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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория ядерных проблем

Лаборатория теоретической физики

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ON PHOTON-NUCLEON SCATTERING

Dubna 1961

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Abstract

An analysis of elastic γ - ray scattering by protons at an energy up to 300 MeV has been made by using the dispersion relations (d. r.) Six dispersion relations were made use of to estimate the real parts of the amplitudes at $Q^2 = 0$. The pion photoproduction has been taken into account in a wider than earlier energy region. It has been proved that the subtraction is necessary in the dispersion relations (at any rate for one amplitude). Five subtraction constants are determined by the low energy limit and expressed in terms of the charge and the nucleon magnetic moment. The differential cross sections and the recoil nucleon polarization have been estimated.

$\gamma - N$ scattering at high energies is being discussed.

1

Following Gell-Mann, Goldberger and Thirring[1] the dispersion relations (d.r.) for γ-N -scattering the validity of which in the e^2 - approximation was proved strictly by Bogolubov and Shirkov[2] were applied to analysis of the experimental data in a number of papers[3-7]. Cini and Strofolini[3] were the first to make the calculations of the cross sections for the forward scattering at the energies of γ-rays up to 210 MeV. Some qualitative features of the energy dependence of the cross section for forward scattering were pointed out earlier in[1], as well as in[8].

Capps[4] considered γ- N scattering at arbitrary angles with account of the minimum number of angular momentum states. In doing this, he made use of some unpublished results of Gell-Mann and J. Mathews.

Akiba and Sato[5] treated the non-zero angle scattering. To calculate the subtraction constants in some dispersion relations the perturbation theory was employed.

In Ref.[6] the dispersion relations were considered in detail for all six invariant functions characterizing the amplitude for γ - N scattering, and the dispersion analysis was made in the energy region up to 200 MeV, some recoil effects being neglected. It has been shown that the account of the photoproduction of pions in the s-state leads to noticeable changes in the threshold region. At the same time the agreement between the dispersion analysis and experimental data improves. Cusp dependence arised in the amplitudes and cross sections near the threshold for pion photoproduction.

In spite of some differences in the published papers which are due to the different assumptions on the number of subtractions in the d.r. and on the maximum moment of the considered states, the common feature for all of them was the impossibility of obtaining good agreement with the experimental data in the energy range of about 160-200 MeV.

In a number of papers[9,7,10] an attempt has been undertaken to do away with the discrepancy by taking into account the Low diagram[11]. However, a direct measurement of the π^0-meson lifetime[12] and the analysis of the problem concerning the sign of the pole amplitude and its magnitude[13] has led to the fact that the account of the Low amplitude cannot affect essentially the result of the analysis.

In view of the differences between the results of the analysis and the available experimental data this paper deals with the analysis of γ-N-scattering based on dispersion relations. In the course of this analysis, besides the account of the pion photoproduction in the s-state, we are considering the contribution of the high energy region more thoroughly. Then we are treating the problem on the number of subtractions in the dispersion relations and by having taken the nucleon recoil into account, we are estimating the quantities. $R_i(\nu)$ *for* $Q^2=0$ introduced earlier.

2

The connection of the invariant functions $Ti(\nu, Q^2)$ with the amplitudes $Ri(\nu, Q^2)$ in the center-of-mass system (c.m.s.) is given in formula (1) of paper/14/ (which is further referred to as paper A). The definitions of $Ti(\nu, Q^2)$ and $Ri(\nu, Q^2)$ see in/13/ (which is further referred to as paper B). The notations adopted in the present paper coincide with those of the papers A and B. Here the amplitudes in the c.m.s. are designated by R_i without additional asterisks.

As far as by the optical theorem

$$Im\ (R_1+R_2) = \frac{\nu_0 \sigma_t}{4\pi} = \frac{W^2-M^2}{2W} \quad \frac{\sigma_t}{4\pi}, \qquad (1)$$

then under the assumption

$$\sigma_t(W) \to Const \qquad \text{by} \qquad W \to \infty$$

asymptotically by $W \to \infty$

$$R_1 + R_2 \to W^2 \sim \nu \ . \qquad (2)$$

Assuming further that by $W \to \infty$ all $R_i \sim \nu$ we get from (1.A), that by $W \to \infty$

$$\begin{array}{lll} T_1 - T_3 \to W^2\ ; & T_1 + T_3 \to W^2; & T_5 \sim W^2 \\ T_2 - T_4 \to W\ ; & T_2 + T_4 \to Const\ ; & T_6 \to W \end{array} \qquad /3/$$

Thus, under the adopted assumptions the dispersion relations for T_1, T_3 and T_5 must contain one subtraction, while for the quantities T_2, T_4 and T_6 the dispersion relations can be written without subtractions.

