ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

Kim Tse Peng, W. Zoellner

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PION PRODUCTION AT LOW ENERGIES

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The cross-sections for the reactions $\pi^{\pm} + p \rightarrow 2\ \pi + N$ are determined by means of static integral equations, which satisfy the properties of crossing-symmetry. It is possible to explain the increase of the cross-section of the reaction $\pi^{-} + p \rightarrow \pi^{+} + \pi^{-} + n$ at low energies without explicit consideration of the $\pi\pi$ -interaction. Available data for other reactions are discussed.

1. Introduction

The fact, that the fixed-nucleon calculations of Franklin[1], Rodberg[2], and Ka zes[3] for the reaction $\pi^- + p \to \pi^+ + \pi^- + n$ at low energies do not agree with the experimental data is usually interpreted as a consequence of neglecting the $\pi\pi$-interaction. The theoretical cross-sections obtained in these papers differs by one order of magnitude from the experimental values.

In the present paper we will use for the calculation of the cross-sections of the reactions $\pi^+ + p \to 2\pi + N$ solutions of the static integral equations, which were derived by means of the dispersion approach [4,5]. The physical differences of these equations and the Chew-Low-equations for the production process[1-3] are discussed in section II. In section III we consider the method of solution of the integral equations; in section IV the physical results are discussed. It turns out, that it is possible at low ener gies to explain the cross-section for the reaction $\pi^- + p \to \pi^+ + \pi^- + n$ by means of these static equations.

2. The Integral Equations

On the basis of a theory with fixed nucleon-sources in the papers[4,5] equations were obtained for the amplitudes of the process $\pi + N \to 2\pi + N$. The investigations were carried out with the help of the Bogolyubov-formalism of field theory, which allows in an elementary manner to show the existence of static dispersion relations. By means of these dispersion relations it was possible to get equations for the process $\pi + N \to 2\pi + N$, which have intrinsic differences in comparison with the analogous equations[1-3] derived with the help of the Wick-Chew-Low-formalism.

The equations obtained have in every approximation the following symmetry properties:

$$\begin{aligned} T(E'', E'; E) &= T(E', E''; E) \\ &= T(-E, E'; -E'') \\ &= T(E'', -E; -E') \\ &= T(-E; -E', -E'') \end{aligned} \qquad (1a-d)$$

E, E', E'' are the energies of the ingoing and outgoing π-mesons, where $E = E' + E''$. Together with the energies the corresponding spin- and isospin-states must be exchanged.

Note the connection between these symmeties and the analytical properties of the amplitudes. In the papers[4,5] it was shown that the static amplitudes of the process $\pi + N \rightarrow 2\pi + N$ have 6 poles. This is in correspondence with the properties (1 a-d). However, the above mentioned Chew-Low-equations do not reflect these analytic properties of the amplitudes, because they do not have the necessary number of poles.

Furthermore, as a consequence of the dispersion approach the amplitudes of the equations used by on the energy shell, whereas for the Chew-Low-amplitudes this is in general not the case.

In the following we use the notation of paper[5], where

$$E' = \nu' E, \quad E'' = \nu'' E; \quad \epsilon' = \nu' \epsilon, \quad \epsilon'' = \nu'' \epsilon .$$

ν', ν'' are parameters which satisfy the conditions $\nu' + \nu'' = 1; \quad \nu', \nu'' > 0$.

Let us consider the expression E-T (E″, E′; E). The inhomogenous terms of the equations for these quantities are proportional to expressions of the kind

$$\lim_{\epsilon \to 0} \{ \epsilon t(\epsilon'; \epsilon) \} = \tfrac{1}{2} (1/\nu' + 1[\,])(j' j - j j'), \tag{3}$$

where t is the amplitude of the elastic πN-scattering and

$$j = \sqrt{4\pi}\; f/_{\mu}\; r(\vec{\sigma}\; \vec{q}/q) \;. \tag{4}$$

In one-meson-approximation we get the following system of equations:

$$T(E'', E'; E) = {}^{0}I(E'', E'; E) + {}^{1}I(E'', E'; E).$$

The inhomogeneous term ${}^{0}I$ has the form:

$$\begin{aligned} {}^{0}I(E'', E'; E) = & \\ = 1/2E\,(1/E' - 1/E'')\{ & j' j'' j - j'' j' j + j j'' j' - j j' j'' \} + \\ + 1/2E'(1/E'' + 1/E)\{ & j j'' j' - j'' j j' + j' j'' j - j' j j'' \} + \\ + 1/2E''(1/E' + 1/E)\{ & j j' j'' - j' j j'' + j'' j' j - j'' j j' \} . \end{aligned} \tag{6}$$

