ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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D-860

ON THE KEMMER SCALAR SYMMETRICAL MODEL

ЖЭТФ, 1962, т42, в6, с 1558-1566

Дубна 1962

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D – 860

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Submitted to JETP

The Kemmer scalar symmetrical model is treated by application of the Lappo-Danilevsky's method and the renormalization principles developed for the problems of the field theory with a fixed nucleon. It turns out that (1) the renormalization coupling constant of the model has an upper limit, (2) the connection between the renormalized and unrenormalized interaction constants is finite within the limit of point interaction.

Introduction

The problem of the origin of ultraviolet divergences is fundamental in modern quantum field theory. We shall not here list all possible attempts to solve this problem in the theory itself as well as in its models.

In the present paper we consider the Kemmer scalar symmetrical model /1/ by the Lappo-Danilevsky's method. /2/ This model is interesting because first, it is a renormalizable theory, i.e. in the case of point interaction all the ultraviolet divergences can be removed by the mass and coupling - constant renormalization, secondly, in spite of the absence of the vacuum polarization there arises the 'pole situation' /3,4,5,6/ in the model.

The application of a method different from the usual ones leads therefore to new results which allow one to understand better the structure of the theory within point interaction.

1. The S - Matrix

The Kemmer scalar symmetrical model describes the interaction of scalar mesons with a fixed nucleon having only two isotopic degrees of freedom (proton and neutron).

The Hamiltonian of the model is written in the form

$$H = H_0 + H_I$$

$$H_0 = m + \tfrac{1}{2}\sum_{j=1}^{3} \int d\vec{x} : [\pi_j^2(\vec{x}) + (\vec{\nabla}\phi(\vec{x}))^2 + \mu^2\phi^2(\vec{x})] : \qquad (1)$$

$$H_I = g\sum_{j=1}^{3} \int d\vec{x}\, \rho(\vec{x})\, \tau_j\, \phi_j(\vec{x}) - \delta m$$

where $\pi_j(\vec{x})$ and $\phi_j(\vec{x})$ are the operators of the scalar meson fields, $\rho(\vec{x}) = \sum_{\vec{k}} v(k)\, e^{-i\vec{k}\vec{x}}$ - form factor of a nucleon, τ_j are matrices of the isotopic spin 1/2 (the Pauli matrices), δm is the counterterm responsible for the nucleon 'mass' renormalization.

We write the Hamiltonian H_I in the interaction representation

$$H_I(t) = \sum_{j=1}^{3} g_j\, \tau_j\, \phi_j(t) - \delta m \qquad (2)$$

where

$$\phi_j(t) = \int d\vec{x}\ \rho(\vec{x})\ \phi_j(\vec{x},t) = \sum_{\vec{k}} \frac{v(k)}{\sqrt{2\omega}} [\, a_{j\vec{k}}\, \ell^{-i\omega t} + a^{+}_{j\vec{k}}\, \ell^{i\omega t} \,]$$

To make further analysis more convenient we assume each field ϕ_j to be entered $H_I(t)$ with the interaction constant g_j.

This Hamiltonian belongs to the class of interaction Hamiltonians considered in/7/, so that all conclusions are directly applicable to the model under consideration.

The S-matrix of the scalar symmetrical model in the Lappo-Danilevsky's method can be represented in the form/2/

$$S = \ell^{\,i\frac{\delta m}{a}}\, S^{a} \tag{3}$$

$$S^{a} = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{m_1=0}^{[\frac{n_1}{2}]} \sum_{m_2=0}^{[\frac{n_2}{2}]} \frac{i^{m_1}(-ig_1 r_1)^{n_1}}{2^{m_1} m_1!\,(n_1-2m_1)!} \cdot \frac{i^{m_2}(-ig_2 r_2)^{n_2}}{2^{m_2} m_2!\,(n_2-2m_2)!} \times$$

$$\times \int_{-\infty}^{\infty} d\xi_1 \ldots \int_{-\infty}^{\infty} d\xi_{n_1} \int_{-\infty}^{\infty} d\zeta_1 \ldots \int_{-\infty}^{\infty} d\zeta_{n_2} \prod_{j_1=1}^{n_1} \prod_{j_2=1}^{n_2} \ell^{-a(|\xi_{j_1}|+|\xi_{j_2}|)} \epsilon(\xi_{j_1}-\zeta_{j_2}) \times$$

