ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

D1 - 4001

K.D.Tolstov

ЛАБОРАТОРИЯ ВЫСОКИХ ЭНЕРГИЙ

A POSSIBLE EXPLANATION OF THE CROSS SECTIONS OF THE P-P ELASTIC SCATTERING AT HIGH ENERGIES IN THE ANGLE INTERVAL FROM $0<\Theta<90°$

1968

D1 - 4001

K.D.Tolstov

A POSSIBLE EXPLANATION OF THE CROSS SECTIONS OF THE P-P ELASTIC SCATTERING AT HIGH ENERGIES IN THE ANGLE INTERVAL FROM $0<\Theta<90°$

Аннотация

Показано, что упругое p-p рассеяние при высоких энергиях в интервале углов $0 < \theta < 90°$ может быть объяснено на основе двух дискретных значений среднеквадратичного поперечного импульса $\langle P_\perp^2 \rangle_1^{1/2} = 0{,}355$ Гэв/с и $\langle P_\perp^2 \rangle_2^{1/2} = 2 \langle P_\perp^2 \rangle_1^{1/2}$. Соответственно с этим интерференция волн с относительным фазовым сдвигом ϕ, от двух областей взаимодействия, приводит для сечений рассеяния к формуле:

$$\frac{d\sigma}{dt} = \left[C_1 \exp\left(\frac{-t + \frac{t^2}{4P^2}}{\langle P_\perp^2 \rangle_1} \right) + C_2 \exp\left(\frac{-t + \frac{t^2}{4P^2}}{4\langle P_\perp^2 \rangle_1} \right) + \right.$$

$$\left. + 2 (C_1 C_2)^{1/2} \cos\phi \exp\left(\frac{-t + \frac{t^2}{4P^2}}{\frac{8}{5}\langle P_\perp^2 \rangle_1} \right) \right] \left(1 - \frac{t}{2P^2} \right),$$

где P -импульс первичного протона; $\phi \approx 120°$, а C_1, C_2 в единицах 10^{-27} (см^2(Гэв/с)$^{-2}$ $C_1 = 80$; $C_2 = 0{,}2$.

Abstract

The differential cross sections of the P-P elastic scattering measured at high energies and in the angle interval $0 < \theta < 90°$ are described on the basis of two discrete values of a transverse momentum:

$\langle P_\perp^2 \rangle_1^{1/2} = 0.355$ Gev/c $\pm$ 0.01 Gev/c and $\langle P_\perp^2 \rangle_2^{1/2} = 2 \langle P_\perp^2 \rangle_1^{1/2}$.

Accordingly, the interference of waves emitted by two regions of interaction with relative phase shift $-\phi$ leads to the following elastic scattering cross section formula

$$\frac{d\sigma}{dt} = \left[C_1 \exp\left(\frac{-t + \frac{t^2}{4P^2}}{\langle P_\perp^2 \rangle_1} \right) + C_2 \exp\left(\frac{-t + \frac{t^2}{4P^2}}{4\langle P_\perp^2 \rangle_1} \right) + \right.$$

$$\left. + 2 (C_1 C_2)^{1/2} \cdot \cos\phi \exp\left(\frac{-t + \frac{t^2}{4P^2}}{\frac{8}{5}\langle P_\perp^2 \rangle_1} \right) \right] \left(1 - \frac{t}{2P^2} \right),$$

where P is the c.m. primary proton momentum,

$\phi \approx 120°$, $C_1 = 80 \cdot 10^{-27}$ cm^2 (Gev/c)$^{-2}$, $C_2 = 0.2 \cdot 10^{-27}$ cm^2 (Gev/c)$^{-2}$

Some models for an explanation of the experimental results of the P-P elastic scattering have been recently proposed in a series of papers /1-4/.

At present, measurements of differential cross sections are extended to even larger ranges of energy and scattering angles. The accuracy of the experimental data has also been increased. This allows a more exact comparison of the experimental data with theoretical predictions. On the basis of this analysis it is also possible to propose new methods for making the model more precise.