For the one-meson-term one gets the expression:

$$(2\pi)^3\ {}^1I\,(E'', E'; E\,) =$$

$$= 1/E \int d^3k\, v^2 k^2 \{ \frac{D(\epsilon'',\epsilon';\epsilon)\, d(\epsilon;\epsilon) + A\, a}{\epsilon - E} + \frac{d(\epsilon;\epsilon)\, \overset{*}{D}(\epsilon'',\epsilon';\epsilon) - a\, \overset{*}{A}}{\epsilon + E} \} +$$

$$+ 1/E' \int d^3k\, v'^2 k'^2 \{ \frac{d(\epsilon';\epsilon')\, D(\epsilon'',\epsilon';\epsilon) + aA}{\epsilon' - E'} + \frac{\overset{*}{D}(\epsilon'',\epsilon';\epsilon)\, d(\epsilon';\epsilon') - \overset{*}{A}\, a}{\epsilon' + E'} \} +$$

$$+ 1/E'' \int d^3k\, v''^2 k''^2 \{ \frac{d(\epsilon'';\epsilon'')\, D(\epsilon'',\epsilon';\epsilon) + aA}{\epsilon'' - E''} + \frac{\overset{*}{D}(\epsilon'',\epsilon';\epsilon)\, d(\epsilon'';\epsilon'') - Aa}{\epsilon'' + E''} \}. \quad (7)$$

Here $T = D + iA$, $t = d + i a$; $k^2 = \epsilon^2 - \mu^2$, v is the Fourier-transform of the fixed nucleon-source.

We write T in the following form

$$T = -(4\pi)^{3/2}\,(4\pi/3)^{3/2}\,\{\Sigma\ P^{T\Pi}_{JL}\ T^{T\Pi}_{JL}\}.$$

$T^{T\Pi}_{JL}$ is the matrix-element for a definite spin and isospin transition, where J and T denote the spin and isospin of the amplitude, L and Π denote the spin and the isospin of the system of the two outgoing π -mesons. The projection operators $P^{T\Pi}_{JL}$ are energy independent and normalized to 1.

We present the explicit expressions for the inhomogenous terms of the various transitions:

$${}^{\circ}I^{3/2\ 2}_{3/2\ 2} = 30\,\frac{f^3}{E'E''} \quad (a); \qquad {}^{\circ}I^{3/2\ 2}_{3/2\ 1} = {}^{\circ}I^{3/2\ 2}_{1/2\ 1} = 0 \quad (a,\beta);$$

$${}^{\circ}I^{3/2\ 2}_{1/2\ 0} = -3\sqrt{10}\,\frac{f^3}{E'E''} \quad (b); \qquad {}^{\circ}I^{3/2\ 1}_{1/2\ 0} = 9\sqrt{2}\,\frac{f^3}{E}\left(\frac{1}{E'} - \frac{1}{E''}\right) \quad (\gamma);$$

$$ {}^{\circ}I^{3/2\ 1}_{3/2\ 1} = -6\,\frac{f^3}{E'E''} \qquad (c)\ ; \qquad {}^{\circ}I^{1/2\ 1}_{1/2\ 0} = -18\sqrt{2}\,\frac{f^3}{E}\left(\frac{1}{E'} - \frac{1}{E''}\right) \qquad (\delta)\ ; $$