$$\times \prod_{\mu_1=1}^{m_1} \Delta(\xi_{2\mu_1-1}-\xi_{2\mu_1}) \prod_{\mu_2=1}^{m_2} \Delta(\xi_{2\mu_2-1}-\zeta_{2\mu_2}) : \prod_{\nu_1=2m_1+1}^{n_1} \phi_1(\xi_{\nu_1}) \prod_{\nu_2=2m_2+1}^{n_2} \phi_2(\zeta_{\nu_2}) : \times \tag{4}$$

$$\times :\exp\{-ig_3 r_3 \int_{-\infty}^{\infty} ds\ \ell^{-a|s|} \phi_3(s) \prod_{\ell_1=1}^{n_1} \prod_{\ell_2=1}^{n_2} \epsilon(\xi_{\ell_1}-s)\,\epsilon(\zeta_{\ell_2}-s)\} : \times$$

$$\times \exp\{-\frac{ig^2}{2} \iint_{-\infty}^{\infty} ds_1\, ds_2\ \ell^{-a(|s_1|+|s_2|)} \prod_{k_1=1}^{n_1} \prod_{k_2=1}^{n_2} \epsilon(\xi_{k_1}-s_1)\epsilon(\zeta_{k_2}-s_1)\,\Delta(s_1-s_2)\,\epsilon(\xi_{k_1}-s_2)\epsilon(\zeta_{k_2}-s_2)\}$$

where

$$\Delta(s_1-s_2) = \sum_{\vec{k}} \frac{v^2(k)}{2i\omega}\ \ell^{-i\omega|s_1-s_2|}.$$

The expression for the S^{α} -matrix is symmetrical with respect to the exchange of the indices 1,2,3. This can be seen if we expand S^{α} -matrix in g_3 and change the order of summation.

It is not difficult to understand the physical meaning of the solution (4) if we take into consideration the fact that the operators of charged mesons (π^+ π^0 π^-) are connected with the scalar fields (ϕ_1 ϕ_2 ϕ_3) by the relations

$$\phi^- = 1/\sqrt{2}\,(\phi_1 + i\phi_2); \quad \phi^+ = 1/\sqrt{2}(\phi_1 - i\phi_2); \quad \phi^\circ = \phi_3 \tag{5}$$

where $\phi^-(\phi^+)$ describes the annihilation of negative (possitive) mesons and the creation of positive (negative) ones. ϕ° is the annihilation and the creation of neutral mesons. So, the solution (4) is such that the contribution from neutral mesons π^0 in taken into account accurately and the expansion is performed in the charged-meson fields.

The renormalization of the Model

In view of the absence of the vacuum polarization in the model only the nucleon 'mass' and the coupling constant are subjected to the renormalization.

The mass renormalization, is performed as is shown in[7], by a simple division by the phase $\langle N|\ S^{\alpha}\ |N\rangle$. The difference lies only in the fact that now the expansion is carried out in g_1 and g_2 with coefficients depending on g_3. The difference is unessential and when calculating the formulas from the paper[7] can be used. Since the constants g_1 and g_2 enter the S-matrix (4) explicitly symmetrically we assume tham to be equal to $g_1 = g_2 = g$.

The coupling constant renormalization is performed in the following way. After the mass renormalization (division by phase) has been performed, the matrix element of any process is a function of the energy and isotopic spin variables ν , the coupling constants $g_1 = g_2 = g$, g_3 and the cuttoff momentum L

$$M = M(\nu, g, g_3, L). \tag{6}$$

In the Lappo-Danilevsky's method (when using the S -matrix (4)) the matrix element M is represented by the power series

$$M = g^m g_3^{m_3} \sum_{n=0}^{\infty} g^{2n} M_n(\nu, g_3^2, L) \tag{7}$$

where m and m_3 are positive integers depending on the process under consideration.

