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$N \vec{K}+N \rightarrow\{(\Sigma)+\gamma$ PROCESSES


#### Abstract

It is shown thet some information on the $\Lambda(\Sigma)+\pi \rightarrow \Lambda(\Sigma)+\gamma$ processes mey be obtained by investigating the reacffons $K+N \rightarrow A(\Sigma)+\gamma$. $A$ detalled phenomenolofical analysis of these processes in the S-state is made.

The Kroll-Ruderman theorem ia conaldered for the pion photoproduction on hyperons near the threahold.


One of the most important problems of the elementary particle physics is the study of the interaction between the unstable particles. As far as there is no target of unstable particles for this purpose we have to make use of the indirect methods.

In $/ 1 /$, we have shown that with the aid of the unitarity condition of the S -matrix it is possible to establish some relations between the matrix elements for the processes $\overrightarrow{\mathrm{K}}+\mathrm{N} \rightarrow \overrightarrow{\mathrm{K}}+\mathrm{N}, \overrightarrow{\mathrm{K}}+\mathrm{N} \rightarrow \pi+\mathrm{N}(\boldsymbol{\Sigma})$, and $\Lambda(\Sigma)+\pi \rightarrow \Lambda(\Sigma)+\pi$. Therefore, one is able to get some data on the processes $\Lambda(\Sigma)+\pi \rightarrow \Lambda(\Sigma)+\pi$ by making the analysis of the cross sections and of the baryon polaization in elastic scottering and in the reactions involving K -mesons and nucleons. Analogous results have been obtained by other authars $/ 2,3 /$. in $/ 2 /$ and especially in $/ 3 / a$ method has been developed for a detalled analysls of elastic scattering and K-meson interaction with nucleons in the S-state. From the avallable experimental data one succeeds in determining the difference between the phase shifts of the $S$-waves of $-\Sigma$ scattering in the states with the isospins $I=1$ and $I=0$.

In arder to obtain some information about the electramagnetic and strong interactions of hyperons we are considering here the processes

$$
\begin{equation*}
\overline{\mathrm{K}}+\mathrm{N} \rightarrow \mathrm{~A}(\Sigma)+\gamma \tag{1}
\end{equation*}
$$

The unitarity conditions of the S -matrix lead to the fact that the matrix elements for the processes $\pi+\Lambda(\Sigma) \rightarrow \Lambda(\Sigma)+\gamma$ are found to be connected with the matrix elements of the processes $\overline{\mathrm{K}}+\mathrm{N} \rightarrow \mathrm{A}(\Sigma)+\boldsymbol{Y}$. For the sake of simplicity we treat the reactions (1) in the S -state only. Let us use the method of a K -matrix developed in ${ }^{/ 3 /}$.

## II

To consider the problem it is convenient to make use of the symmetrical and hermition K -matrix which is expressed in terms of the T-matrix by means of the relation

$$
\begin{equation*}
K=T-i \pi K \rho T=T-i \pi T \rho K, \tag{2}
\end{equation*}
$$

where $\rho$ is the density of the phase volume for the intermediate states with the fixed total energy. For the two-particle (hinary) reactions with a definite value of the momentum the matrix $\rho$ has a diaganal form.

With the relativistic nomalization of the wave functions the diagonal elements of the $\rho$ matrix are equal to

$$
\begin{equation*}
\rho_{n n}=\frac{\|_{n} k}{\pi F_{2}} \tag{3}
\end{equation*}
$$

where $k$ is the relative momentum of particles in the $c . \pi$..s.; In is the baryon mass in the intermediate sta. te; $\mathfrak{b}$ is the total energy of the systers

$$
\begin{equation*}
E=\left(x^{2}+M_{n}^{2}\right)^{4 / 2}+\left(x^{2}+m^{2}\right)^{x} \tag{4}
\end{equation*}
$$

