ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ
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1. Small angle elastic proton-proton scattering at 5.2 BeV was studied using the emulsions and employing the technique like in /1-4/. A 10 x 10 x 2 cm^3 chamber made up of type NIKFI-R emulsions was exposed in the Joint Institute synchrophasotron by the internal 5.2 BeV proton beam, the primary particles being incident perpendicular to the emulsion plane. Area scanning was made with an immersion objective at a magnification of 630 x. All the events due to the beam protons, with the number of prongs less or equal to two, were registered. To increase the reliability of the results, as well as the efficiency of finding the events, these emulsions were scanned twice. A total of 722 events were detected which outwardly resembled elastic scattering. However, after all the events had been reconsidered, most of them were excluded at once since they did not satisfy the elastic scattering criteria, while 235 events were measured with particular care. The accuracy of measureing the scattering angle and the angle of the recoil proton comprise approximately 3 and 1.5°, respectively. The measurement procedure was described in detail in /1/. As a result of measurements, 113 elastic scattering events at 5.2 BeV have been selected (the energy of the primary particle beam was made more accurate by means of the kinematics of the selected events).

The intensity of the primary particle beam amounted to $(4.88 \pm 0.10).10^5$ cm^{-2}. The kinematic measurements of the events have shown that in the primary beam there is an admixture of protons with lower energies. This admixture has been determined to be $(0.37 \pm 0.10).10^5$ cm^{-2} in the manner described in /4/.

The scanning we carried out provided for the high efficiency of registering the elastic scattering events in the small scattering angle region of interest. It is well-known (see, e.g., /5/) that if the effectiveness of finding the events is not the same, the true number of events determined according to the results of the double scanning is underestimated. This systematic error is small if the efficiencies of recording different events are high, though not the same, since in this case the difference in the efficiencies cannot be great. Therefore, the area, where the scanning efficiency turned out to be not very high for some reasons, was scanned for the third time, and in calculating the true number of the events the results of the two worse scannings were combined. In the scattering angle range 4.5°–10.5° (c.m.s.), the efficiency of finding the events was found to be 0.84 as a result of the two first scannings, and – 0.96 after the three scannings. This means that the true number of events in this angular interval which was calculated according to the triple scanning is about 9% more than that calculated by the results of the double scanning. This divergence determined a possible systematic inaccuracy in similar experiments when the effectiveness of finding the events is not too high (~ 0.85). The working efficiencies in different angle intervals were found to be 0.95–0.99, and one can hope that the indicated inaccuracy was negligibly small.

When the true number of events was sought, the cases located at a distance of less than 20 μ from the glass and the surface (in the undeveloped emulsion) were ruled out from the analysis. The question concerning the efficiency of recording the events located under the marker lines was also considered. It was found that the number of such events amounts to (6.2 ± 2.0) % of the total number, while the marking occupies $\sim 6.5\%$ of the entire area. This indicates that systematical missing of events due to marking is not essential .

The statistical fluctuations of the efficiency and of the true number of events were calculated by the formulae given in[6]. Let N_1 be the number of events of the given type which were found in the first scanning, N_2 - the number of events obtained in the second scanning, and N_{12} -the number of events found both in the first and the second scannings ('coincidences'). Then, if the events of the given type be equally well recorded, for determining the real number of events and the efficiency of finding them , one can use the following expressions

$$N = \frac{N_1 N_2}{N_{12}} \quad .$$

$$\epsilon = \frac{N_{12}}{N_1} + \frac{N_{12}}{N_2} - \frac{N_{12}^2}{N_1 N_2} \quad .$$

For the fluctuations of the magnitudes of N and ϵ , the expressions have been obtained[6]

$$\sqrt{\frac{\overline{(\Delta N)^2}}{N^2}} = \sqrt{\frac{(N_1 - N_{12})(N_2 - N_{12})}{N_1 N_2 N_{12}} + \frac{1}{N}} \quad ;$$

$$\sqrt{\overline{(\Delta \epsilon)^2}} = \sqrt{N_{12}^2 [a^3 + b^3 + (a+b)^2 c - 2(a+b)(a^2 + b^2)]} \, ,$$

where $a = \frac{1-\epsilon_1}{N_1}$, $b = \frac{1-\epsilon_2}{N_2}$, $c = \frac{1-\epsilon_1 \epsilon_2}{N_{12}}$.

The efficiencies of finding the events for different scattering angle intervals (this experiment) are listed in Table 1.

According to the NIKFI data[7] the hydrogen content was estimated to be $(2.95 \pm 0.23) . 10^{22}$ atoms per cubic centimeter of exposed emulsion.