Unlike perturbation theory where in the limit of point interaction $(L \to \infty)$ the integrals in M diverges logarithmically, the coefficients M_n in (7) for $L \to \infty$ tend to zero. For example, the first term of (7) for the elastic scattering amplitude of π°-meson is of the form:

$$M_0(\omega_f) = \frac{16\, g^2 g_3^2}{i \omega_f^2} \exp\{-2 g_3^2 \sum_{\vec{k}} \frac{v^2(k)}{\omega^3}\} \int_0^{\infty} dx\, (1 - \cos \omega_f x) \Delta(x) \exp\{2 g_3^2 \sum_{\vec{k}} \frac{v^2(k)}{\omega^3}\, e^{-i\omega x}\} \qquad (8)$$

where ω_f is the energy of a scattered meson. For $L \to \infty$ M tends to zero as $\exp\{-g^2/\pi^2 \ln L\}$.

A similar situation takes place for the other terms of this series and for amplitudes of other processes. So, the task of the renormalization theory in the Lappo-Danilevsky's method consists in the removal of zeros but not infinities.

The renormalizability of the theory means, as was pointed out in /7/, that the matrix element M must be of the form

$$M = M^{(r)}(\nu, g_r, g_{3r}) = g_r^{m} g_{3r}^{m_3} \sum_{n=0}^{\infty} g_r^{2n} M_n^{(r)}(\nu, g_{3r}). \qquad (9)$$

It is necessary to find $M_n^{(r)}$. This can be done easily, since the dependence of the renormalizable coupling constants g_{jr} on g_j and L is known:

$$g_{jr} \langle N_1 | r_j | N_2 \rangle = g_j \langle N_1 | T(r_j(0) S) | N \rangle = g_j \lim_{a \to 0} \frac{\langle N_1 | T(r_j(0)\, S^a) | N_2 \rangle}{\langle N | S^a | N \rangle} \qquad (10)$$

from here

$$g_r = g_{1r} = g_{2r} = g \sum_{n=0}^{\infty} g^{2n} C_n(g_3^2, L) \qquad (11)$$

$$g_{3r} = g_3 \sum_{n=0}^{\infty} g^{2n} D_n(g_3^2, L). \qquad (12)$$

By inserting now (11) and (12) into (9) and comparing it with (7), we obtain the equality:

$$\sum_{n=0}^{\infty} g^{2n} M_n(\nu, g_3^2, L) = \Big(\sum_{k=0}^{\infty} g^{2k} D_k(g_3^2, L)\Big)^{m_3} \times$$
$$\times \sum_{n=0}^{\infty} g^{2n} \Big(\sum_{\ell=0}^{\infty} g^{2\ell} C_\ell(g_3^2, L)\Big)^{2n+m} M_n^{(r)}\Big(\nu, g_3^2 \Big[\sum_{m=0}^{\infty} g^{2m} D_m(g_3^2, L)\Big]^2\Big) \qquad (13)$$

By expanding the right hand side in a power series in g and equaling the coefficients for equal powers of g^2 we obtain the system of linking equations which connect the known functions M_n, C_k, D_m with unknown ones $M_n^{(r)}$ and their derivatives. This set of equations is solvable and the functions $M_n^{(r)}$ can be consequently found (See Appendix).

So, we are able to represent the renormalized matrix element of any process as a series partially summed up over the renormalized coupling constant.

We apply the aforementioned renormalization procedure to the meson-nucleon elastic scattering amplitude. We shall consider the scattering amplitudes of the particles ϕ_1 ϕ_2 ϕ_3 since it is more convenient in our approach and the scattering amplitudes of the charged particles π^+ π^0 π^- are simply their linear combinations. By restricting ourselves to the first terms of the series*, we get

$$f_{1-1}(\omega_f) = f_{2-2}(\omega_f) = 2 g_r^2 i \int^{\infty} dx \; \mathrm{Cos}\, \omega_f x \; I(x) + \ldots$$

$$f_{3-3}(\omega_f) = \frac{16 g_r^4}{\omega_f^2} \int_0^{\infty} dx \, (1 - \mathrm{Cos}\, \omega_j x) \, \Delta(x) \; I(x) + \ldots \qquad (14)$$