If we introduce the notations

$$
\begin{equation*}
\mathrm{K}^{4}=\pi \rho^{1 / 2} \mathrm{~K}^{1 / 2}, \quad \mathrm{~T}^{4}=\pi \rho^{1 / 2} \mathrm{~T} \rho^{1 / 2} \tag{5}
\end{equation*}
$$

then E.q.(2) may be put as

$$
\begin{equation*}
K^{4}=T^{4}-i K^{4} T^{4}=T^{4}-i T^{4} K^{4} . \tag{6}
\end{equation*}
$$

Fron (6), we get

$$
\begin{equation*}
T^{4}=\left(1-i K^{4}\right)^{-1} K^{4}=K^{4}\left(1-i K^{4}\right)^{-t} \tag{7}
\end{equation*}
$$

Being expressed in terms of the $T^{\prime \prime}$-matrix, the cross section for reaction (1) in the state with the definite value of the total momentum $J$ and the parity is equal to

$$
\begin{equation*}
\sigma(\mathrm{i} \rightarrow \mathrm{j})=\frac{4 \pi}{\mathrm{x}^{2}}\left(\mathrm{j}+\frac{1}{2}\right\}|<\mathrm{j}| \mathrm{T}^{4}|\mathrm{i}>|^{2} \tag{8}
\end{equation*}
$$

Let us consider the submatrices of the introduced $K$ - and $T$-matrices. We designate them by

$$
\begin{aligned}
& a=\langle\overline{\mathrm{K}} \mathrm{~N}| \mathrm{K}|\overrightarrow{\mathrm{~K}} \mathrm{~N}\rangle \\
& \mathrm{T}_{\boldsymbol{K} \overline{\mathrm{K}}}=\langle\overline{\mathrm{K}} \mathrm{~V}| \mathrm{T}|\overline{\mathrm{~K}} \mathrm{~N}\rangle \\
& \beta=\langle\overrightarrow{\mathbf{K}} \mathbf{N}| \mathbf{K}\left|\mathbf{Y}_{\pi}\right\rangle \\
& \beta^{+}=\left\langle\mathrm{Y}_{\pi}\right| \mathrm{K}|\overline{\mathrm{~K}} \mathrm{~V}\rangle \\
& \left.\mathrm{T}_{\boldsymbol{K} \overline{\mathbf{Y}}}<\overline{\mathbf{K}} \mathbf{N}|\mathrm{T}| \mathrm{Y}_{\pi}\right\rangle \\
& \mathrm{T}_{\mathbf{Y K}}=<\mathrm{Y}_{7}|\mathrm{~T}| \overline{\mathrm{K}} \mathrm{~V}> \\
& \gamma=\langle\mathrm{Y} \|| \mathrm{K}\left|\mathrm{Y}_{\pi}\right\rangle \\
& \mathrm{T}_{\mathbf{Y} \mathbf{Y}}=\left\langle\mathrm{Y}_{\pi}\right| \mathrm{T}\left|\mathrm{Y}_{\pi}\right\rangle \\
& \xi_{s}=\langle\overline{\mathrm{X}} \mathrm{~N}| \mathrm{K}\left|\mathrm{Y}_{\gamma}\right\rangle \\
& \mathrm{T}_{\boldsymbol{K} \boldsymbol{y}}=\langle\overleftarrow{\mathbf{K}} \mathbf{V}| \mathbf{T}|\mathbf{Y} y\rangle \\
& \xi^{+}-\langle\mathrm{Y} \gamma| 《|\overline{\mathrm{~K} N}\rangle \\
& \mathrm{T}_{\boldsymbol{\gamma} \boldsymbol{K}}=\langle\mathrm{Y} \boldsymbol{\gamma}| \mathrm{T} \mid \overline{\mathrm{K}} \mathrm{~V}> \\
& { }_{f} \cdots \mathbf{Y}_{\pi} \mathbf{K}^{\prime} \mathbf{Y}_{7} . \\
& \mathrm{T}_{\boldsymbol{r y}}=<\mathrm{Y}_{\pi}|\mathrm{T}| \mathrm{Y}_{\gamma} \gg
\end{aligned}
$$