To estimate the amplitudes $R_1 + R_2$, R_3, R_4, $R_5 + R_6$
it is sufficient to write down the dispersion relation for T_i at $Q^2=0$. When $Q^2=0$, the invariant ν

$$\nu = \nu_L - Q^2/M$$

is known to pass into the energy of γ - rays in the lab.system ν_L (which is further designated by ν).

As is seen from (1.A), for the forward scattering the function T_5 and $T_2 + T_4$ are reduced to $R_4 - R_3$, so that the dispersion relations for T_5 and $T_2 + T_4$ at $Q^2 = 0$ are equivalent.

Let us consider the functions

$$\begin{aligned}
F_1(\nu_o) &= \frac{1}{2}[T_1 - T_3 - \nu_o(T_2 - T_4)] = \frac{W_o}{M}(R_1 + R_2) \\
F_2(\nu_o) &= \gamma\ T_6 = \frac{W_o}{M}[R_3 + R_4 + 2R_5 + 2R_6] \\
F_3(\nu_o) &= \frac{\nu_o}{2M}(T_1 + T_3) = (\frac{W_o}{M})^2 (R_3 - R_4) \\
F_4(\nu_o) &= \frac{1}{2}(T_1 - T_3) = \frac{W_o^2}{M\nu_o}(R_3 + R_4) - \frac{2W_o}{M + W_o}(R_1 + R_2)\,.
\end{aligned} \tag{4}$$

It is clear from the discussion given above that the dispersion relations for the functions $F_1, \ldots, F_4$ must contain one subtraction. All the quantities in the right-hand side of (4) are in the c.m.s. If we take into account that (at $Q^2 = 0$) the amplitudes in the lab.system (' Λ ') are connected with the corresponding quantities in the c.m.s. by

$$\begin{aligned}
(R_1 + R_2)^\Lambda &= \frac{W_o}{M}(R_1 + R_2) \\
[R_3 + R_4 + 2R_5 + 2R_6]^\Lambda &= \frac{W_o}{M}[R_3 + R_4 + 2R_5 + 2R_6] \\
(R_4 - R_3)^\Lambda &= (\frac{W_o}{M})^2\ (R_4 - R_3),
\end{aligned} \tag{5}$$

then from the dispersion relations for $F_1, \ldots F_4$ we get

$$\begin{aligned}
D^\Lambda_{1,4}(\nu_o) - D_{1,4}(0) &= \frac{2\nu_o^2}{\pi} \int_{\nu_t}^{\infty} \frac{d\nu}{\nu^2 - \nu_o^2} \cdot \frac{A_{1,4}(\nu)}{\nu} \\
D^\Lambda_{2,3}(\nu_o) - \nu_o D'_{2,3}(0) &= \frac{2\nu_o^3}{\pi} \int_{\nu_t}^{\infty} \frac{A_{2,3}(\nu)\, d\nu}{\nu^2(\nu^2 - \nu_o^2)}\,,
\end{aligned} \tag{6}$$

where

$$D_1^\Lambda = Re\,(R_1 + R_2)^\Lambda \qquad D_1(0) = -\,e^2/M$$

$$D_2^\Lambda = Re\,[R_3 + R_4 + 2R_5 + 2R_6]^\Lambda \qquad D'_2(0) = -2\mu_a^2$$

$$D_3^\Lambda = Re\,(R_4 - R_3)^\Lambda \qquad D'_3(0) = -2\,[\mu^2 - (\frac{e}{2M})^2]$$

$$D_4^\Lambda = Re\,F_4(\nu) \qquad D_4(0) = -\frac{e^2}{2M}\lambda\,(2 + \lambda)\,,$$

and the imaginary parts of the corresponding amplitudes are designated by $A_i(\nu_0)$, the magnetic moment by $\mu = \frac{e(1+\lambda)}{2M}$ and the anomalous magnetic moment of a nucleon by μ_a.

