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G.I. Kopylov, I.V. Polubarinov

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The analytical calculations of the Feynman graphs often turn out to be cumbersome and lead to immense expressions. It is difficult therefore to find desired dependences without considering limiting special cases or, in a general case, without applying further numerical computations.

Hence, it is natural to begin the computation procedure directly from Feynman graphs.

The differential and total cross sections are expressed by the integral of the squared module of the amplitude. We may integrate the squared module of the amplitude numerically in different ways, but it is convenient, especially in the case of a large number of particles in the final state to make use of 'the method of random stars'[1]. According to this method the state indices are picked randomly and the weights $\Phi(\vec{p}_1 \dots \vec{p}_n ; s_1 \dots s_m)$ of the states are averaged over state space. Here $\vec{p}_k$ - are momenta, s_k - are spin indices of the initial and final states, and

$$\sigma = \sum_{s_1 \dots s_m} \int |M|^2 \delta(p_1 + \dots + p_n - P)\, d\vec{p}_1 \dots d\vec{p}_n \cong 1/N \sum_{1}^{N} \Phi(\vec{p}_1 \dots \vec{p}_n ; s_1 \dots s_m) \qquad (1)$$

(N - is the number of samples of state indices).

The weights of the states are of the form:

$$\Phi = |M|^2 K(\vec{p}_1 \dots \vec{p}_n ; s_1 \dots s_m) \qquad (2)$$

where K does not depend on M, but depends only on the choice of random variables (spin and momentum). In[1] it was shown how the laws of conservation in (1) could be satisfied for the choice of independent random indices from the unit cube.

The differential cross sections in this method are represented by histograms. They are obtained by averaging the weights, sorted according to values of corresponding variable.

This method is directly applicable to the calculation of the contourless graphs, as well as of graphs in which the integration over internal lines is performed beforehand (analytically or numerically)*. In this case the matrix element M, written according to graphs, is the sum of the products of matrices and columns and rows (the wave functions of initial and final states). A random choice of definite state determines numerically the elements of the matrices.

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* For example, for self-energy and vertex parts we may use the well-known expressions.

To calculate the complex number M all the operations with the matrices can be carried out with the aid of the electronic computer for all dummy spin indices. The summation over those indices and the calculation of $|M|^2$ are easily carried out on the computer too. This completes the calculation of the weight for the picked set $\vec{p}_1 \dots \vec{p}_n, s_1 \dots s_m$. By repeating N times this procedure we are able to calculate the cross section by means of the 'method of random stars' (formula (1)).

The disadvantages of proposed method are seen clearly: the calculation should be repeated anew for each set of parameters; result is obtained with statistical errors; the calculation of histograms (differential cross sections) requires very large N's; the calculation becomes more complicated for a large number of dummy spin indices.

However:

1. The method does not require any transformations of the amplitude after it has been written directly according to graphs and the meaning of symboles 'interpreted' for the computer;

2. The complexity of calculations is proportional to the number of graphs, but not to their square as in the case of the conventional trace method. The numerical method may therefore turn out to be useful in calculating processes to which some graphs correspond (for example, the calculation of the bremsstrahlung and pair production, especially in the case of higher spins, the calculation of the multiple production according to the Chew-Low scheme);

3. The computation of the cross sections for polarized particles is curiously simpler than for unpolarized ones, because the number of summations over spin states decreases;

4. The insertion of the form factors in diagrams does not complicate calculations, since, a.e. it only changes the form of the matrix at the vertex ;

5. The computation of the total cross section can be made simultaneously with the computation of several differential cross sections - the same weights are sorted according to the values of various variables; the computation with several form factors can also be made simultaneously;

6. The calculations of secondary processes (particle decay) may be inserted in the general calculation scheme, giving desired differential cross sections without any increase of the computer time.

The method is, in principle, applicable to the case, when there are integrations to be performed over the internal lines. But it is not clear what the increase of the computational work should be expected. Rough estimates show that in calculations which are very difficult in usual technique, the computation of one term of the sum (1) will require about 1-10 sec for the electronic computer (type 'M–20'). Therefore, in this way we can get one point on the curve of the energy dependence of the cross section

with suitable accuracy in about ten minutes. To decrease the computation time (to speed up to convergence of the sum in (1) to the integral) one may use the method of levelling summand in (1) (which is equivalent to well-known importance sampling in Monte-Carlo method). So, if we imagine qualitatively state indices distribution (i.g., from experiment), then picking of random numbers in the most probable region of phase space leads to the levelling of states.

At present the method suggested is testing by concrete examples.

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References

1. G.I. Kopylov. JETP, 35, 1426 (1958); JFTP, 39 1091 (1960); Preprint ОИЯИ,Е -528(1960).

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