$$ {}^{\circ}I^{3/2\ 1}_{1/2\ 1} = 12\,\frac{f^3}{E'E''} \qquad (d)\ ; $$

$$ {}^{\circ}I^{1/2\ 1}_{1/2\ 1} = -24\,\frac{f^3}{E'E''} \qquad (e)\ ; $$

$$ {}^{\circ}I^{1/2\ 0}_{1/2\ 0} = -24\,\frac{f^3}{E'E''} \qquad (f)\ . $$

The amplitudes $T^{3/2\,2}_{3/2\,1}$, $T^{3/2\,2}_{1/2\,1}$, $T^{3/2\,1}_{1/2\,0}$, $T^{1/2\,1}_{1/2\,0}$ are antisymmetric in E', E'' and vanish for $E' = E''$ $(\nu' = \nu'' = 1/2)$. In the following we restrict ourselves to the special case $8'\ \nu' = \nu'' = 1/2$ and consider the coupled system of equations for the six amplitudes $T^{3/2\,2}_{3/2\,2}$, $T^{3/2\,2}_{1/2\,0}$, $T^{3/2\,1}_{3/2\,1}$, $T^{3/2\,1}_{1/2\,1}$, $T^{1/2\,1}_{1/2\,1}$, $T^{1/2\,0}_{1/2\,0}$ with the inhomogenous terms $(a - f)$.

3. *Numerical Calculations*

For $\nu' = \nu'' = 1/2$, the system of integral equations can be written schematically in the form $(1 \leq E < +\infty)$:

$$D(E) = \lambda/E^2 + 1/\pi E \; P\int_i^{\infty} d\epsilon \left\{ \frac{\epsilon A(\epsilon)}{\epsilon - E} + \frac{\epsilon B(\epsilon)}{\epsilon + E} \right\} , \qquad (9a)$$

$$A(E) = G^1(E)\, D(E) + G^2(E)\, A(E) , \qquad (9b)$$

$$B(E) = G^3(E)\, D(E) + G^4(E)\, A(E) .$$

P is the symbol for the principal value; D, A, B, λ are vectors with 6 columns, G^i are matrices depending on the amplitudes of the elastic π N -scattering. We have used here the Salzman-solutions[6] of the static π N -equations with a cut-off parameter P = 7.

With $E = 1/t$ the system (9) has been transformed to the following form ($0 \le t \le 1$):

$$u(t) = \lambda t^2 - \frac{t^2}{\pi} P \int_{-1}^{+1} \frac{v(\tau)}{\tau^2(\tau - t)} d\tau , \qquad (10a)$$

$$v(t) = \Gamma_1(t)\, u(t) + \Gamma_2(t)\, v(t) + \Gamma_3(-t)\, u(-t) + \Gamma_4(-t)\, v(-t) , \qquad (10b)$$

where

$u(t) = D(1/t)$,

$v(t) = A(1/t)$,

$\Gamma_i(t) = G^i(1/t) = 0$ for $t < 0$.

To get a solution of the system (10) the following development has been made:

$$v(t) \approx (1 - t^2)^{3/2} e^{-\frac{1}{196 t^2}} \sum_{n=0}^{N} a_n t^n \equiv v^N(t) , \qquad (11a)$$

$$u(t) \approx \lambda t^2 - t^2/\pi \; P \int_{-1}^{+1} d\tau \frac{v^N(\tau)}{\tau^2(\tau - t)} \equiv u^N(t) . \qquad (11b)$$

The coefficients $a_n^{(i)}$ have been determined by means of the least square method

$$\frac{\partial}{\partial a_n^{(i)}} \sum_{j=1}^{6} \int_{-1}^{+1} |\epsilon_j(t, a)|^2 dt = 0 , \qquad (12)$$

where

$$\epsilon(t,a) \equiv v^N(t) - \Gamma_1(t)\, u^N(t) - \Gamma_2(t)\, v^N(t) - \Gamma_3(-t)\, u^N(-t) - \Gamma_4(-t)\, v^N(-t), \qquad (13)$$

The equations (12) represent a system of linear algebraic equations for the determination of the $a_n^{(1)}$. This system was solved with the help of the electronic computer M–20 of the Joint Institute for Nuclear Research.

The upper limit for the power series development was chosen to be $N = 10$. For the practical calculation the region $0 \le t \le 1$ was divided into 130 intervals. The results of the calculations justified our method of solution for the system of integral equations, because the quantities ϵ turned out to be two orders of magnitude smaller than the amplitudes. The unambiguousness of the results obtained, however, must be considered as an open question.

4. *Discussion of the Results*

The expressions for $T_{JL}^{T\Pi}$, obtained from the integral equations by the calculation method of section 3 give the cross-sections for the reactions $\pi^+ + p \to 2\pi + N$ represented in Table 1, where we have put $\nu' = \nu'' = 1/2$, assuming the amplitudes to have only a weak ν -dependence. The experimental cross-section[7–11] for the reaction $\pi^- + p \to \pi^+ + \pi^- + n$ are given in Fig. 1. The curve represents the theoretical values of Table 1. One can see that the obtained crossing-symmetrical amplitudes reflect the increase of the cross-sections at low energies. We will discuss this fact more in detail.