If the elements of the amplitude of the pion photoproduction in the states with $J \leq 3/2$ are denoted in the c.m.s, by

E_1 - /el. $1/2^-$/; M_1 - /magn. $1/2^+$/; E_2 - /el. $3/2^+$/;
E_3 - /el. $3/2^-$/; M_3 - /magn. $3/2^+$/; M_2 - /magn. $3/2^-$/,

then the unitarity relations will lead to the equalities

$$\begin{aligned} Im\, R_1 &= \nu_c \{ |E_1|^2 + 2|E_3|^2 + \frac{1}{3} |E_2|^2 \cos\theta - \frac{1}{6} |M_2|^2 \} \\ Im\, R_3 &= \nu_c \{ |E_1|^2 + \frac{1}{3} |E_2|^2 \cos\theta - |E_3|^2 + \frac{1}{12} |M_2|^2 + Re\,(E_3^* M_2) \} \\ Im\, R_5 &= -\nu_c \{ \frac{1}{6} |E_2|^2 + Re\,(E_2^* M_3) \}, \end{aligned} \tag{7}$$

which are the generalization of the corresponding equalities in /6/. The expressions for ImR_2 differ from ImR_1 by the replacement $E_1 \rightleftarrows M_1$. Similarly, the expression for ImR_4 can be obtained from ImR_3, and ImR_6 from ImR_5.

In (7), the total contribution of the photoproduction of π^+ and π° -mesons is meant by the module of the amplitude in the right--hand side. Let us note, that if the pion and nucleon mass differences are neglected, the interference terms, e.g., in $|E_0|^2 + |E_+|^2$ are vanished due to the isotopic symmetry of the pion photoproduction matrix elements.

At the same time

$$\begin{aligned} A_1(\nu) &= \frac{\nu\,\sigma_t}{4\pi} = \nu \{ |E_1|^2 + |M_1|^2 + 2|E_3|^2 + 2|M_3|^2 + \frac{1}{6} |M_2|^2 + \frac{1}{6} |E_2|^2 \} \\ A_2(\nu) &= \nu \{ |E_1|^2 + |M_1|^2 + \frac{1}{3} |E_2|^2 + \frac{1}{3} |M_2|^2 - |M_3 + \frac{1}{2} E_2|^2 - |E_3 + \frac{1}{2} M_2|^2 \} \\ A_3(\nu) &= -\nu \frac{W}{M} \{ |E_1|^2 - |M_1|^2 + |M_3 - \frac{1}{2} E_2|^2 - |E_3 - \frac{1}{2} M_2|^2 \} \\ A_4(\nu) + \frac{W-M}{4\pi}\sigma_t &= W \{ |E_1|^2 + |M_1|^2 + \frac{2}{3} |M_2|^2 + \frac{2}{3} |E_2|^2 - |E_3 - \frac{1}{2} M_2|^2 - |M_3 - \frac{1}{2} E_2|^2 \} \end{aligned} \tag{8}$$

The necessity of considering the dispersion relations with the subtraction for the amplitude $R_1 + R_2$ can be proved.

Let us prove that the low energy limit of the amplitude $R_1 + R_2$ contradicts the value of this quantity obtained with the aid of dispersion relations without the subtraction.

As is well-known, for $\nu \to 0$ ($Q^2 = 0$)

$$R_1 + R_2 \to - e^2/M < 0 \tag{9}$$

the dispersion relations without the subtraction for $R_1 + R_2$ are of the form

$$Re\,[R_1(\nu_0) + R_2(\nu_0)] = \frac{1}{2\pi^2} \int_{\nu_t}^{\infty} \frac{\nu^2 \, d\nu}{\nu^2 - \nu_0^2} \, \sigma_t(\nu). \tag{10}$$

For $\nu_0 \to 0$, it follow from (10), that

$$Re\,(R_1 + R_2) \to + \frac{1}{2\pi^2} \int_{\nu_t}^{\infty} \sigma_t(\nu)\, d\nu > 0 \ ,$$

what contradicts equality (9)*.

Thus, the dispersion relation without the subtraction for the amplitude $R_1 + R_2$ contradict the relativistic and gauge invariance requirements on which the low energy limit is based.

Let us note, that possible sum rules connected with the square of the magnetic moment, if the dispersion relations without subtraction are assumed for $F_2(\nu)$, are not in a direct contradiction with the low-energy limit. As is seen from (6) and (8), the contribution of the resonance state proportional to $|M_3|^2$ is especially essential here. The same result holds if one takes into account (numerically important) contribution of the photoproduction in the s-state which decreases the effective contribution $|M_3|^2$.