In the angle interval where the real part of the scattering amplitude as well as the Coulomb amplitude are not important, as is seen from /4/, the differential cross sections of the small angles of the elastic scattering of different particles on protons are in good agreement with the formula:

$$\frac{d\sigma}{dt} = C \exp\left(- \frac{P_\perp^2}{\langle P_\perp^2 \rangle} \right) \left(1 - \frac{P_\perp^2}{P^2} \right)^{1/2}, \tag{1}$$

where P is the proton momentum in c.m.s.

Formula (1) corresponds to the Gaussian distribution of the transverse momentum components by coordinate axes.

In the case of the P-P elastic scattering the root-mean-square transverse momentum $\langle P_\perp^2 \rangle^{1/2}$ in (1) appears to be independent of the proton energy in the interval $\approx$ 5-20 Gev and is equal to: $\langle P_\perp^2 \rangle_1^{1/2} = 0.35 \pm 0.01$ Gev/c .

The author has proposed in /4/ that in a range of large angle scattering when $P_\perp \gg 0.35$ Gev/c, $\frac{d\sigma}{dt}$ is also described by formula (1), but with another parameter which is larger $- \langle P_\perp^2 \rangle_2^{1/2}$. The values $\langle P_\perp^2 \rangle_2^{1/2}$ calculated accordingly (1) from any two values of $\frac{d\sigma}{dt}$ from paper /5/fall within the limits $\langle P_\perp^2 \rangle_2^{1/2} = 0.70 \pm 0.05$ Gev/c, i.e. $\langle P_\perp^2 \rangle_2^{1/2} = 2 \langle P_\perp^2 \rangle_1^{1/2}$.

The data of Ref. /6/allow us to perform even more accurate analysis.

In the range of the scattering angles 40-65° the values of $\frac{d\sigma}{dt}$ calculated from formula (1) using $\langle P_\perp^2 \rangle_2^{1/2} = 0.72$ Gev/c and the experimental data for three values of the proton momenta: 10.1, 11.1, 12.1 Gev/c are plotted in Fig.1.

The plot has been normalized to the experimental values at $\langle P_\perp^2 \rangle^{1/2} = 1.49$ Gev/c for a primary momentum of 11.1 Gev/c and $\theta = 43°$. Consequently $\frac{d\sigma}{dt}$ in the range of the scattering angles up to 65° can be described by the formula:

$$\frac{d\sigma}{dt} = \left[C_1 \exp\left(-\frac{P_\perp^2}{\langle P_\perp^2 \rangle}\right) + C_2 \exp\left(-\frac{P_\perp^2}{4\langle P_\perp^2 \rangle}\right)\right]\left(1-\frac{P_\perp^2}{P^2}\right)^{1/2} . \quad (2)$$

The curve for protons with a momentum of 11 Gev/c calculated from formula (2) at $\langle P_\perp^2 \rangle^{1/2} = 0.355$ Gev/c and the experimental data from /6,7,8/ are shown in Fig.2.

It should be stressed that there is an agreement of the calculation with the experimental data over seven orders of magnitude of the cross sections. It should also be remarked that a linear scale was used.

Parameters C_1 and C_2 are equal to 80 and 0.22,respectively, with 10% accuracy in units of 10^{-27} cm ^{2} (Gev/c)$^{-2}$.

An agreement of calculations performed using formula (2) seems to indicate the discrete structure of the values of the root-mean-square transverse momentum because $\langle P_\perp^2 \rangle_2^{1/2} = 2 \langle P_\perp^2 \rangle_1^{1/2}$.

On the basis of the uncertainty principle this can be explained by the existence of two discrete regions of interactions in the P-P elastic scattering.

In this case the wave function after a collision should be a superposition of at least two waves connected with these regions. Because $C_2 \ll C_1$ for $P_\perp \leq 2\langle P_\perp^2 \rangle^{1/2}$ the term containing C_2 in formula (2) can be neglected. At $P_\perp \geq 2 \langle P_\perp^2 \rangle^{1/2}$ term $C_1 \exp\left(\frac{-P_\perp^2}{\langle P_\perp^2 \rangle}\right)$ becomes very small. Consequently at sufficiently small and sufficiently large $P_\perp$ values the interference of waves coming from two discrete interaction regions can be disregarded. On the other hand, the interference should be essential if both terms are equal to each other. If $C_1 = 80$, $C_2 = 0.2$ and $\langle P_\perp^2 \rangle^{1/2} = 0.355$ this happens at $P_\perp = 0.99$ Gev/c.