The differential cross section obtained is presented in Table 1. Since the efficiency of recording the events for the angles larger than 10.5^o in the c.m.s. drops essentially, the data on the differential cross section in this angle region are not given. The elastic scattering cross section in the angle interval 1.8^o–10.5^o in the c.m.s. at 5.2 BeV was found to be (5.2 ± 0.5) mbs.

Table 1.

θ c.m.s.	$1.8^{\circ} - 4.5^{\circ}$	$4.5^{\circ} - 6.5^{\circ}$	$6.5^{\circ} - 8.5^{\circ}$	$8.5^{\circ} - 10.5^{\circ}$
Recording efficiency	0.977 ± 0.019	0.992 ± 0.014	0.934 ± 0.034	0.956 ± 0.026
Differential cross section mb/ster.	78.1 ± 17.0	57.0 ± 13.0	50.7 ± 11.0	$35,9 \pm 7.7$

The data the differential cross sections obtained in this paper were compared with results of [8] and [9], in which the elastic pp scattering at 6.2 BeV was studied. To this end, the data $\frac{1}{k^2} \frac{d\sigma}{d\Omega}$ were plotted against the transverse momentum P. It has been shown [4], that for the small transverse momenta, the difference in the magnitude of $\frac{1}{k^2} \frac{d\sigma}{d\Omega}$ for different energies (2–9 BeV) is not great The data of the present paper are in satisfactory agreement with the results [9], where the differential cross section was measured in the same region of the transverse moments.

A much more underestimated value of the differential cross section has been obtained in [8] for the point at an angle of 7.6° in the c.m.s. May be, this is connected with the difficulties we face in recording small angle scattering events by the technique used in [8]. According to the results of [8],[9] and of this paper, the total cross section for elastic pp interaction at 6.2 BeV is found to be about (9.7 ± 1.0) mbs.

2. The problem was treated as to whether the approximation, in which the scattering phases are considered to be purely imaginary and the spin dependence is neglected, can be applied to describe the elastic scattering in the energy range of 5 –6 BeV. For this purpose, a calculation was made according to the optical model which takes into account the Coulomb interaction [3] with purely imaginary and spin independent interaction potential. As for the dependence of the interaction potential on the distance, it was taken to be Gaussian. Using the method of the least squares, we calculated the parameters of the model (the magnitude of the potential and its root-mean-square radius) which are best consistent with the experimental data, and the total cross section for pp interaction was computed. It was found to be (44.5 ± 6.2) mb according to this paper, and (46.4 ± 2.5) mb according to [9]. The value of the experimental measured total cross section of pp interaction is equal to (42.0 ± 1.0) mb [14,15]. A comparison of the calculated and measured total cross section shows that the results of the present paper and those of [9] do not contradict the data of [3] and [4] which point out that in describing pp interaction at 2.8 and 8.5 BeV one cannot neglect the real part of the scattering amplitude and its dependence on the spin state.

3. A comparison of the elastic scattering data shows[4], that in the high energy region the values of the differential cross section plotted on the graph $\frac{1}{k^2}\frac{d\sigma}{d\Omega}(P)$ are not described by one curve. At 2.24 BeV[8] up to 8.5 BeV[3], a systematic decrease in the magnitude of $\frac{1}{k^2}\frac{d\sigma}{d\Omega}$ with energy is observed for all P (see Fig. 1). This difference is not great in the region of small P (as alredy mentioned, this was confirmed by the results of the present paper and[9]), in the energy interval of 600–800 MeV/c the magnitudes of $\frac{1}{k^2}\frac{d\sigma}{d\Omega}$ differ by a factor of approximately 5.

One can state, therefore, that the average transverse momentum in elastic pp interaction decreases with energy. This should affect the dimension of the interaction region. We have calculated the root-mean-square radius of the proton-proton interaction for different energies by the data of[3,4,8-10,12]. The calculation was carried out in the quasi-classical approximation in a manner similar to that of[11,3,4].

A complex spin-independent interaction potential was chosen in the Gaussian form. According to this potential, the proton-proton scattering phases were calculated, as well as the magnitudes of the differention elastic interaction cross section and the total cross section. Using the method of the least squares, the parameters of the potential were chosen which are in best agreement with the experimental data. The total cross section for pp interaction was taken from[13,14,15,16].