$$
\begin{array}{ll}
\eta^{+}=\left\langle\mathrm{Y}_{y}\right| \mathrm{K}\left|\mathrm{Y}_{\pi}\right\rangle & \mathrm{T}_{Y}=\left\langle\mathrm{Y}_{y}\right| \mathrm{T}\left|\mathrm{Y}_{\pi}\right\rangle \\
\zeta=\left\langle\mathrm{Y}_{Y}\right| \mathrm{K}\left|\mathrm{Y}_{Y}\right\rangle & \mathrm{T}_{y \gamma}=\left\langle\mathrm{Y}_{y}\right| \mathrm{T}\left|\mathrm{Y}_{Y}\right\rangle \tag{9}
\end{array}
$$

We denote the subnatrices of the $\mathrm{K}^{\prime}$ - and $\mathrm{T}^{*}$-matrices by the corresponding letters with the primes. In what follows the submatrix $\zeta$ which is, at least, less than other ones by an order, is neglected.

If introduce the notations

$$
\mathrm{K}_{0}=\left(\begin{array}{cc}
a & \beta  \tag{10}\\
\beta^{+} & \gamma
\end{array}\right), \quad \delta=\binom{\xi}{\eta}
$$

then we can write down

$$
\mathbf{K}=\left(\begin{array}{cc}
\mathbf{K}_{p} & \delta  \tag{11}\\
\delta^{+} & 0
\end{array}\right)
$$

It is easy to get from (5), (7), (10) and (11) that

$$
\begin{align*}
& \mathrm{T}_{\mathbf{K} \boldsymbol{K}}=\left(1-\mathrm{i} \mathrm{X}^{6}\right)^{-1} \mathrm{X}^{t} ; \mathrm{T}_{\boldsymbol{Y} \boldsymbol{Y}}=\left(1-\mathrm{i} \mathrm{Z}^{6}\right)^{-t} \mathrm{Z}^{*} \\
& \mathrm{~T}_{K Y}^{6}=\left(1-i X^{s}\right)^{-1} \beta^{6}\left(1-i \gamma^{6}\right)^{-1}=\left(1-i a^{6}\right)^{-1} \beta^{4}\left(1-i Z^{4}\right)^{-8} \\
& \mathrm{~T}_{\mathrm{YK}}^{*}=\left(1-i Z^{6}\right)^{-1} \beta^{\cdot}{ }^{\top}\left(1-i a^{4}\right)^{-1}=\left(1-i \gamma^{6}\right)^{-1} \beta^{\top T}\left(1-i X^{6}\right)^{-i} \\
& T_{K y}=\left(1-i X^{*}\right)^{-1} \xi^{t}+i\left(1-i X^{*}\right)^{-t} \beta^{0}\left(1-i y^{*}\right)^{-1} \eta^{6} \\
& \mathrm{~T}_{Y Y}^{*}=\mathrm{i}\left(1-\mathrm{i} \mathrm{Z}^{4}\right)^{-1} \beta^{\prime T}\left(1-\mathrm{i} \alpha^{4}\right)^{-t} \epsilon_{s}^{4}+\left(i-i Z^{6}\right)^{-t} \eta^{6} \tag{12}
\end{align*}
$$

$$
\begin{align*}
& \mathrm{X}^{4}=a^{4}+\mathrm{i} \beta^{4}\left(1-\mathrm{i} y^{4}\right)^{-1} \beta^{\mathrm{T}} ; \quad \mathrm{Z}^{4}=\gamma^{4}+\mathrm{i} \beta^{4}\left(1-\mathrm{i} a^{4}\right)^{-1} \beta^{4} . \tag{13}
\end{align*}
$$

where

## II

In our discussion it suffices to take into account the electromagnetic interaction in the first order of the perturbation theory by considering separately the contributions of the isoscalar and isovector parts of the electromagnetic interaction.