The sum rule for the square of the magnetic moment is very sensitive to the relationship between the photoproduction amplitudes E_2 and M_3. For some relations (for instance, with $E_2 = M_3$ /5/) one can arrive to a contradiction. However, the information on the analysis of the photoproduction is not so accurate at present to state that the experimental data contradict the sum rule. A further specification of the photoproduction analysis is extremely desirable for obtaining the data on the amplitudes E_2, E_3 and M_2.

The fact that the dispersion relations without subtractions lead to definite sum rules can be of great interest for some processes. So, for $\pi\,\pi$ scattering similar considerations (if applied to the disper-

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* This result has been obtained also by W.N. Gribov.

sion relations at $Q^2 = 0$) lead one to the conclusion that the scattering lengths in the s-state at low energies a_0 and a_2 are positive. The same holds for π-K and K-K scattering.

5

If we consider the properties of the functions*

$$F_5(\nu_0) = (T_2 - T_4)' \tag{11}$$

$$F_6(\nu_0) = (T_2 + T_4)' \tag{12}$$

$$F_7(\nu_0) = T_6' \;, \tag{13}$$

additionally to those introduced earlier, then one can become sure that $F_{5,6}(\nu)$ are the odd functions of ν, which have no poles, and $F_7(\nu)$ is the even function with the second-order pole. At $\nu \to \infty$,

$$F_{5,6,7} \to \nu^{-1/2} \;,$$

so that the dispersion relations for these functions do not contain the subtraction. The use of these dispersion relations may prove useful since in taking into account the photoproduction in the states with $J \leq 3/2$ the angular dependence of the amplitudes $R_i(\nu, Q^2)$ in the c.m.s. is represented as

$$\begin{aligned}
R_1 &= \mathcal{E}_1 + 2\mathcal{E}_3 + 2\mathcal{E}_2 \cos\theta - \mathcal{M}_2 \\
R_2 &= \mathcal{M}_1 + 2\mathcal{M}_3 + 2\mathcal{M}_2 \cos\theta - \mathcal{E}_2 \\
R_3 &= \mathcal{E}_1 - \mathcal{E}_3 + 2\mathcal{E}_2 \cos\theta + \tfrac{1}{2}\mathcal{M}_2 + C(\mathcal{E}_3 \mathcal{M}_2) \\
R_4 &= \mathcal{M}_1 - \mathcal{M}_3 + 2\mathcal{M}_2 \cos\theta + \tfrac{1}{2}\mathcal{E}_2 + C(\mathcal{M}_3 \mathcal{E}_2) \\
R_5 &= -\mathcal{E}_2 - C(\mathcal{M}_3 \mathcal{E}_2) \\
R_6 &= -\mathcal{M}_2 - C(\mathcal{E}_3 \mathcal{M}_2)
\end{aligned} \tag{14}$$

and characterized by the eight function of energy $\mathcal{E}_i$, $\mathcal{M}_i$, $C(\mathcal{E}_3, \mathcal{M}_2)$ $C(\mathcal{M}_3 \mathcal{E}_2)$, which are expressed in terms of $R_i(\nu, 0)$ and $R'_i(\nu, 0)$.

It follows from (14), that if we restrict ourselves to the contribution of the states with $J \leq 3/2$

* The sign ' means the differentiation with respect to Q^2 and a subsequent transtion to $Q^2 = 0$.

$$R'_1 = R'_3 = 2\mathfrak{S}_2\left(\frac{\partial \cos\theta}{\partial Q^2}\right)_{Q^2=0} = -4\mathfrak{S}_2 \frac{W_0^2}{M^2\nu_0^2}$$

$$R'_2 = R'_4 = -4\mathfrak{M}_2 \frac{W_0^2}{M^2\nu_0^2}$$

$$R'_5 = R'_6 = 0$$

and

$$(R_1+R_2)' = (R_3+R_4)' = (R_3+R_4+2R_5+2R_6)' \qquad (15)$$

In the low energy limit /14/

$$(R_1+R_2)' = -2\,\frac{e^2}{M^2}\,\frac{1}{\nu} + O(1)$$

$$(R_3+R_4)' = -\frac{e^2}{2M^3}\left[3+2(1+\lambda)^2\right] + O(\nu) \qquad (16)$$

$$(R_3+R_4+2R_5+2R_6)' = -\frac{e^2}{2M^3}(2\lambda^2-2\lambda-1) + O(\nu).$$

The contradiction of (15) with the low-energy limit (16) implies that the restriction to the states with $J \leqslant 3/2$ is not correct even in the low-energy region. The crossing-symmetry conditions introduce the kinematic corrections of the order of ν/M, what corresponds to the account of the states with higher values of J.

