two free particles with arbitrary spins taking into account only two-particle states in $\sigma_{1,2}$

$$\tilde G_0^{(2)}(P;p_1;q_1)=\frac{2\pi i}{|P^+|^2}\,\delta^{(2)}(p_{1\perp}-q_{1\perp})\,\delta(\xi_1-\eta_1)\,\theta(\eta_1)\,\theta(1-\eta_1)\,\frac{\theta(P^+)\Lambda_1^{(+)}(p_1)\Lambda_2^{(+)}(p_2)\pm\theta(-P^+)\Lambda_1^{(-)}(p_1)\Lambda_2^{(-)}(p_2)}{\eta_1(1-\eta_1)\left[P^2+P_\perp^2-\sum_{j=1}^{2}\frac{p_{j\perp}^2+m_j^2}{\eta_j}+i\varepsilon\right]}\,,$$

where

$$p_2=P-p_1\,.$$

In the general case of n-particles, the free "two-time" Green function has the form:

$$\tilde G_0^{(n)}(P^2;p_1\dots p_n;q_1\dots q_n)=i\,\frac{(2\pi)^{n-1}}{|P^+|^{2n-1}}\prod_{j=1}^{n}\frac{\delta^{(2)}(p_{j\perp}-q_{j\perp})\,\delta(\eta_j-\xi_j)\,\theta(\eta_j)}{\eta_j}$$

$$\frac{\theta(P^+)\prod_{j=1}^{n}\Lambda_j^{(+)}(p_j)\pm\theta(-P^+)\prod_{j=1}^{n}\Lambda_j^{(-)}(p_j)}{P^2-\sum_{j=1}^{n}\frac{(p_{j\perp}-\eta_jP_\perp)^2+m_j^2}{\eta_j}+i\varepsilon}\,. \qquad (23)$$

4. <u>The Equations for the System of Two Particles</u>

It is seen from the previous consideration that both the free and total two-dimensional Green functions have the projection properties in the scale variables η_i , ξ_j . Just they are different from zero only in the range $0<\eta_i,\xi_j<1$. Besides, if the particles have spins, the free and full Green functions being considered as operators have eigenvalues different from zero only in the subspace where all the particles are either with positive or negative energies. Since, as is known, the definition of the quasipotential is connected with the inversion of the free Green function, we turn to subspaces of positive energies of all particles. The transition to the subspace $0<\eta_i,\xi_j<1$ is natural as beyond this region all values are equal to zero.

Let us define the projected Green function of two particles

$$g^{(2)}_{i_1i_2j_1j_2}(P^2;p_1p_2;q_1q_2)=\frac{|P^+|^2}{2\pi i}\,\bar U_1^{(i_1)}(p_1)\,\bar U_2^{(i_2)}(p_2)\,\tilde G^{(2)}(P^2;p_1p_2;q_1q_2)\,U_1^{(j_1)}(q_1)\,U_2^{(j_2)}(q_2)\,. \qquad (24)$$

Then, the free one is as follows:

$$g^{(1)}_{0\,i_1i_2j_1j_2}(P^2;\underline{p}_1\underline{p}_2;\underline{q}_1\underline{q}_2)=\frac{\delta^{(2)}(p_{11}-q_{11})\,\delta^{(2)}(p_{21}-q_{21})\,\delta(\eta_1-\xi_1)\,\delta(\eta_2-\xi_2)\,\delta_{i_1i_2}\,\delta_{j_1j_2}}{|P^+|\,\eta_1\eta_2\left[P^2-\frac{(p_{11}-\eta_1P_\perp)^2+m_1^2}{\eta_1}-\frac{(p_{21}-\eta_1P_\perp)^2+m_2^2}{\eta_2}+i\varepsilon\right]}\,.$$