Le' is start with the isoscalar current. In this case the total isospin $\mathbf{I}=0$ for the system $A+y$ and $\dot{I}=1$ Sor $\Sigma+y=\cdots t o m$. We denote the matrix elements with the isoscalar current for the processes $\overline{\mathbf{K}}+\mathrm{N} \rightarrow \mathrm{A}(\Sigma)+y$ $\therefore$ ' $\|(\Sigma), \pi \rightarrow N(\Sigma)+y$ by $\xi_{i}^{0}, \xi_{\Sigma}^{1}, \eta_{A}^{0}$ and $\eta_{\Sigma}^{1}$, respectively. In the case of the isovector current the tota! : sospin $\mathrm{I}=1$ for the system $\mathrm{A}+\gamma$ and $\overline{\mathrm{I}}=0,1$ for the system $\mathbf{\Sigma}+\gamma$. The corresponding matrix elements are disiqnated by $\xi_{1}^{1}, \xi_{\Sigma}^{0}, \xi_{\Sigma}^{\prime 1}, \eta_{A}^{1}, \eta_{\Sigma}^{0}$ and $\eta_{\Sigma}^{1 s}$.

Let us consider the thannels with the isospin $\dot{I}=0$. In this case the subnatrices $a, \beta$ and $y$ are sirmply nurhors, it the satie time (3) can be reduced to the form.

$$
\begin{align*}
& \mathrm{X}=a+\mathrm{i} \pi \beta^{2} \rho_{\Sigma} /\left(1-i \pi \gamma^{\rho}{ }_{\Sigma}\right)=a+i b \quad,  \tag{14}\\
& a=a-\quad \frac{\pi^{2} \beta^{2} \gamma q_{i}^{2}}{l^{2}+\pi^{2} \rho \Sigma^{2} \gamma^{2}} \\
& \mathrm{~b}=\frac{\pi \beta^{2} \rho}{1+\pi^{2} \rho \Sigma y^{2}}>0 . \tag{15}
\end{align*}
$$

$\because \eta$ substituting (14) into (12), we get

$$
\begin{align*}
& \mathrm{T}_{K K}^{*}=\left(1-i Y^{6}\right)^{-1} \mathrm{X}^{\prime}=\pi \rho_{K}\left(\mathrm{a}^{0}+i b^{0}\right) 1_{0}^{-1} \\
& \mathrm{~T}_{\Sigma_{K}}=\pi^{3 / 4} \rho_{K}^{4 /}\left(b^{0}\right)^{1 / 2} \mathrm{e}^{9 \lambda} \mathrm{~S}_{0}^{-1} \tag{16}
\end{align*}
$$

$\because$ here

$$
\operatorname{tg} \lambda_{\Sigma}=\pi \gamma \rho \Sigma ; \quad \lambda_{0}=1-\mathrm{i} \pi \rho_{\boldsymbol{K}}\left(\mathrm{a}^{0}+\mathrm{ib} b^{0}\right)
$$

Pornulae (16) for the processes $\overline{\mathrm{K}}+\mathrm{V} \rightarrow \overline{\mathrm{K}}+\mathrm{V}$ and $\overline{\mathrm{K}}+\mathrm{V} \rightarrow \Sigma+\pi$ have been obtained by many authors/3/.
Let us write down

$$
\begin{align*}
& \mathrm{T}_{Y K}^{s}=\xi^{T}\left(1-i X^{*}\right)^{-1}+i \eta^{T}\left(1-i y^{\prime}\right)^{-1} \beta^{r}{ }^{r}\left(1-i X^{s}\right)^{-1}= \\
& =\left(T^{*} \operatorname{lin}^{\prime} k\right.  \tag{17}\\
& \mathrm{T}_{\mathrm{S}}^{\mathrm{H}} \mathrm{~K}
\end{align*}
$$

in i

$$
\eta^{r}=\binom{\eta^{0}}{\eta_{S S}^{0}}, \quad \xi^{\top}=\left(\begin{array}{l}
\xi_{N K}^{0} \\
\xi_{0} \\
\Sigma_{\mathcal{K}}
\end{array}\right) .
$$

From (14), (15), (16) and (17) it is easy to get that
 gion where (under the assumption that the relative hyperon parity is positive) it is possible to neglect the energy dependence of the guantities $\rho_{\Sigma}, \rho_{A}, \rho_{Y \Lambda}$ and $\rho_{Y \Sigma}$.