In order to make the analysis with such a high accuracy it is necessary to introduce new functions of energy and to consider a great number of dispersion relations. The introduction of the Low diagram fails to solve the contradiction mentioned above.

All the estimates of the amplitudes are given here, $R'_i(\nu,0)$ being neglected.

6

The results of the calculations for the amplitudes at $Q^2 = 0$ are plotted in the graphs. The energy of γ-rays is expressed in the part of the threshold energy $\nu_t = 150$ MeV, the values of the amplitudes in the part of e^2/Mc^2.

To calculate the differential cross section for the forward scattering

$$\sigma(0°) = |R_1+R_2|^2 + |R_3+R_4+2R_5+2R_6|^2$$

as is well-known, it suffices to have the amplitudes R_1+R_2 and $R_3+R_4+2R_5+2R_6$.

To estimate $D_1(\nu_o)$ the data on the total cross for γ-ray interaction with protons have been used. These data incorporated also the second maximum and the cross section for production of pion pairs. The dependence $A_1(\nu_o)$ is plotted in Fig. 1. Earlier we neglected the contribution of the energy region higher than 500 MeV. The result of the estimation of the amplitude $R_1 + R_2$ is given in Fig. 2. The main difference with the previous results takes place in the range $1 < \nu_o < 2$, where the values of $D_1(\nu_o)$ decrease considerably due to the compensation of the low-energy limit and of the dispersion contribution. Note, that it is this energy region which turns out to be sensitive to the change of $A_1(\nu_o)$. The second maximum in $A_1(\nu_o)$ corresponds to the second maximum in the photoproduction cross section.

When estimating the real parts of the amplitudes different from $R_1 + R_2$, which require more detailed experimental data on the photoproduction we restrict ourselves to the energy region up to 300 MeV. As for the amplitude $R_1 + R_2$, it was found possible to go further, although with energy the uncertainly of the contribution of the photoproduction of pairs (and of larger number) of pions is demonstrated noticeably.

In some papers[15,16] the process of $\gamma - p$ scattering at 300–800 MeV was treated as a diffraction one with $\mathrm{Re}\, R_i \ll \mathrm{Im}\, R_i$. An experimental investigation of $\gamma - p$ scattering in the range of the second maximum is of interest as a sen. .tive method for studying the maximum itself.

If, by neglecting all $\mathrm{Re}\, R_i$, we restrict ourselves to the imaginary parts of the amplitudes and consider only the contribution proportional to $|E_3|^2$, then from (7) we obtain immediatcly that

$$R_2 = R_4 = R_5 = R_6 = 0\,; \qquad R_1 = \mathrm{Im}\, R_1 = -2\, \mathrm{Im}\, R_3 = 2\nu_c |E_3|^2,$$

and the differential cross section[6] is equal to

$$\sigma(0) = \frac{R_1^2}{8}(7+3\cos^2\theta) = \frac{R_3^2}{2}(7+3\cos^2\theta)\,, \qquad (17)$$

what coincides with the Minami's results. The same result holds for the angular distribution, if in (7) only M_3 ($R_1 \to R_2$, $R_3 \to R_4$) is different from zero. For E_3 and M_3. simultaneously different from zero (at $\mathrm{Re}\, Ri = 0$)

$$\sigma(\theta) = (R_3^2 + R_4^2)\,\frac{7+3\cos^2\theta}{2} + 10 R_3 R_4 \cos\theta. \qquad (18)$$

However, as our estimates show the values $\mathrm{Re}\,(R_1 + R_2)$ cannot be neglected in the region of the second maximum. From this point of view, the second maximum is much different from (3.3) resonance in whose region $\mathrm{Re}\,(R_1 + R_2) \ll \mathrm{Im}\,(R_1 + R_2)$.