Let us define the two-particle quasipotential $V^{(2)}$ in the given subspace

$$V^{(2)}=g_0^{(2)\,-1}-g^{(2)\,-1}$$

which results, as a rule, in its construction by perturbation theory in quantum field theory on the null plane /2*/. The equation for the Green function in the symbolic operator form is the following

$$g^{(2)}=g_0^{(2)}+g_0^{(2)}V^{(2)}g^{(2)}\,, \tag{25}$$

where by the product we mean the three-dimensional integration

$$\int_0^1 d\eta_1\,d\eta_2\,\delta(\eta_1+\eta_2-1)\int dp_{11}^{(2)}\,dp_{21}^{(2)}\,\delta(p_{11}+p_{21}-P_\perp)\,.$$

We introduce the off-shell quasipotential T-matrix $T^{(2)}(P,\underline{p},\underline{q})$

$$g^{(2)}=g_0^{(2)}+g_0^{(2)}T^{(2)}g_0^{(2)}\,, \tag{26}$$

$\underline{p},\underline{q}$ are the relative momenta of the two-particle system in the initial and final states.

As was shown in paper /2*/, it coincides with the physical scattering amplitude of two considered particles on the mass shell

$$P^2+P_\perp^2=\sum_{i=1}^{2}\frac{p_{i1}^2+m_i^2}{\eta_i}=\sum_{i=1}^{2}\frac{q_{i1}^2+m_i^2}{\xi_i}\,.$$

The equation for the T-matrix according to (25) and (26) has the form:

$$T^{(2)}=V^{(2)}+V^{(2)}g_0^{(2)}T^{(2)}\,. \tag{27}$$

Considering the spectral representation (19) for the two-particle Green function and separating the contribution of the bound state near the corresponding pole, we have

$$g^{(2)}(P,\underline{p},\underline{p}')=2(2\pi)^3\,\frac{\varphi_P(\underline{p})\times\bar{\varphi}_P(\underline{p}')}{P^2-M^2+i\varepsilon}\,, \tag{28}$$

where $\varphi_P(\underline{p})$ is the wave function of the bound state with mass M in the momentum representation

$$(2\pi)^6\,\delta^{(3)}(\underline{P}-\underline{P}')\,\varphi_{P'}^{i_1i_2}(\underline{p})=P^+\,\bar{U}_1^{(i_1)}(\underline{p}_1)\,\bar{U}_2^{(i_2)}(\underline{p}_2)\int d\underline{x}_1\,d\underline{x}_2\,e^{i\sum \underline{p}_j\underline{x}_j}\,\langle 0|\psi_1(\underline{x}_1)\psi_2(\underline{x}_2)|P'\rangle.$$

Taking into account (28) in (25), we deduce the equation for the bound states

$$\left(M^2+P_\perp^2-\sum_{i=1}^{2}\frac{p_{i1}^2+m_i^2}{\eta_i}\right)\varphi_P^{i_1i_2}(\underline{p})=\frac{1}{\eta_1\eta_2}\sum_{j_1j_2}\sum_{i_1i_2j_1j_2}V^{(2)}(P,\underline{p},\underline{p}')\,dp_\perp'\,d\eta'\cdot\varphi_P^{j_1j_2}(\underline{p}')$$

and the normalization condition

$$\int d\eta\,dp_\perp\,\eta_1\eta_2\sum_{i_1i_2}\bar{\varphi}_P^{i_1i_2}(\underline{p})\,\varphi_P^{i_1i_2}(\underline{p})-\left(\bar{\varphi}\,\frac{\partial V^{(2)}}{\partial P^2}\,\varphi\right)=\frac{1}{2(2\pi)^3}\,.$$