$$f_{1-3}(\omega_f) = -\tau_1 \tau_3 \{ \frac{2 g_r^2}{\omega_f} - \frac{8 g_r^4}{i \omega_f} \iint_0^{\infty} dx_1 \, dx_2 [(\mathrm{Cos}\, \omega_f x_2 - \mathrm{Cos}\, \omega_j (x_1 + x_2)) \times$$

$$\times \Delta(x_1) I(x_1) \left(\frac{I(x_1)}{I(x_1 + x_2)} - 1 \right) - (1 - \mathrm{Cos}\, \omega_f x_1) \Delta(x_1 + x_2) I(x_1 + x_2)] + \ldots$$

where

$$I(x) = \exp \{ 2 g_r^2 \sum_{\vec{k}} \frac{e^{-i\omega x}}{\omega^3} \} \qquad (15)$$

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* The amplitudes $f(\omega)$ are connected with the cross section by the relation : $\sigma(\omega) = 1/4\pi \, |f(\omega)|^2$

Here we put $g_{3r}^2 = g_r^2$.

First of all and what is evident the amplitude f_{3-3} differs from the amplitudes f_{1-1} and f_{2-2} and f_{1-3} from f_{1-2}. This contradicts the isotopic invariance of the model. As it has already been pointed out this fact follows from the apparent non-symmetry of the S-matrix (4) in ϕ_1 -, ϕ_2 - and ϕ_3 - fields. Of course, if we consider total series for the amplitude f_{1-1} and f_{3-3} they are to be coincident. It is difficult to say which approximation is better f_{1-1} or f_{3-3} especially as the model is of a purely methodical interest, it is therefore impossible to make use of any physical arguments .

The following conclusions may be drawn on the basis of these approximations.*

The integrals converge for small x in (14) provided

$$g_r^2 / \pi^2 < 1 . \qquad (16)$$

Thus, it turns out that renormalized coupling constant is restricted. This result agrees with the Khalfin's conclusion[8] who has shown that the restriction to g_r can appear if the amplitude consists of a finite number of partial waves. In the model under consideration there is only the S-scattering.

We note also that for high energies the amplitudes (14)* have the same behaviour.

$$f(\omega_f) \sim \omega_f^{-1+g_r^2/\pi^2} \qquad (\omega_f >> 1).$$

With increasing energy the amplitudes fall off more slowly than in the case of perturbation theory.

The Renormalized Coupling Constant

We consider more in detail the connection between 'bare' g and 'observable' g_r interaction constants. This connection is represented by the relation (10) . Since the series for the S^a -matrix (4) is apparently non-symmetrical with respect to indices 1,2 and 3, then depending on the choice of r_1 , r_2 or r_3 in (10) series for g_r (11) and (12) will turn out to be apparently different but their sum is the same. Here we may speak about a successful or ansuccessful choice of partial summation (which field ϕ_1 ϕ_2 or ϕ_3 is considered rigorously in the expression for the S -matrix).

We consider first the expression for g_{3r} (12), i e. we study the effect from a field which is considered rigorously. By calculating[2] we obtain the following expressions for the D_n -coefficients*

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* In paper [2] an error has been made in the formulas (4.10) and (4.11) when calculating the terms of the power series for g_1^4 and g_1^6 which led to a wrong conclusion that in going to the limit $L \to \infty$ the restrictions to g^2 grow from one term of the power series to another.

$$D_0 = 1$$

$$D_1 = -2\,F(0)\int_0^\infty dx\ x\ R(x)$$

$$D_2 = -2F^2(0)\iiint_0^\infty dx_1\,dx_2\,dx_3\{(x_1+x_3)\,R(x_1)\,R(x_3)\,[\frac{F(x_1+x_3)F(x_2+x_3)}{F(x_2)F(x_1+x_2+x_3)}-1]+$$

$$+(x_1+x_3)\,R(x_2)\,R(x_1+x_2+x_3)\,[\frac{F(x_1+x_2)\,F(x_2+x_3)}{F(x_1)\,F(x_3)}-1]- \qquad (17)$$