The results of the calculations for $R_3 + R_4$, $R_3 + R_4 + 2R_5 + 2R_6$, and $R_5 + R_6$ are presented in Fig. 2–4. In calculating the dispersion integrals $|E_1|^2$, $|M_3|^2$ and $|E_3|^2$ where taken to be different from zero. At the same time, for $|E_1|^2$ and $|M_3|^2$ the energy dependence given in /6/ was used, and $|E_3|^2$ was taken to be different from zero in the energy range of $3.1 < \nu_0 < 5.8$.

Note, that even if the imaginary part of $R_5 + R_6$ is absent, the real part of this quantity is different from the low energy limit, as far as the invariant functions $T_i(\nu)$ satisfy the dispersion relations.

The values of $\sigma(0^\circ)$ are plotted in Fig. 5 where the result of Cini and Stroffolini for $\sigma_c(0^\circ)$ in the c.m.s. is given for comperison. As appreciable difference is observed in the threshold region.

7

To estimate $R_1 - R_2$ and $R_5 - R_6$ dispersion relations (6) are not sufficient. Let us consider the function

$$F(\nu) = \frac{1}{W^2}\phi(\nu) = \frac{M^2\nu^2}{2W^2}[(T_1+T_3)' - \nu(T_2+T_4)']. \tag{19}$$

As one can see from (B.4)

$$F(\nu) = \frac{W}{M}\{R_1 - R_2 - \frac{2M\nu}{W(W+M)}(R_3 - R_4)\}. \tag{20}$$

The consideration of the dispersion relations for $F(\nu)$ at the known $R_3 - R_4$ enables us to estimate $R_1 - R_2$. In the energy region we are considering the coefficient before $(R_3 - R_4)$ in (20) is of the order of ν/M. However, as far as the value $R_3 - R_4$ is great (if compared with $R_1 - R_2$), the second component in (20) cannot be neglected.

The function $\phi(\nu)$ introduced in (19) is an analytical function of ν with the cuts along $\nu_t < \nu < \infty$ which satisfies the following property of the crossing-symmetry

$$\phi(\nu) = \phi^*(-\nu). \tag{21}$$

Thus, for $\nu \ll \nu_t$, $\phi(\nu)$ is a real function and

$$\phi(\nu) \cong a + b\nu^2, \tag{22}$$

whereas

$$F(\nu) = \frac{\phi(\nu)}{M^2+2M\nu} \cong \frac{a}{M^2}(1 - \frac{2\nu}{M}) + b\nu^2 + \dots \tag{23}$$

We see that the linear term in $F(\nu)$ is wholly determined by the first term in (22). Then, it follows from (20), that for small

$$R_1 - R_2 = -\frac{e^2}{M}\left(1-\frac{3\nu}{M}\right) + O(\nu^2)$$

and the linear term in $R_1 - R_2$ and in $F(\nu)$ is fully determined by the crossing-symmetry requirement what is discussed in detail in B.

$F(\nu)$ introduced in (19) is an analytical function of ν with the cuts along $\nu_t < \nu < \infty$ and $-\infty < \nu < -\nu_t$ and a (kinematic) pole at

$$W^2 = M^2 + 2M\nu = 0 .$$

The crossing-symmetry requirements lead to

$$F(-\nu) = \frac{M^2+2M\nu}{M^2-2M\nu} F^*(\nu) \quad ,$$

and for small ν

$$F(\nu) \simeq -\frac{e^2}{M}\left(1-\frac{2\nu}{M}\right) + O(\nu^2).$$

By applying the Cauchy formula with a contour drawn in Fig. 6 with $\rho \to \infty$, to $F(\nu_0)$ and writing down the dispersion relations with subtraction, we get

$$F(\nu_0) = -\frac{e^2}{M}\left(1-\frac{2\nu_0}{M}\right) + \frac{\nu_0^2}{2\pi i}\int_C \frac{F(\nu)\,d\nu}{\nu^2(\nu-\nu_0)} = -\frac{e^2}{M}\left(1-\frac{2\nu_0}{M}\right) + \frac{\nu_0^2}{\pi} P\int_{\nu_t}^{\infty}\frac{Im\,F}{\nu^2}\left[\frac{1}{\nu-\nu_0} + \frac{M^2+2M\nu}{M^2-2M\nu}\,\frac{1}{\nu+\nu_0}\right]d\nu +$$