5. <u>The Structure of the Quasipotential of the Three-Body System</u>

Let us show that the quasipotential of the three-body system

$$V=g_0^{-1}-g^{-1} \tag{29}$$

is represented in the form

$$V=\sum_{i=1}^{3}\eta_i\,\delta^{(2)}(p_{i\perp}-p'_{i\perp})\,\delta(\eta_i-\eta'_i)\,V_i^{(2)}(P-\bar p_i;p_{kl},p'_{kl})+V_T\,, \qquad (30)$$

where $V_i^{(2)}$ are the two-particle quasipotentials of the interaction of k-th and l-th particles ($i\neq k\neq l\neq i$) introduced in the previous section, V_T corresponds to purely three-particle forces. Here,for the sake of definiteness we introduce the momentum variables of the Jacobi i-th system

$$P=p_1+p_2+p_3\,,\quad p_{ikl}=\frac{m_i(p_k+p_l)-(m_k+m_l)p_i}{m_1+m_2+m_3}\,,\quad p_{kl}=\frac{m_l p_k-m_k p_l}{m_l+m_k}$$

$$\bar p_i=\left(\bar p_i^-=\frac{p_{i\perp}^2+m_i^2}{p_i^+},\ p_i^+,\ p_{i\perp}\right)$$

(i,k,l) - is the cyclic permutation from (1,2,3).

Really, the three-dimensional Green function of three particles has the structure represented by the following diagrams

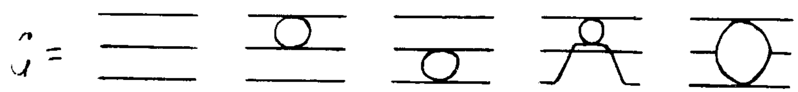

Fig. 1

where the blocks correspond to the sum of all connected diagrams. To the transition to the quasipotential Green function in the momentum space there corresponds the integration

$$g(P,p,p')=\int dp^-_{ijk}\,dp^-_{jk}\,G(P,p;p')\,dp'^-_{ijk}\,dp'^-_{jk}$$

and the above mentioned projection for all three particles. Let us perform this procedure on one of the two-particle connected diagrams. For the sake of definiteness let the first particle spread freely

$$g_1(P,p;p')=\frac{|P^+|^4}{i\pi^2}\bar U_1(p_1)\bar U_2(p_2)\bar U_3(p_3)\int dp^-_{123}\,dp^-_{23}\,G^{(2)}_{23}(P-p_1;p_{23},p'_{23})$$

$$S_1(p_1)\,\delta^{(4)}(p_1-q_1)\,dp'^-_{123}\,dp'^-_{23}\,U_1(p'_1)U_2(p'_2)U_3(p'_3)$$

$G^{(2)}_{23}$ is the total Green function of the system consisting of the second and third particles. Using the explicit form of the single-particle propagator S_1 and the spectral representation (15) for g_{23}, we have

$$g_{1\,i_1i_2i_3j_1j_2j_3}(P,p;p')=\eta_1^{-1}\delta^{(2)}(p_{1\perp}-p'_{1\perp})\,\delta(\eta_1-\eta'_1)\,g^{(2)}_{23}(P-\bar p_1;p_{23},p'_{23})\,\delta_{i_1j_1}\,. \qquad (31)$$

It is important to note, that the two-particle connected diagrams in the three-body system are simply expressed in terms of the two--particle quasipotential Green function with the shifted variable of the total 4-momentum by complete analogy with the non-relativistic case. In the usual equal-time approach it was necessary to separate additionally the "retarded" part/6/. Due to this fact the pair interactions in (30) exactly coincide with the two-particle quasipotentials. The algorithm of constructing the three-particle forces in terms of the two- and three-particle connected diagrams in quantum field theory on the null plane follows from definition (29) and is given in paper/6/ in a symbolic form.

6. The Wave Function of the Bound States of Three Particles

The quasipotential Green function of three particles g near the pole of the bound state $P^2 \simeq M^2$ has the form:

$$g(P;\underline{p};\underline{p}') = 4(2\pi)^6 \frac{\chi_P(\underline{p})\,\bar{\chi}_P(\underline{p}')}{P^2-M^2+i\varepsilon} + \cdots ,$$

where $\chi_P(\underline{p})$ is the quasipotential wave function of the bound state of three particles in the momentum representation

$$(2\pi)^9 \delta^{(3)}(P-P')\chi_P^{i_1 i_2 i_3}(\underline{p}) = |P'|^2 \bar{U}_1^{(i_1)}(\underline{p}_1)\bar{U}_2^{(i_2)}(\underline{p}_2)\bar{U}_3^{(i_3)}(\underline{p}_3)\int e^{i\underline{p}_j \underline{x}_j} \langle 0|\prod_{j=1}^{3}\psi_j(\underline{x}_j)|P'\rangle \prod_{j=1}^{3} d\underline{x}_j^3 . \quad (31)$$