For the cross section determination in this region one should calculate the square of waves corresponding to $\langle P_\perp^2 \rangle^{1/2}$ and $2\langle P_\perp^2 \rangle^{1/2}$. Obviously a sum or difference of this amplitudes correspond to the extreme cases.

The full lines in Fig.3 were plotted on the basis of the experimental data on P-P scattering at 11 Gev/c coming from Refs. /6,7,8/ in the angle region 24 ÷ 40° there are no experimental data and the dotted lines show the behavior of the $\frac{d\sigma}{dt}$ for the sum and the difference of amplitudes of the partial waves.

Let us obtain the cross sections in a general case of the superposition of partial waves. In accordance with the formula (2) and taking into account the relative phase shift ϕ the partial wave amplitude are evidently equal to

$$A_1(P_\perp) = \left[C_1^{1/2} \exp\left(-\frac{P_\perp^2}{2\langle P_\perp^2 \rangle}\right)\right]\left(1-\frac{P_\perp^2}{P^2}\right)^{1/4}$$

$$A_2(P_\perp) = \left[C_2^{1/2} \exp\left(-\frac{P_\perp^2}{8\langle P_\perp^2 \rangle}\right)\right]\left(1-\frac{P_\perp^2}{P^2}\right)^{1/4} e^{i\phi} .$$

The summary amplitude is

$$A(P_\perp)=\left[C_1^{1/2}\exp\left(-\frac{P_\perp^2}{2\langle P_\perp^2\rangle}\right)+C_2^{1/2}\exp\left(-\frac{P_\perp^2}{8\langle P_\perp^2\rangle}+i\phi\right)\right]\left(1-\frac{P_\perp^2}{P^2}\right)^{1/4} \quad (3)$$

$$\frac{d\sigma}{dt}=\left|A(P_\perp)\right|^2$$

and from (3) we obtain

$$\frac{d\sigma}{dt}=\left[C_1\exp\left(-\frac{P_\perp^2}{\langle P_\perp^2\rangle}\right)+C_2\exp\left(-\frac{P_\perp^2}{4\langle P_\perp^2\rangle}\right)+2(C_1C_2)^{1/2}\cos\phi\,.\right. \quad (4)$$

$$\left.\cdot\exp\left(-\frac{P_\perp^2}{\frac{8}{5}\langle P_\perp^2\rangle}\right)\right]\left(1-\frac{P_\perp^2}{P^2}\right)^{1/2}.$$

The differential cross sections for the region where the interference is important were measured in /9/. In Fig.4 and 5 data from /9/ is shown for protons of momenta 8.5 Gev/c and 12.4 Gev/c correspondingly. The curves were computed by formula (4) for three values of ϕ. The agreement of these curves with experiment is better for $\phi=120^\circ$.

Thus the interval of $P_\perp$ corresponding to the partial wave interference also confirmes the existence of two regions of interaction. The r.m.s. radii of these regions are equal: $\langle r^2\rangle_1^{1/2}=(0.55\pm0.015)$ fer., $\langle r^2\rangle_2^{1/2}=(0.27\pm0.01)$ fer.

At scattering angles close to 90° $\frac{d\sigma}{dt}$ approaches a minimum value for the given energy. The minimum value $-C(E)$ decreases with the increase of energy. Formula (2) gives $\frac{d\sigma}{dt}(90^\circ)=0$ and therefore one can assume that in order to find the $\frac{d\sigma}{dt}$ dependence in the angle range up to 90° it is necessary to add $C(E)$ to $\frac{d\sigma}{dt}$.