$$+\frac{\nu_0^2}{2\pi i}\int_{C_+}\frac{F(\nu+i\epsilon)}{\nu^2(\nu-\nu_0)}\,d\nu + \frac{\nu_0^2}{2\pi i}\int_{C_-}\frac{F(\nu-i\epsilon)}{\nu^2(\nu-\nu_0)}\,d\nu$$

and

$$Re\,F(\nu_0) = -\frac{e^2}{M}\left(1-\frac{2\nu_0}{M}\right) + K(\nu_0) + \frac{4\nu_0^2\,Re\,F(\,M/2\,)}{M(\nu_0 + M/2\,)} \;, \qquad (24)$$

where

$$K(\nu_0) = \frac{\nu_0^2}{\pi}\int_{\nu_t}^{\infty}\frac{Im\,F(\nu)}{\nu^2}\left[\frac{1}{\nu-\nu_0} + \frac{M^2+2M\nu}{M^2-2M\nu}\,\frac{1}{\nu+\nu_0}\right]d\nu \qquad (25)$$

Since

$$K(M/2) = 0 ,$$

it is impossible to determine Re F (M/2) from (24), and this quantity enters as a free parameter which is necessary to determine from the experimental data. If we restrict ourselves to the photoproduction in the states with $J \leq 3/2$, then

$$Im\,F(\nu) = \nu \{ \frac{M}{W} (|E_1|^2 - |M_1|^2) + 2(|E_3|^2 - |M_3|^2)(1 + \tfrac{1}{2} \frac{W-M}{W}) +$$

$$+ \tfrac{1}{4} \frac{W+M}{W} (|E_2|^2 - |M_2|^2) - \frac{W-M}{W} Re(E_3^* M_2 - E_2 M_3^*) \} .$$

In Fig. 7 are given the results of the estimate of Re $(R_1 - R_2)$ (24), the contribution proportional to Re F (M/2) being neglected.

As is seen from (4.B), in order to estimate $R_5 - R_6$ at $Q^2 = 0$ it suffices to consider the function

$$\psi(\nu_0) = \frac{\nu_0^2}{2} [T_5' + \tfrac{1}{2} (T_1 + T_3)'] = (\frac{W_0}{M})^3 \{ \frac{W_0}{\nu_0} (R_5 - R_6) + \frac{M}{W_0 + M} [R_1 - R_2 - (R_3 - R_4)] \} , \qquad (26)$$

the dispersion relations for which are of the form

$$Re\,\psi(\nu_0) - \psi(0) = \frac{2\nu_0^2}{\pi} \int_{\nu_t}^{\infty} \frac{Im\,\psi(\nu)\, d\nu}{\nu(\nu^2 - \nu_0^2)} , \qquad (27)$$

where, as it follows from (2.B)

$$\psi(0) = - \frac{e^2}{M} \cdot \frac{2+\lambda}{2} , \qquad (28)$$

and $Im\,\psi(\nu) = (\frac{W}{M})^2 \{ - W [\frac{1}{6} (|E_2|^2 - |M_2|^2) + Re(E_2^* M_3 - M_2^* E_3)] + \frac{M\nu}{(M+W)} [3 (|E_3|^2 - |M_3|^2) +$

$$+ \tfrac{1}{4} (|E_2|^2 - |M_2|^2) + Re(E_2^* M_3 - M_2^* E_3)]\} . \qquad (29)$$

The results of the estimation for Re $(R_5 - R_6)$ at $Q^2 = 0$ are given in Fig. 4 . The estimates of the quantities $R_3 + R_4$ and $R_5 - R_6$ which play the main part in the differential cross section at $\nu \gtrsim 1$ are not much different from those obtained earlier /6/.

These results are of interest for studying the dependence of the amplitudes near the threshold for a new reaction. /6/. In the case under consideration all the estimates can be made completely. We stress the dependence of the amplitude Re $(R_1 + R_2)$ the value of which keeps on decreasing higher than the threshold. This result points out that for other processes the sharp dependence of the imaginary parts of the amplitudes higher than the threshold may lead to the displacement of the threshold minimum (or maximum) of the cross section from the reaction threshold.

The results of the calculations with the aid of $R_i(\nu, 0)$ for the angular distributions

$$\sigma(\theta) = \sum_{l=0}^{3} B_l \cos^l \theta$$

for $\theta = 90^\circ$, 135°, 139° and 180°, as well as for the total elastic scattering cross section

$$\frac{\sigma_s}{4\pi} = B_0 + \tfrac{1}{2} B_2$$

and for the recoil nucleon polarization at $\theta = 90^\circ$ are presented in Figs. 5, 8 – 12.