Considering the equation for g, $g = g_0 + g_0 V g$ near this pole, we derive the equation for the wave function (31)

$$\left(P^2+P_\perp^2-\sum_{i=1}^{3}\frac{p_{i\perp}^2+m_i^2}{\eta_i}\right)\chi_P^{i_1 i_2 i_3}(\underline{p}) = \frac{1}{\eta_1\eta_2\eta_3}\int V^{i_1 i_2 i_3 j_1 j_2 j_3}(P;\underline{p},\underline{p}')\,dp'_{1\perp}dp'_{2\perp}d\eta'_1 d\eta'_2\,\chi_P^{j_1 j_2 j_3}(\underline{p}') \quad (33)$$

and the normalization condition

$$\sum_{i_1 i_2 i_3}\int |\chi_P^{i_1 i_2 i_3}(\underline{p})|^2 \eta_1\eta_2\eta_3\, dp_{1\perp}dp_{2\perp}d\eta_1 d\eta_2 + \left(\bar{\chi}_P \frac{\partial V}{\partial P^2}\chi_P\right) = \frac{1}{4(2\pi)^6} .$$

As is known, eq. (32) is not mathematically correct due to singularity of kernel, thus, we reduce it to the form of the Faddeev equation . In the approximation of pair interaction, we have $\chi_P = \sum_{i=1}^{3}\chi_i$

$$\left(P^2-\sum_{i=1}^{3}\frac{(p_{i\perp}-\eta_i P_\perp)^2+m_i^2}{\eta_i}\right)\chi_i(\underline{p}) = \sum_{j\neq i}\int \frac{1}{\eta_j\eta_k} T_i^{(2)}(P-\underline{p}_i;\underline{p}_{jk}\underline{p}'_{jk})\,\delta^{(2)}(p_{i\perp}-p'_{i\perp})\,\delta(\eta_i-\eta'_i)\,\chi_j(\underline{p}) .$$

where $T_i^{(2)}(P;\underline{p}\,\underline{p}')$ is the off-shell scattering amplitude of j-th and k-th particles determed by eq. (27). The wave

functions of bound states of two and three particles, introduced above, may be used, in particular, for the investigation of form factors and the scattering amplitudes of relativistic composite systems.

7. The Scattering Amplitudes in the Three-Particle System

In the case of two particles , having the expression for the Green function, one can determine the T-matrix by eq. (26) which gives the on-shell physical scattering amplitude. In the three-body problem, one can construct the analogous T-matrices corresponding to the possible 16 processes. Among these processes there is the elastic scattering of three particles, the quasi-elastic scattering on the bound state and also the disintegration of bound states. In the case of elastic scattering, the T-matrix is determined in a usual way

$$g = g_0 + g_0 T g_0 .$$

On the mass shell

$$P^2 = \sum_{j=1}^{3}\frac{(p_{j\perp}-\eta_j P_\perp)^2+m_j^2}{\eta_j} = \sum_{j=1}^{3}\frac{(q_{j\perp}-\xi_j P_\perp)^2+m_j^2}{\xi_j}$$

it gives the physical amplitude of the elastic scattering of three particles. It will be shown below that the determined "two-time" Green function of three particles has a complete information both on the elastic scattering of three particles and on the processes involving the bound states. In the general case 16 operators of the transition from the state (β) to the state (α)

$\alpha, \beta = 0,1,2,3$ $\quad A_{\alpha\beta}(P^2;\underline{p}_1\underline{p}_2\underline{p}_3;\underline{q}_1\underline{q}_2\underline{q}_3)$

are determined[*)]

$$A_{\alpha\beta} = g_\alpha^{-1}\, g\, g_\beta^{-1} - \delta_{\alpha\beta}\, g_\alpha^{-1}, \tag{34}$$

where g_0 is the free quasipotential Green function of three particles, $g_i \ (i=1,2,3)$ is the two-particle Green function of j-th and k-th interacting particles.