Accordingly for the argument t the formula (4) changes to the formula (5)

$$\frac{d\sigma}{dt}=\left[C_1\exp\left(\frac{-t+\frac{t^2}{4P^2}}{\langle P_\perp^2\rangle}\right)+C_2\exp\left(\frac{-t+\frac{t^2}{4P^2}}{4\langle P_\perp^2\rangle}\right)+2(C_1C_2)^{1/2}\cos\phi\right. \quad (5)$$

$$\left.\cdot\exp\left(\frac{-t+\frac{t^2}{4P^2}}{\frac{8}{5}\langle P_\perp^2\rangle}\right)\right]\left(1-\frac{t}{2P^2}\right)+C(E)$$

The values of $\frac{d\sigma}{dt}$ calculated from formula (5) for protons with a momentum of 11.1 Gev/c are shown in Fig.6. The experimental points are plotted from data from /6/ for the scattering angles $40\div80^\circ$.

From formula (5), neglecting relatively small term $C(E)$, one can calculate a total elastic scattering cross section σ_{el}. Making use of the relations $t=4p^2\sin^2\frac{\theta}{2}$ and integrating over θ from 0° to 90° one obtains:

$$\sigma_{el}=\left[C_1+\frac{16}{5}(C_1C_2)^{1/2}\cos\phi+4C_2\right]\langle P_\perp^2\rangle. \quad (6)$$

A present knowledge of the σ_{el} dependence on energy does not allow us to give an unambiguous answer whether formula (6) is exactly right. However we can state that the formula (6) fits the experimental with 10% accuracy in C_1 and $\approx 3\%$ in $\langle P_\perp^2\rangle^{1/2}$.

On the basis of formula (6) one can attempt to compare the cross sections of so called "peripheral" collision with that for a "central" one in $P-P$ inelastic interactions. In fact if the contribution of the wave connected with $\langle P_\perp^2\rangle^{1/2}$ to the σ_{el} is a diffraction shadow of "peripheral" inelastic interactions the relevant cross section σ_p will be proportional to $C_1\langle P_\perp^2\rangle$. Furthermore if the other contribution $\sigma_{e\ell}$ is a diffraction shadow of the "central" inelastic interactions the inelastic "central" cross section σ_c will be proportional to $4\,C_2 < P_\perp^2 >$. Hence putting $C_1 = 80$, $C_2 = 0.2$ one obtains $\sigma_c = 0.01\ \sigma_p$. Denoting $\sigma_{p,c}$ the cross section in the interference region we obtain from (6) $\sigma_{p,c} = 3.2\,(C_1 C_2)^{1/2} \cdot \mathrm{Cos}\,\phi < P_\perp^2 >$.

For $\phi = 120^\circ$, $\sigma_{p,c} = 6.4 < P_\perp^2 > = 0.08\ \sigma_p$.

Qualitatively this is confirmed by a fact that "central" collisions have not been observed in the study of inelastic interactions. In fact there is no rapid change of transverse momentum $< P_\perp^2 >^{1/2}$ and the angular distribution of surviving baryons with the increase of, for example, multiplicity of secondary particles.

In the study of inelastic collisions it is necessary to obtain much better statistics in order to attempt to separate the central collisions, the cross sections of which can be small analogously to the large angle elastic scattering.

References

1. I.Orear. Phys. Lett., 13, 190 (1964).
2. A.D.Krisch. Phys. Rev., 135B, 1456 (1964).
3. K.D.Tolstov. Jadern. Phys., 1, 832 (1965).
4. K.D.Tolstov Izv. AN Ser. Phys., 31, 1480 (1967).
5. I.V.Allaby et al. Phys. Lett., 23, 389 (1966).
6. I.V.Allaby et al. Conf. on High-Energy Collisions of Hadrons CERN January 15-18, 1968.
7. K.I.Foley et al. Phys. Rev. Lett., 11, 425 (1963).
8. L.F.Kirillova. V.A.Nikitin et al. Jadern. Phys., 1, 533 (1965).
9. D.Harting, P.Blackkall et al. Nuovo Cim., 38, 60 (1965).

Received by Publishing Department
on July 25, 1968.