In the analysis of the experimental data of[3] we considered two cases. In the first case all the experimental data were made use of, and in the second case we ruled out the differential cross sections at the angles of 15.5°, 17.5°, 19.5°, since they seemed to be not so reliable, as concerns the technique used, as the other points. At the same time, the value of the root-mean-square radius and the magnitude of its error do not practically change. The values of the root- mean-square radius of the proton-proton interaction obtained in the calculations are listed in Table II for different energies. In the same Table are also given the data on the elastic interaction cross section at 6.2 and 2.85 BeV. It can be seen from this Table (see also Fig. 2) that in the framework of the model used the root-mean-square radius of the proton-proton interaction increases with energy. This conclusion is likely independent of which model is used. This can be understood from the fact that if the real part of the scattering amplitude is essentially smaller than the imaginary one, the elastic scattering cross section in the energy region under consideration drops rather rapidly while the total cross section for pp interaction remains practically constant.

Table II.

El., Bev	σ_{el}, mb	reference	$\langle r^2 \rangle^{1/2}$ (fermi)
2,24	–	/8/	0,69 ± 0,02
2,85	18,0 ± 0,8*	/4/	0,70 ∓ 0,03
		/10/	
3,0	–	/12/	0,75 ± 0,11
4,40	–	/8/	0,84 ± 0,04
6,20	9,7 ± 1,0	/8/ /9/ and the present paper	0,91 ± 0,04
8,5	8,7 ± 0,4	/3/	1,20 ± 0,05

* The value (18 – 0.8) mbs has been obtained by combining the data of[4] and[10].

An increase of the interaction radius and a decrease in the elastic scattering cross section with ener-energy is likely to point out that the contribution from the multi-pion exchange becomes smaller if compared with the exchange between, for instance, two pions in the nucleon-nucleon scattering.

It should be emphasized that if in the high energy region the behaviour of the total and elastic pp interaction cross sections is similar to that at 2-9 BeV, then it is very likely, that the asymptotic behaviour of the scattering amplitude is such as it follows from Gribov's paper/17/.

4. An analysis was made of the data on elastic pp interaction at 8.5 BeV/3/ to choose the radial dependence of the optical potential which would satisfy the experimental data best of all. For this purpose the purely imaginary potential was chosen to be

$$w(r) = \begin{cases} w_0, & 0 \le r \le r_{01} \\ w_0 \exp\left[-\dfrac{(r-r_{01})^2}{2r_{02}^2}\right], & r > r_{01}. \end{cases}$$

For $r_{01} = 0$, this potential has the purely Gaussian form, in the case of $r_{02} = 0$ this is a homogeneous sphere, in the intermediate cases - this a homogeneous sphere with the Gaussian decreasing on the edge. By the formulas of the optical model including the Coulomb interaction (by analogy with/3/), the differential elastic scattering cross section was calculated. Using the method of least squares, the parameters of the model were chosen which are in best agreement with the experimental data. According to the chosen parameters the total cross section and the root-mean-square interaction radius were calculated.

Best of all, the experimental data are satisfied by the variant $r_{01} = 0$ ($r_{02} = 0.653$ fermi), i.e., the purely Gaussian potential. The sum of the variations $\chi^2 =$ 7.63 for $\bar{\chi}^2 = 7$ ($\bar{\chi}^2 = n - m$, where n is the number of experimental points, m is the number of non-fixed parameters of the model). If r_{02} is taken to be 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0 (fermi), then r_{01} is chosen to be 0.192, 0.479, 0.715, 0.933, 1.085, 1.209, 1.397 (fermi). Here χ^2 is equal to 8.47 9.55, 11.85, 11.71, 12.22, 10.55, 11.37, respectively. From these data, one can draw a conclusion that agreement of the experimental data with the potential as a homogeneous sphere is worse than with a purely Gaussian form of the potential.

It should be noted that under the assumptions we have made the root-mean-square interaction radius appears to be independent of the model and equal to (1.10 ± 0.05) fermi. The total pp interaction cross section calculated by the best parameters for different models is found to be practically the same. For the homogeneous sphere it equals (47.04 ± 1.24) mb, for the Gaussian distribution – (45.40 ± 1.49) mb.