The integral equations of the papers[1–3] in the approximation used are not invariant with respect to the symmetry properties (1). The Chew-Low-equation used by Rodberg[2] has, for instance, the following form:

$$T_{pq,k} = -\sum_n \left\{ \frac{\langle q|V_p^+|n\rangle\langle n|V_k|0\rangle}{\omega_n - \omega_k} + \frac{\langle q|V_k|n\rangle\langle n|V_p^+|0\rangle}{\omega_n + \omega_p} \right\}. \qquad (13)$$

We will consider the consequences of a violation of the symmetry laws (1) for example by comparing the pole contributions corresponding to the equations used by Kazes[3] with the crossing-symmetric pole contributions (6). For $\nu' = \nu'' = 1/2$ we get the following expressions:

$$^{\circ}I_{JL}^{T\Pi} = a_{JL}^{T\Pi} \frac{f^3}{E^2}, \qquad (14a)$$

$$\{{}^{\circ}I^{T\Pi}_{JL}\}_{non\text{-}sym.} = \beta^{T\Pi}_{JL}\,\frac{6f^3}{E^2} \quad . \tag{14b}$$

α and β are given in Table 2. We see that the pole contributions to the non-crossing-invariant amplitudes are much smaller than those satisfying the symmetry laws.

Franklin/1/ and Rodberg/2/ symmetrize the obtained expressions with respect to E', E'':

$$f(E', E'') \to \tfrac{1}{2}\{\, f(E', E'') + f(E'', E')\,\} .$$

Franklin uses for instance the following symmetrized pole approximation

$$T_{pq,k} = \frac{1}{2\omega_p}\{V_p^+ T_k(q^+) - T_k(q)\,V_p^+\} +$$

$$+ \frac{1}{2\omega_q}\{V_q^+ T_k(p^+) - T_k(q)\,V_q^+\} + \frac{1}{\omega}\{V_k T_p(q^+) - T_p(q)\,V_k\} . \tag{15}$$

The symmetrizing factors 1/2 of the first two expressions on the right-hand-side of (15) must be cancelled. Then the amplitude has all the necessary symmetry properties. This means that the cross-sections given by Franklin must be multiplied by factors $\lesssim 4$. At low energies they then obviously come into the region of the experimental values.

The analysis has shown that it is possible to explain the increase of the cross-section for the reaction $\pi^- + p \to \pi^+ + \pi^- + n$ (1) at low energies by means of crossing - invariant amplitudes of the static theory witho ut explicit consideration of the $\pi\pi$ -interaction.

For the reactions $\pi^- + p \to \pi^- + \pi^\circ + p$ (2), $\pi^+ + p \to \pi^+ + \pi^+ + n$ (3), $\pi^+ + p \to \pi^+ + \pi^\circ + p$ (4) only very little experimental data are avaible. Results of the investigations/7,13/ indicate that in the region ~ 300 MeV $\sigma(2)$ is much smaller than $\sigma(1)$. The present analysis gives $\sigma(2) \approx 1/2\, \sigma(1)$. At 500 MeV the experimental cross-section/14,15/ for the reactions (3) and (4) is smaller than the cross-section of reaction (1). Table 1 gives a theoretical result which is higher than the experimental cross-section. One must emphasize, however, that the energy region 500 MeV lies already outside the region of application of a static approximation.

It is to be hoped that the existing discrepances can be explained by including into our system of equations the s-waves and the $\pi\pi$ -interaction.

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Table I

Cross-section for the reactions

$\pi^{\pm} + p \rightarrow 2\pi + N$ (in [mB])

$E_{kin\ lab}$ [MeV]	$\pi^+ p \rightarrow$ $\pi^+\pi^+ n$	$\pi^+ p \rightarrow$ $\pi^+\pi^0 p$	$\pi^- p \rightarrow$ $\pi^+\pi^- n$	$\pi^- p \rightarrow$ $\pi^-\pi^0 p$	$\pi^- p \rightarrow$ $\pi^0\pi^0 n$
246	0,022	0,0II	0,023	0,0I2	0,005
272	0,I0	0,05	0,I2	0,063	0,023
3I5	0,36	0,I7	0,42	0,23	0,084
367	I,I	0,48	I,2	0,66	0,24
45I	3,6	I,4	2,7	I,9	0,78

Table II

Comparison of the symmetrical (α) and non-symmetrical (β) pole- contributions.