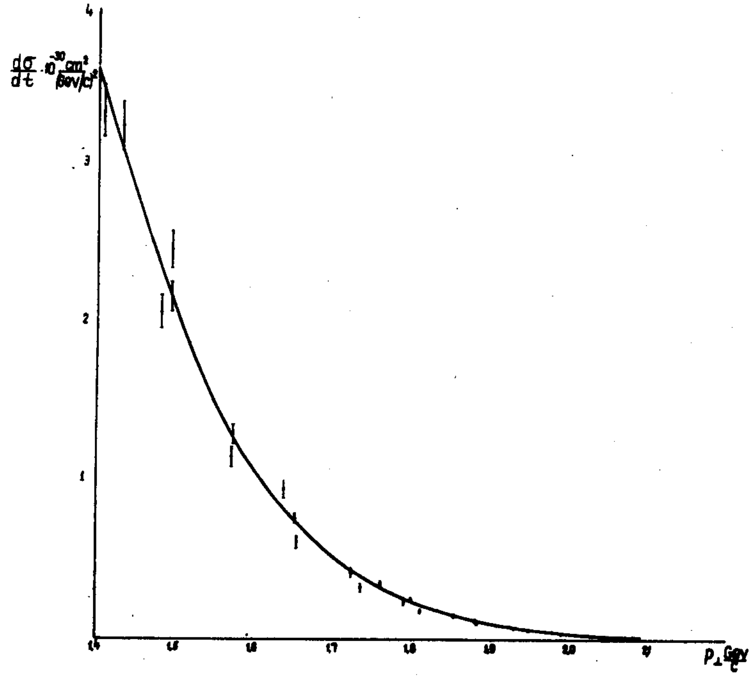

Fig.1. Elastic P-P scattering cross section at 10.1, 11.1, 12.1Gev/c. Experimental points from /6/. The curve is computed according to (2).

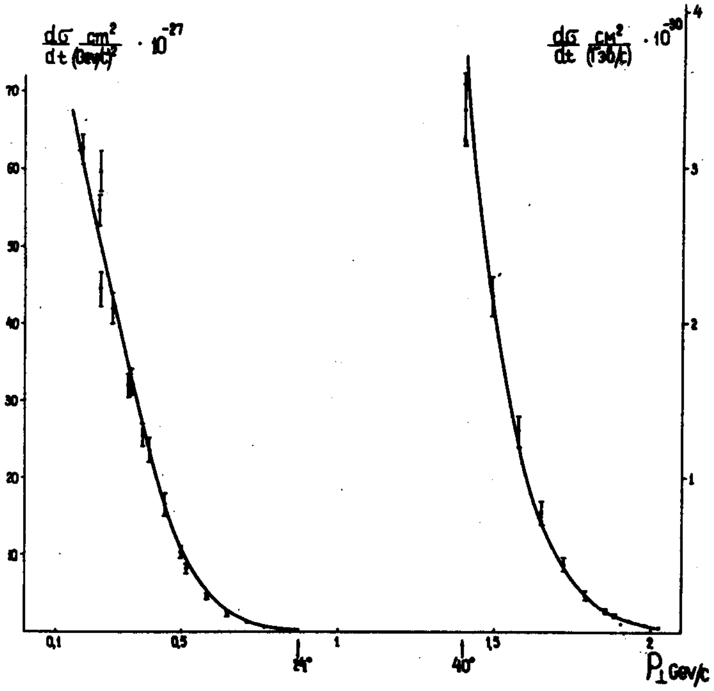

Fig.2. Elastic P-P Scattering cross **section** at 11 Gev/c. Experimental points from /6,7,8/. **The curve is computed according to (2).**