$$-2\,x_2R(x_2)\,R(x_1+x_2+x_3)-(x_1+2\,x_2+x_3)\,R(x_1+x_3)\,R(x_2+x_3)\}$$

where

$$F(x) = \exp\{-2g^2\sum_{\vec{k}}\frac{v^2(k)}{\omega^3}\,\ell^{-\omega x}\}$$

$$R(x) = \sum_{\vec{k}}\frac{v^2(k)}{\omega}\,\ell^{-\omega x}\cdot F^{-1}(x).$$

A remarkable feature of the formulas (17) consists in the fact that there exists a finite limit for $L\to\infty$

$$\lim_{L\to\infty} D_1 = -\frac{1}{\pi^2\lambda(1+\lambda)}$$

$$\lim_{L\to\infty} D_2 = -\frac{1}{2\pi^4}\iiint_0^\infty dx_1\,dx_2\,dx_3\{\frac{(x_1+x_3)}{(1+x_1)^{2+\lambda}\,(1+x_3)^{2+\lambda}}\,[\left(\frac{(1+x_1+x_2)(1+x_2+x_3)}{(1+x_2)(1+x_1+x_2+x_3)}\right)^{\lambda}-1]+$$

$$+\frac{(x_1+x_3)}{(1+x_1+x_2+x_3)^{2+\lambda}\,(1+x_2)^{2+\lambda}}\,[\left(\frac{(1+x_1+x_2)(1+x_2+x_3)}{(1+x_1)(1+x_3)}\right)^{\lambda}-1]\}+$$

$$+\frac{1}{2\pi^4}\ \frac{3+10\lambda+9\lambda^2}{\lambda^2\,(1+\lambda)^3\,(1+2\lambda)}$$

where

$$\lambda = \frac{g^2}{\pi^2}.$$

Thus, it turns out that the integrals for D_n as functions of g_3^2 have a pole at the point $g_3 = 0$ and, consequently, can not be explanded in Taylor's series. The same situation has been tested up to D_4 .

We failed to study terms of higher order because of great technical difficulties although there exists algorithm for obtaining each term D_n . To all appearence a similar situation takes place in each term of the series.

The limit g_r/g for $g \to 0$ is not equal to unit

$$\lim_{g \to 0} g_r/g = 1 - 1 + ½ - ... \neq 1 \qquad (19)$$

contrary to perturbation theory. The logarithmic divergences may therefore be explained as follows. Let $f(x)$ be some function analytical in $x = 0$ and equal to $f(x) = a \neq 1$

Then, if we seek for this function expansion in Taylor's series in the form

$$f(x) = 1 + \sum_{n=1}^{\infty} a_n x^n$$

then for the coefficients a_n we shall obtain, of course, senseless expressions.

Probably, a similar situation takes place in the model under consideration. The ratio $g_r/g = Z_2 Z_1^{-1}$ having a finite limit for $L \to \infty$ at the point $g = 0$ has a finite value unequal to unit.

This result agrees with the conclusion obtained from the assumption on the completeness of the set of the Hamiltonian eigenfunctions (1). By repeating the calculations of the paper/9/ we get for the considered model

$$g^2 = g_r^2 + 1/\pi \int_{\mu}^{\infty} d\omega_k \, k \, \sigma_{N\pi} (\omega_k) \qquad (20)$$

where $\sigma_{N\pi} (\omega)$ is the total cross section of the reaction $(N\pi)$. Hence, it appears that always $g_r/g < 1$.

Since we failed to find an n-th term of the series (12) the problem of the convengence of this series remains open.

It should be noted that with increase of g the terms D_n decrease and if we assume the series to be convergent then

$$\lim_{g^2 \to \infty} g_r/g = 1. \qquad (21)$$

Drawing an analogy of the series (12) with that of perturbation theory it hould be noted that a partial summation in (4) distributes all the Feynmann graphs among the terms of the Lappo-Danilevsky's series, in this case the same graph can enter the different terms of the series (12) with the coefficient $0 < a < 1$; and in each term of the series it is possible to go to the limit for $L \to \infty$. In the renormalization group method /6,10/ only a part of p graphs is summed up, these are socalled 'main' graphs although the remaining part diverges no less strongly.