${}^{0}I_{JL}^{T\pi}$	α	β
${}^{0}I_{3/2\,2}^{3/2\,2}$	20	I0
${}^{0}I_{1/2\,0}^{3/2\,2}$	$-2\sqrt{I0}$	$-\sqrt{I0}$
${}^{0}I_{3/2\,1}^{3/2\,1}$	- 4	- 6
${}^{0}I_{1/2\,1}^{3/2\,1}$	8	6
${}^{0}I_{1/2\,1}^{1/2\,1}$	-I6	0
${}^{0}I_{1/2\,0}^{1/2\,0}$	-I6	-8

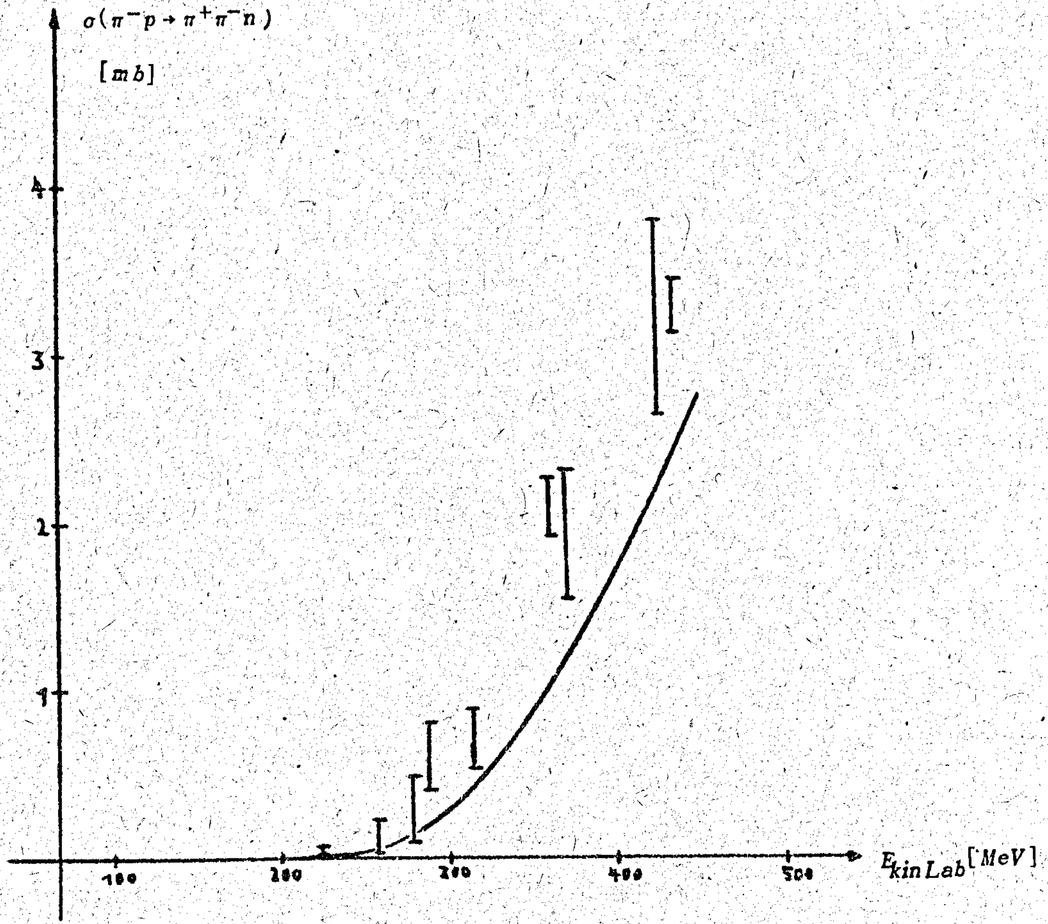

Fig. 1.

Experimental[7-12] and theoretical cross-sections for the reaction
$\pi^- + p \to \pi^+ + \pi^- + n$.

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