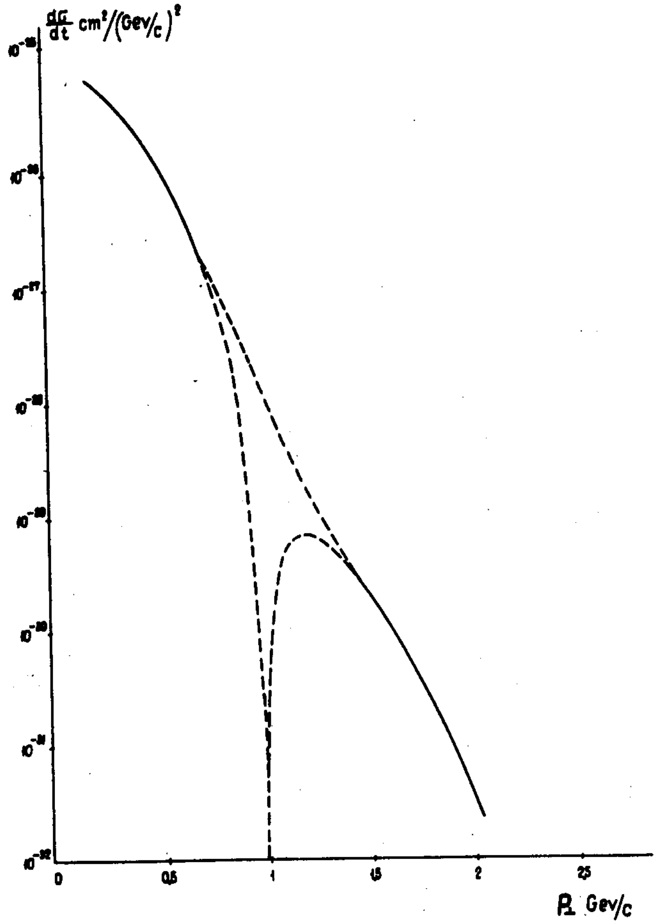

Fig.3. Elastic P-P scattering cross section 2 11.0 Gev/c **Continuous** curves are the same as in Fig.2. Dashed curves are computed according to (4). The upper curve refers to $\phi = 0°$, the lower one - to $\phi = 180°$.

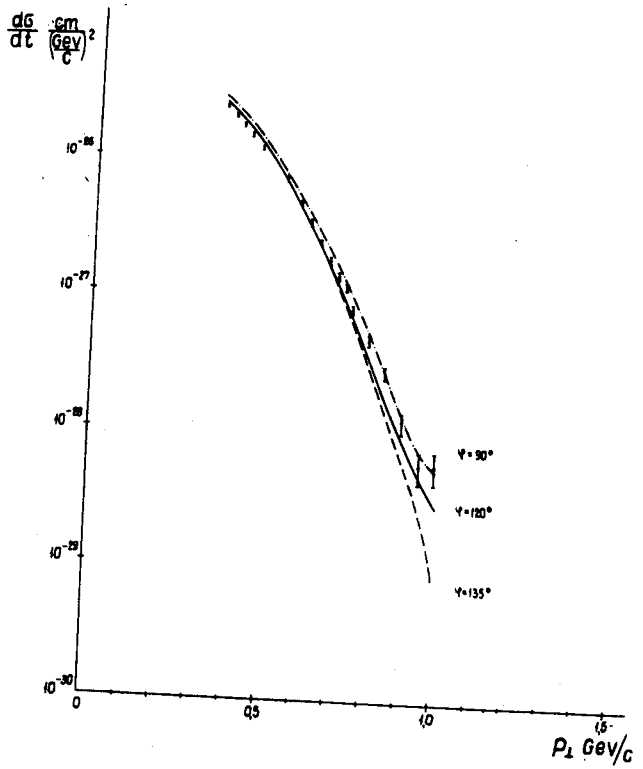

Fig.4. Elastic P-P scattering cross section at 8.5 Gev/c. Experimental points from /6/. The curve is computed according to (4).

