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ON THE ASYMPTOTIC BEHAVIOUR OF THE ELASTIC $\pi\pi$ AND πN SCATTERING AMPLITUDE

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ON THE ASYMPTOTIC BEHAVIOUR OF THE ELASTIC $\pi\pi$ AND πN SCATTERING AMPLITUDE

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In a previous paper[1] we studied the asymptotic behaviour of the elastic scattering amplitude of neutral pseudoscalar particles. The present work offers an alternative and more accurate approach to the problem. Let us start once again with the scattering of pseudoscalar particles on each other.

The partial wave amplitudes A_ℓ possess analytic properties in ℓ, similar to those in potential scattering[2]. (This is an almost trivial consequence of the Mandelstam representation for the amplitude and of the properties of the Legendre functions[3]).

Knowing the analytic properties of the A_ℓ , one can perform the same operations is in ref.[2]; thus we find that the asymptotic behaviour in $\cos\vartheta$ of the third channel will be determined by a Regge-pole. By crossing symmetry, we expect that the same pole will dominate the high energy, low momentum transfer region of the first channel.

Let us write down the generalized unitarity condition in elastic approximation in the third channel * .

$$A_{13}(z,t)=\frac{\vartheta(z-z_0)}{4\pi^2}\sqrt{\frac{t-4}{t}}\iint_{z_0}^{\infty}\frac{dz_1dz_2\,\vartheta(k)}{\sqrt{k(z,z_1,z_2)}}A_1^*(z_1,t)\,A_1(z_2,t) \tag{1}$$

$$z\equiv 1+2s(t-4)^{-1}>z_0\equiv 1+8(t-4)^{-1}$$

and assume that for $s\to\infty$,

$$A_1(z,t)\sim\alpha(t)\,P_{L(t)}(z) \tag{2}$$

Here $L(t)$ is the position of the Regge-pole, $\alpha(t)$ – essentially the residue of the partial wave amplitude at the pole L . From the dispersion relations for $s=const$ it follows that $\alpha(t)$ and $L(t)$ must be analytic in the cut t plane, the cut running from $t=4+O(s^{-1})$ to infinity along the real axis.

Let us now divide the integration region in (1) into three parts:

$$\text{I.)}\ z_1=O(1),\ z_2=O(1);\qquad \text{II.)}\ z_1=O(z),\ z_2=O(1)$$

$$\text{and}\ z_1=O(1),\ z_2=O(z);\qquad \text{III.)}\ z_1=O(z),\ z_2=O(z)$$

A closer investigation of (1) together with (2) shows that the largest contribution to the integral comes from the region II. Inserting (2) into (1) for the regions where the integration variables are large, after some calculation we arrive at the expression:

$$A_{13}(z,t)\sim\frac{-1}{2\pi^2}\sqrt{\frac{t-4}{t}}\,\mathrm{Re}\left\{\alpha(t)\,P_L(z)\int_{z_0}^{\infty}dz_1A_1^*(z_1,t)\,Q_L(z_1)\right\} \tag{3}$$

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* We follow the notations of ref.[1].

Taking into account (2), we see that (3) will be satisfied identically in z, if

$$\frac{i}{2\pi^2}\sqrt{\frac{t-4}{t}}\int_{z_0}^{\infty} dz_1 A_1(z_1,t)\, Q_{L^*}(z) = 1 \tag{4}$$

Let us notice that the integral in (3) and (4) is – apart from constant factors – just the generalized partial wave amplitude in the third channel. Introducing the corresponding phase shifts, (4) is equivalent to:

$$\operatorname{ctg}\delta_L(t) = i \tag{4'}$$

(4′) is obviously the condition for a Regge-pole, and knowing $\operatorname{ctg}\delta_L(t)$ as an analytic function of L and t, we can determine therefrom the position of the pole as a function of t.

Going over to πN scattering, we write the invariant amplitude in standard notation as:

$$\mathfrak{M}^{(\pm)} = A^{(\pm)} + \gamma Q\, B^{(\pm)}$$

It follows from the requirement of asymptotic γ_5 – invariance/4/ that for $s \to \infty$ only B will survive. Carrying out analogous operations as those leading to (3) and (4) and taking into account the partial wave expansion of B in the third channel/5/, we find:

$$B_1(s,t) \sim 16\pi^{3/2}\frac{L+1/2}{\sqrt{L(L+1)}}\,\frac{\Gamma(L+1/2)}{\Gamma(L)}\,\tau(t)\, s^{L-1} \qquad /5/$$

Here, $\tau(t)$ is once again the residue of the partial wave amplitude at L and in (5) we have made use of the well-known asymptotic expansion for the Legendre functions. If the total πN cross section is to tend to a constant value at high energies, we must have:

$$\lim_{t\to 0} L(t) = 1.$$

In this case, $B^{(-)} \to 0$, in consequence of the Pomeranchuk-theorem, and $L(t)$ will be determined by an equation, analogous to (4′), with the $I=0$ pion-pion phase shift.

Thus we see that a Regge-pole in the πN amplitude is 'induced' by a corresponding one in the $\pi\pi$ amplitude, through the generalized unitarity condition.

Calculations to determine $L(t)$ are in progress; however, without the detailed knowledge of the latter, we can mention some interesting properties of the theory sketched above.

1. The two-particle unitarity condition does not say anything about the residue of the Regge-pole. In particular, it follows therefrom that $\alpha(t)$ and $\tau(t)$ do not have a branch point at $t=4$ and the values of the total cross sections remain undetermined.

2. The function $L(t)$ is the same for both $\pi\pi$ and πN scattering. For $t<0$, $L(t)$ satisfies the inequality: $0 \leq L \leq 1$ and is a monotonously increasing function of t.

3. $B_1(s,t) \to 0$ for $t \to -\infty$.

4. In the case of a constant total cross section, the total elastic cross section decreases logarithmically with s. (The latter property has been first noticed by Lovelace for a special case [6]).

The above mentioned statements can be easily deduced from the general properties of the scattering amplitudes, and from eqs. (1) $\div$ (5).

Details of these calculations together with numerical results will be published soon.

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