Now we are prnceeding to the consideration of the channels with the isospin $\mathrm{I}=1$ In this case y and $\beta$ are the matrices

$$
\gamma=\left(\begin{array}{ll}
\gamma_{A A} & \gamma_{\Sigma N}  \tag{19}\\
\gamma_{A \Sigma} & \gamma_{\Sigma \Sigma}
\end{array} \quad \beta=\left(\beta_{A K}, \beta_{\Sigma K}\right) .\right.
$$

It can be easily checked that in this case also X is simply a complex number

$$
\begin{equation*}
\mathrm{x}=\mathrm{a}^{\mathrm{t}}+\mathrm{i} \mathrm{~b}^{1} \quad, \tag{20}
\end{equation*}
$$

where

$$
\begin{align*}
& a^{I}=a-\pi \beta \rho_{y}^{*}-1+y_{z}^{\prime} \underset{y}{1} \gamma^{\prime} p_{r}^{2} \beta^{\top} \\
& b^{2}=\pi \beta \rho_{y}^{1 / 4}-\frac{1}{1+y^{j}}{ }^{-1} \rho_{y}^{1 / 2} \beta^{T} \tag{21}
\end{align*}
$$

It follows from (12), (13), (19), (20) and (21) that

$$
\begin{align*}
& \mathrm{T}_{K K}^{s}=\pi \rho_{K}\left(\mathrm{a}^{s}+\mathrm{i} \mathrm{~b}^{2}\right) \mathrm{t}_{\mathrm{I}}^{-1} \tag{22}
\end{align*}
$$

where

$$
\begin{aligned}
& 1_{2}=1-i \pi \rho_{K}\left(a t+i h^{\prime}\right)
\end{aligned}
$$

$$
\begin{align*}
& \pi_{\pi}^{\prime 2} \rho_{K}^{1 / 2} b_{K} \sum_{K} e^{\lambda_{K}} \Sigma_{K}<\Sigma\left|\left(1-i \gamma^{2}\right)^{-1} 3^{s^{T}}\right| K>. \tag{23}
\end{align*}
$$

while the quantities $\mathbf{b}_{\mathbf{A k}}$ and ${ }^{\mathrm{b}} \mathbf{\Sigma}_{\mathrm{k}}$ are connected with b

$$
\begin{equation*}
b_{\Lambda_{X}}+b_{\Sigma_{X}}=b \tag{24}
\end{equation*}
$$

If we represent the matrices $\xi$ and $\eta$ in the form

$$
\begin{align*}
& \xi=\left(\xi_{\mathrm{A}_{K}} \cdot \xi_{\Sigma_{K}}\right) \\
& \eta=\left(\begin{array}{l}
{ }^{\eta} \mathrm{A} \\
{ }^{\eta} \mathrm{I} \\
{ }^{\eta} \mathrm{A} \\
{ }^{\eta} \mathrm{\Sigma} \Sigma
\end{array}\right), \tag{25}
\end{align*}
$$

then the matrix elements $T_{\gamma / k}, T_{\gamma}^{4} \Sigma_{k}$ take on the form

$$
\begin{aligned}
& +i \eta_{\Delta \Sigma} \pi^{h / n} \rho_{\Sigma}^{\gamma /} b_{\Sigma K}^{1 / 2} e^{\left.i \lambda \Sigma_{K}\right]}
\end{aligned}
$$

and

$$
\begin{align*}
& +i \eta_{\Sigma \Sigma} \pi^{1 / 2} \rho_{\Sigma}^{2 / 2} b_{\Sigma_{K}}^{1 / 2} e^{\left.i \lambda \Sigma_{K}\right]} . \tag{27}
\end{align*}
$$