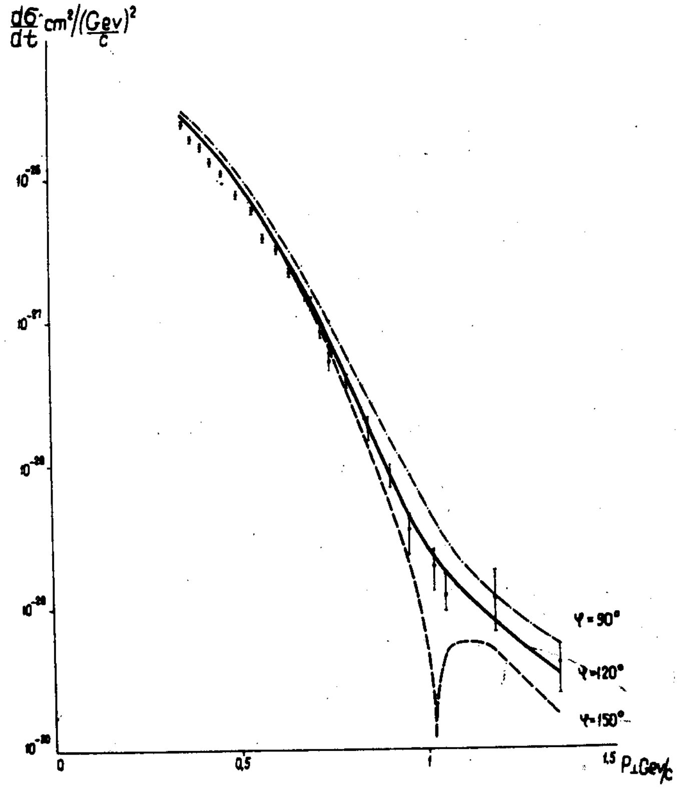

Fig.5. Elastic P-P scattering cross section at 12.4 Gev/c. Experimental points are taken from /6/. The curve is computed according to /4/.

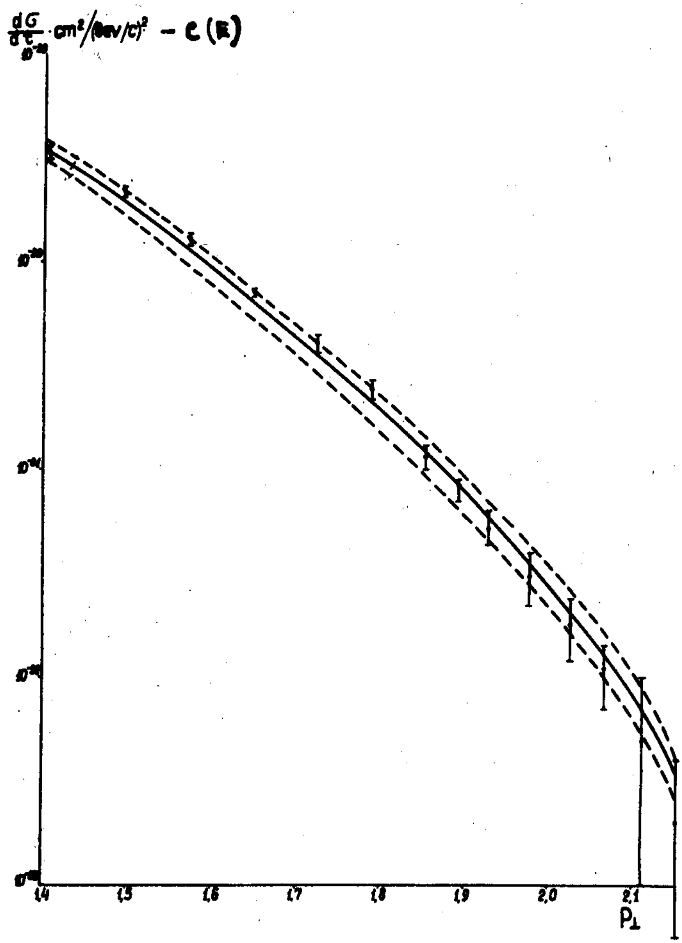

Fig.6. Elastic scattering cross section at 11.0 Gev/c. Experimental points are taken from (6). The continuous curve is computed according to formula (5) at $\langle P_\perp^2\rangle^{1/2}$ =0.355 Gev/c. The upper dashed curve correponds to $\langle P_\perp^2\rangle^{1/2}$ =0.365 Gev/c. the lower one-to $\langle P_\perp^2\rangle^{1/2}$ =0.345 Gev/c.

$$C_1 = 80\cdot 10^{-27}\ \mathrm{cm}^2(\mathrm{Gev/c})^{-2}\ ,\quad C_2 = 0.2\cdot 10^{-27}\,\mathrm{cm}^2(\mathrm{Gev/c})^{-2}$$