Now we consider the expression for $g_r = g_{1r} = g_{2r}$. After the division by phase in (11) has been made we get $C_0 = F(0)$

$$C_1 = F^3(0) \iint_0^\infty dx_1 \, dx_2 \{ R(x_1) [\frac{F(x_1 + x_2)}{F(x_2)} - 1] + R(x_1 + x_2) [\frac{F^2(x_1 + x_2)}{F(x_1) F(x_2)} - 1] \} \qquad (22)$$

For rather large L the ratio g_r/g can be written in the form:

$$g_r/g = 1/L^{\lambda/2} \{ 1 - \frac{\lambda}{2(1+\lambda)} \ln L + \dots \}. \qquad (23)$$

In going over to the limit for $L \to \infty$ each term of this expression is equal to zero. However, this transition is not apparently correct. Indeed, we consider an example

$$1 = L^\lambda / L^\lambda = 1/L^\lambda + \lambda \frac{\ln L}{L^\lambda} + \lambda^2/2 \, \frac{\ln^2 L}{L^\lambda} + \dots \underset{L \to \infty}{\to} 0. \qquad (24)$$

In (23) we can not go to the limit in each term too because of the presence of incertainties of the kind $\ln^n L / L^\lambda$. In going into (17) we had not similar uncertainties.

Thus, nothing can be said about the behaviour of the series (23) in the limit of point interaction as well as nothing can be said basing on the knowledge of some approximations in perturbation theory.

Conclusion

The application of the Lappo-Danilevsky's method to the Kemmer scalar symmetrical model alloved us to obtain two important results:

1. The renormalized coupling constant is restricted to the condition (16). Note, that in investigating the Low equations for this model /11/ the restriction to the coupling constant $\frac{g_r^2}{2\pi} < 1$ is close to our restriction.

2. The connection between g_r and g is finite in the limit of point interaction. The logarithmic divergences in perturbation theory are related to the expansion in the coupling constant g. In this case the function $g_r = g_r(g)$ has no special point for $g = 0$ and $\lim_{g \to 0} g_r/g \neq 1$ as it is assumed in perturbation theory.

It should be noted that these conclusions are based on the assumption that the investigated series converge. But the problem of the convergence of series remains open. It can be said only that the study of higher approximations in the Lappo-Danilevsky's series offers no difficulties in principle, but is connected with cumbersome calculations.

In conclusion I express my gratitude to prof. D.I. Blokhintsev and acad. N.N. Bogolubov for unceasing attention and valuable advice, I thak also B.M. Barbashov for useful discussions.

Appendix

From (13) we have the following recurrent relations

$$M_n^{(r)}(g_3^2) = C_o^{-(2n+m)}(g_3^2) \{ M_n(g_3^2) -$$

$$- \sum_{\substack{p+k+\ell+s=n \\ p<n-1}} T_{ps}(g_3^2) \sum_{k_1+\ldots+k_{m_3}} D_{k_1} \ldots D_{k_{m_3}} \sum_{\ell_1+\ldots+\ell_{2p+m}} C_{\ell_1} \ldots C_{\ell_{2p+m}}$$

where

$$T_{ps}(g_3^2) = \sum_{j+2j_2+\ldots+sj_s=s} g_3^{2(j_1+\ldots+j_s)} \frac{d^{(j_1+\ldots+j_s)}}{d\, g_3^{2(j_1+\ldots+j_s)}} M_p^{(r)}(g_3^2) \times$$

$$\times \frac{(\bar{D}_1(g_3^2))^{j_1}}{j_1!} \frac{(\bar{D}_2(g_3^2))^{j_2}}{j_2!} \ldots \frac{(\bar{D}_s(g_3^2))^{j_s}}{j_s!} .$$

$$\bar{D}_k(g_3^2) = \sum_{k_1+k_2=k} D_{k_1}(g_3^2) D_{k_2}(g_3^2), \qquad D_0 = 1 .$$

The summation is performed over all integral non-negative roots of equations, written down under the summation symbol.

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Received by Publishing Department
on December 16, 1961.