Then the scattering matrices $T_{\alpha\beta}$ corresponding to these transitions $(\beta) \to (\alpha)$ are written as:

$$T_{i'i}(P, \underline{p}_i; \underline{q}_i) = \int \bar{\Phi}^{(2)}_{i', P-\underline{p}_{i'}}(\underline{q}_{j'k'})\, d\underline{p}_{j'k'}\, A_{i'i}(P^2, \underline{p}, \underline{q})\, d\underline{q}_{jk}\, \Phi^{(2)}_{i, P-\underline{q}_i}(\underline{q}_{jk}) \tag{35}$$

$$T_{0i}(P^2, \underline{p}_1 \underline{p}_2; \underline{q}_i) = \int A_{0i}(P^2, \underline{p}_1 \underline{p}_2; \underline{q}_i, \underline{q}_{jk})\, d\underline{q}_{jk}\, \Phi^{(2)}_{i, P-\underline{q}_i}(\underline{q}_{jk}),$$

where $\Phi^{(2)}_i$ is the wave function of the bound state of j-th and k-th particles determined in section 4. When passing to the mass shell the above T-matrices coincide with the physical scattering amplitudes. Note, that to the disintegration of the bound state as a result of the collision with the third particle $(12)+3 \to 1+2+3$, there corresponds the following condition of the mass shell:

$$P^2 = \sum_{i=1}^{3} \frac{(p_{i\perp} - \eta_i P_\perp)^2 + m_i^2}{\eta_i} = \frac{(q_{3\perp} - \xi_3 P_\perp)^2 + m_3^2}{\xi_3} + \frac{(q_{3\perp} - \xi_3 P_\perp)^2 + M_{12}^2}{1-\xi_3} \tag{36}$$

---

*) $\alpha = 0$ corresponds to the state of three free particles, $\alpha = 1,2,3$ α-th particle is free, and the rest of two are connected between themselves.

Let us demonstrate the validity of the latter statement by the same process. Really, one can show that the "two-time" Green function of three particles has different poles corresponding to the contribution of any definite initial or final state. In particular, near the pole corresponding to the initial state $(12)+3$ and the final state $1+2+3$, it has the form

$$g(P; \underline{p}; \underline{q}) \sim \frac{F_{03}(\underline{p}_1 \underline{p}_2 \underline{p}_3, \underline{q}_3) \times \bar{\Phi}^{(2)}_{3, P-\underline{q}_3}(\underline{q}_{12})}{\left(P^2 + P_\perp^2 - \sum_{i=1}^{3} \frac{p_{i\perp}^2 + m_i^2}{\eta_i}\right)\left(P^2 + P_\perp^2 - \frac{q_{3\perp}^2 + m_3^2}{\xi_3} - \frac{(P_\perp - q_{3\perp})^2 + M_{12}^2}{1-\xi_3}\right)},$$

where F_{03} is the physical amplitude of the considered disintegration process.

On the other hand, having rewritten definition (34) as

$$g = \delta_{\alpha\beta}\, g_\alpha + g_\alpha\, A_{\alpha\beta}\, g_\beta$$

and taking into account the pole contribution of the bound state to the two-particle Green function, one can easily see that the on-shell scattering matrix T_{03} coincides with the physical amplitude on the mass shell (36).