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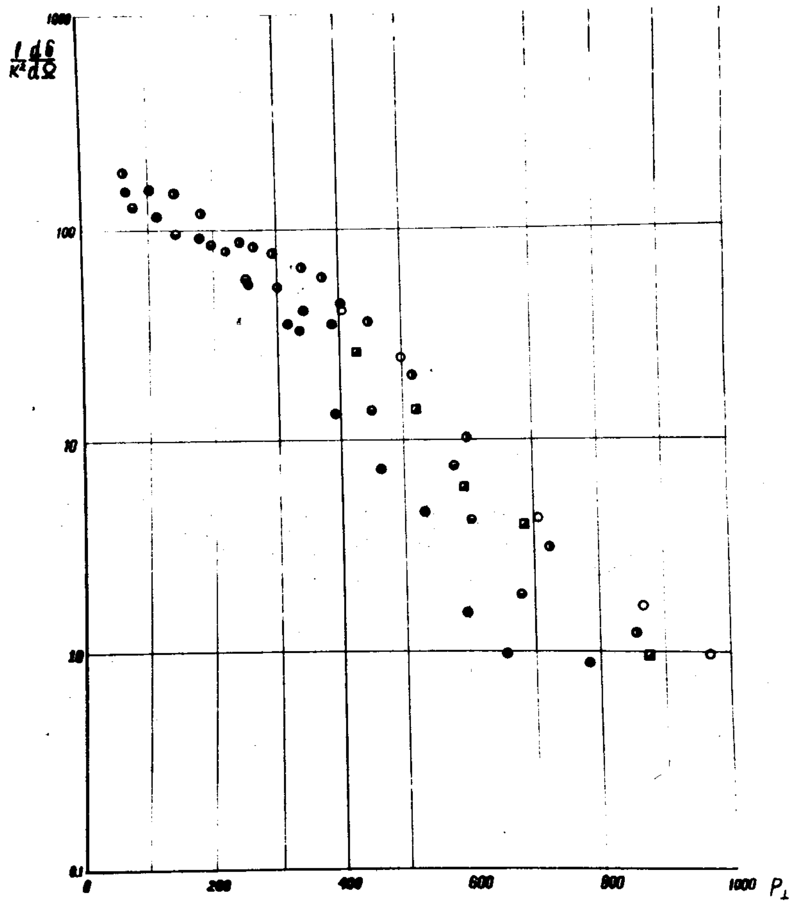

Fig. 1. *Differential elastic pp interaction cross section data*[3,4,8-10] *are plotted on graph* $\frac{1}{k^2}\frac{d\sigma}{d\Omega}(P)$ *, where* k *is the wave number in the c.m.s.,* P *is the transverse momentum.*

o – *2.24 BeV*

● – *2.8 and 2.85 BeV*

▣ – *4.40 BeV*

● – *6.20 BeV*

● – *8.5 BeV*

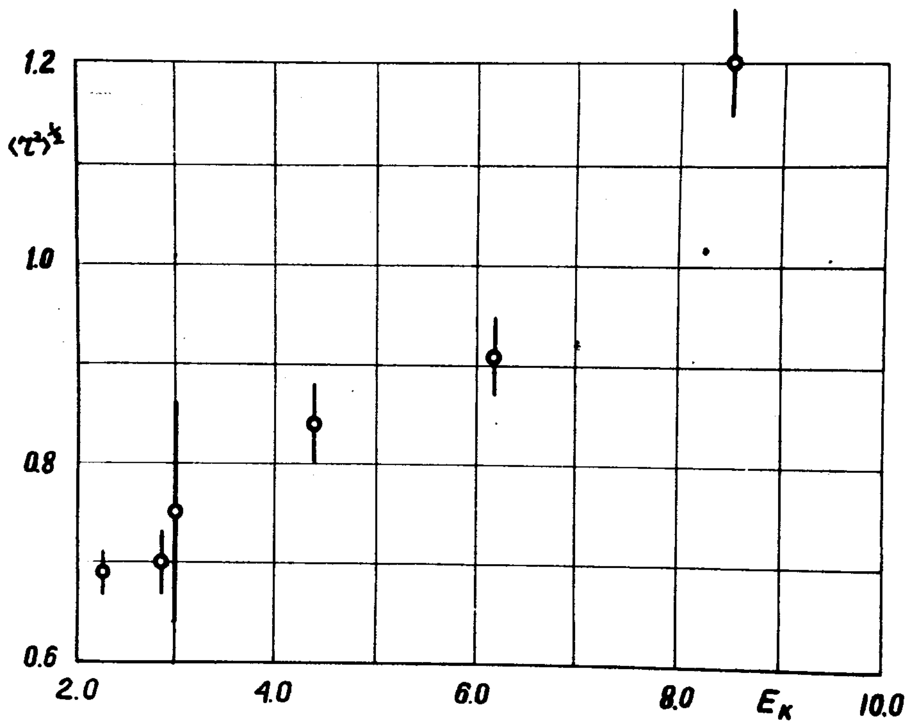

Fig. 2. Root- mean-square radius of pp interaction vs energy. The abscissa axis shows the kinetic energy of the incident proton in the lab.system (BeV), the ordinate axis the root-mean-square radius in fermi.

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