For simplification, we introduce new notations

$$
\begin{align*}
& \left.+i \eta_{\Sigma \Sigma}^{z} \pi^{H /} \rho_{\Sigma}^{H}\left(b_{\Sigma K}^{N}\right)^{H} e^{M} \Sigma K\right] \tag{28}
\end{align*}
$$

with the aid of which the cross sections for processes (1) may be put as

Process
$K^{-}+p+1^{0}+y$
$\bar{K}^{0}+n \cdot l^{0}+y$
$\mathrm{K}^{-}+\mathrm{p} \rightarrow \Sigma^{0}+\gamma$

$\mathbf{K}^{-}+n \rightarrow 5+\gamma$
$\overline{\mathbf{K}}^{0}+\Gamma \cdot \mathbf{\Sigma}^{\dagger}+\gamma$

## Cross section

Thus, the experimentat investigation of the $\mathbf{K}+\boldsymbol{V} \rightarrow \mathbf{M}(\Sigma)+y$ processes in $\overline{\mathbf{K}}+\boldsymbol{p}$ and $\overline{\mathbf{K}}+\mathbf{d}$ collisions may yield some information about the matrix elements $a_{A}$ and $a_{\Sigma}$. Of course, it is insufficient for reconstructing the matrix elements $\xi$ and $\eta$ which describe the photoproduction of pions on hyperons. Vote the less, they may prove to be useful for studying the hyperon interaction with pions and photons.

## IV

4 poweeful method for analysing strong interactions is that of dispersion relations (d.r.), the use of which allows in some cases to get interesting results in the' low energy region. One may think that the method of d.r. is applicated to the photoproduction of pions on hyperons.

In this note we restrict ourselves to the generalization of the Kroll-Ruderman theorem for the photoproduction of pions near the threshold $/ 4 /$.

Let us assurne that $\operatorname{I}$ and $\mathbf{\Sigma}$ hyperons hrve positive relitive parity and the $\mathbf{K}$-meson is pseudoscalar. In the low energy region of the produced particles it suffices to take into account the dipole radiation. The generalized theorem of Kroll-Ruderman states that up to $m \pi M \approx 15 \%$ the matrix $\eta$ for the electric dipole transition is completely determined by the coupling constints of pions with hyperons.

We write Jown the pion-hyperon interaction Hamiltonian as

$$
\begin{aligned}
& H=i g \Sigma \sum^{\stackrel{\rightharpoonup}{\Sigma}} y_{y} \wedge \vec{\pi}+\mathrm{ig}_{\Sigma \Sigma}\left(\left[\vec{\Sigma}_{y_{s}} \vec{\Sigma} \mid \vec{\pi}\right)+\text { h.c. }=\right. \\
& =i g_{\Sigma}\left[\bar{\Sigma}_{f} y_{g} \lambda_{0}+\bar{\Sigma}_{+} y_{g} A_{n^{+}+} \bar{\Sigma}_{-} y_{g} \Lambda_{\pi}-1+\right.
\end{aligned}
$$

$$
\begin{gather*}
+\mathrm{ig} \Sigma \Sigma^{[ }\left[\left(\bar{\Sigma}_{-} \gamma_{s} \Sigma_{-}-\bar{\Sigma}_{+} y_{s} \Sigma_{+}\right) \pi^{0}+\left(\bar{\Sigma}_{+} y_{s} \Sigma_{0} \pi^{+}-\bar{\Sigma}_{-} y_{s} \Sigma_{0} \Pi^{-}\right)+\right. \\
\left.+\left(\bar{\Sigma}_{0} y_{s} \Sigma_{\pi+-} \bar{\Sigma}_{0} y_{s} \Sigma^{+} \pi^{-}\right)\right]+ \text {h.c. } \tag{29}
\end{gather*}
$$

Following the Low method $/ 5 /$ one can obtain that
where $m \pi$ is the pion mass, M is the hyperon mass and $a=e^{2} / 4 \pi=1 / 137, \quad \mathrm{f}^{2}=\mathrm{g}^{2} / \mathrm{g} \pi \mathrm{V}$.

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