For the transition matrices, one can deduce the equation following from definition (34) and equation for the three-particle "two-time" Green function

$$A_{\alpha\beta} = (1 - \delta_{\alpha\beta})\, g_0^{-1} + \delta_{\beta 0}\, V_T + \sum_{\gamma \neq \beta} A_{\alpha\gamma}\, g_0\, T_\gamma + A_{\alpha 0}\, g_0\, V_T,$$

where $$T_i(P;\underline{p},\underline{q}) = \eta_i\,\delta^{(2)}(p_{i\perp}-q_{i\perp})\,\delta(\eta_i-\xi_i)\,T_i^{(2)}(P-\bar{p}_i;\underline{p}_{jk};\underline{q}_{jk}) \quad i=1,2,3$$

$$T_c = 0 .$$

It is important to note, that the kernels of the obtained system of equations in the approximation of the pair interaction $V_T = 0$ are purely the off-shell two-particle scattering amplitudes. The two-particle quasipotentials do not appear in them in the explicit form.

Note, that the investigation of the structure of the quasipotential for the system of many particles and derivation of many-particle equations can analogously be performed taking into account that we have already obtained the expression for the free Green function of n particles with arbitrary spin.

In the conclusion we should like to note the importance of the transformation properties (19), (20) which we have obtained for the "two-time" Green functions. It is easily seen, that the quasipotentials, off-shell scattering amplitudes as well as the wave functions of the bound states have the analogous properties. Using these properties, one can considerably simplify the problem of definition of the kernels of the three-particle equations in the approximation of pair interactions. In particular, it is sufficient to perform the consideration made in sect.4 in the system where the total transverse momentum of two particles is equal to zero, in spite of the fact that in the three-particle equations it is arbitrary.

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References

1. A.A. Logunov, A.N.Tavkhelidze. Nuovo Cim.29,380 (1963).
2. V.A. Matveev,R.M. Muradyan,A.N.Tavkhelidze.JINR preprints, E2-3498 (1967); P2-3900 (1968), Dubna;
V.G. Kadyshevsky. Nucl. Phys. E6, 125 (1968);
C. Fronsdal, L.E. Lundberg. Phys. Rev. D1, 3247 (1970).
А.А. Логунов, В.И. Саврин, Н.Е. Тюрин, О.А. Хрусталев. ТМФ, 6, 157. 1971.
I. Todorov. Phys. Rev. D3, 2351 (1971).
R.N. Faustov. Ann. Phys. 78, 176 (1973).
В.Р.Гарсеванишвили, А.Н.Квинихидзе,В.А.Матвеев, А.Н.Тавхелидзе, Р.Н. Фаустов. ТМФ, 23, 3 (1975);
А.А.Хелашвили. Препринт ОИЯИ Р2- 8750, Дубна (1975).
3. D.Stoynov, A.N.Tavkhelidze. Phys. Lett. 13,76 (1964).
V.Shelest,D. Stoyanov. Phys. Lett. 13, 253 (1964).
А.Н.Квинихидзе, Д. Цв. Стоянов. ТМФ, 3, № 3,332 (1970); ТМФ, II, № I, 23 (1973).
В.М. Виноградов. ТМФ, 8, 343 (1971).
А.А. Архипов, В.И.Саврин. Препринт ИТЭФ СТФ 72-19 (1972).
4. S, Weinberg. Phys. Rev. 150, 1313 (1966).
S.P. Kuleshov,A.N. Kvinihidze,V.A. Matveev,A9N9 A.N. Sissakian, L.A.Slepchenko.JINR Preprint E2-8128, Dubna,1974.
J.F.Gunion.S.J.Brodsky,R. Blankenbecler. Phys. Rev. D8, 287 (1973).
5. P.A.M.Dirac. Rev. Mod. Physics. 21, 392 (1949).
F.Rohrlich. Acta Phys. Austr. 32, 87 (1970).
H. Leutwyler. Nucl. Phys. B76, 413 (1974).
6. А.Н. Квинихидзе, Д. Цв. Стоянов, лекции на школе молодых ученых. Сухуми (1972); Препринт ОИЯИ Р2-8667, стр. 215, Дубна, 1972.

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