The experimental data are summarized in[10] and[17]. The coefficient

$$B_3(\nu_0) = 2\,[\,|R_5 + R_6|^2 - |R_5 - R_6|^2\,]$$

is close to zero throughout the energy region $\nu_0 \leqslant 2$.

The experimental data are seems to indicate, that the quantity $\mathrm{Re}\,(R_5 - R_6)$ is positive. However, we could not do it by introducing $\mathrm{Re}\,F(M/2) \neq 0$. The requirement for $\mathrm{Re}\,(R_5 - R_6)$ to be positive leads to large (negative) values of $\mathrm{Re}\,F(M/2)$ what increases considerably the contribution $|R_1 - R_2|^2$ to the cross section and fails to improve the agreement with the experimental data.

It is worth while noting that outside the region $1 < \nu_0 < 1.3$ there is a satisfactory agreement between the dispersion analysis and experimental data. In the region $1 < \nu_0 < 1.3$ especially sensitive to the dispersion effects if is, probably, necessary to consider the contribution of higher states for which more information on the pion photoproduction in the wide energe region is required.

V_0	0	0,5	0,8	0,90	0,95	0,97	I,00	I,05	I,I0	I,20	I,50	2,00	2,50
$Re\,(R_1+R_2)$	-I	-0,8	-0,55	-0,38	-0,29	-0,23	-0,I7	-0,I4	-0,I2	-0,09	-0,06	-0,I6	-0,52
$Im\,(R_1+R_2)$	0	0	0	0	0	0	0	0,I5	0,23	0,35	0,83	2,83	2,2I
$\|R_1+R_2\|^2$	I	0,64	0,30	0,I4	0,08	0,05	0,03	0,04	0,06	0,I3	0,69	8,0	5,I5
$Re\,(R_1-R_2)$	-I	-0,74	-0,54	-0,44	-0,38	-0,34	-0,20	-0,23	-0,26	-0,29	-0,43	-0,05	+I,7I
$Im\,(R_1-R_2)$	0	0	0	0	0	0	0	0,I4	0,20	0,26	0,22	-I,03	-0,48
$\|R_1-R_2\|^2$	I	0,54	0,29	0,I9	0,I5	0,I2	0,04	0,07	0,II	0,I5	0,23	I,06	3,I5
$Re\,(R_3+R_4)$	0	-0,29	-0,43	-0,44	-0,43	-0,4I	-0,34	-0,42	-0,50	-0,68	-I,I2	-I,78	-I,70
$Im\,(R_3+R_4)$	0	0	0	0	0	0	0	0,I4	0,2I	0,28	0,38	-0,I5	+0,I3
$\|R_3+R_4\|^2$	0	0,08	0,I8	0,I9	0,I8	0,I7	0,I2	0,20	0,25	0,54	I,4I	3,20	2,87
$Re\,(R_3-R_4)$	0	0,26	0,50	0,60	0,68	0,72	0,83	0,88	0,93	I,0I	I,I2	I,20	0,85
$Im\,(R_3-R_4)$	0	0	0	0	0	0	0	0,I5	0,27	0,32	0,63	I,6I	I,35
$\|R_3-R_4\|^2$	0	0,07	0,25	0,36	0,46	0,52	0,69	0,80	0,94	I,I2	I,65	4,04	2,54
$Re\,(R_5+R_6)$	0	0,08	0,II	0,II	0,II	0,II	0,I2	0,I2	0,I0	0,II	0,I3	0,25	-0,7
$\|R_5+R_6\|^2$	0	0,006	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,06	0,49
$Re\,(R_5-R_6)$	0	-0,08	-0,I0	-0,II	-0,II	-0,II	-0,I2	-0,II	-0,II	-0,II	-0,II	-0,09	-0,08
$\|R_5-R_6\|^2$	0	0,006	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,0I	0,006
$Re\,(R_3+R_4+2R_5+2R_6)$	0	-0,I3	-0,20	-0,2I	-0,20	-0,I8	-0,I0	-0,I9	-0,26	-0,45	-0,86	-I,29	-3,06
$\|R_3+R_4+2R_5+2R_6\|^2$	0	0,02	0,04	0,05	0,04	0,03	0,0I	0,06	0,II	0,28	0,90	I,68	9,36

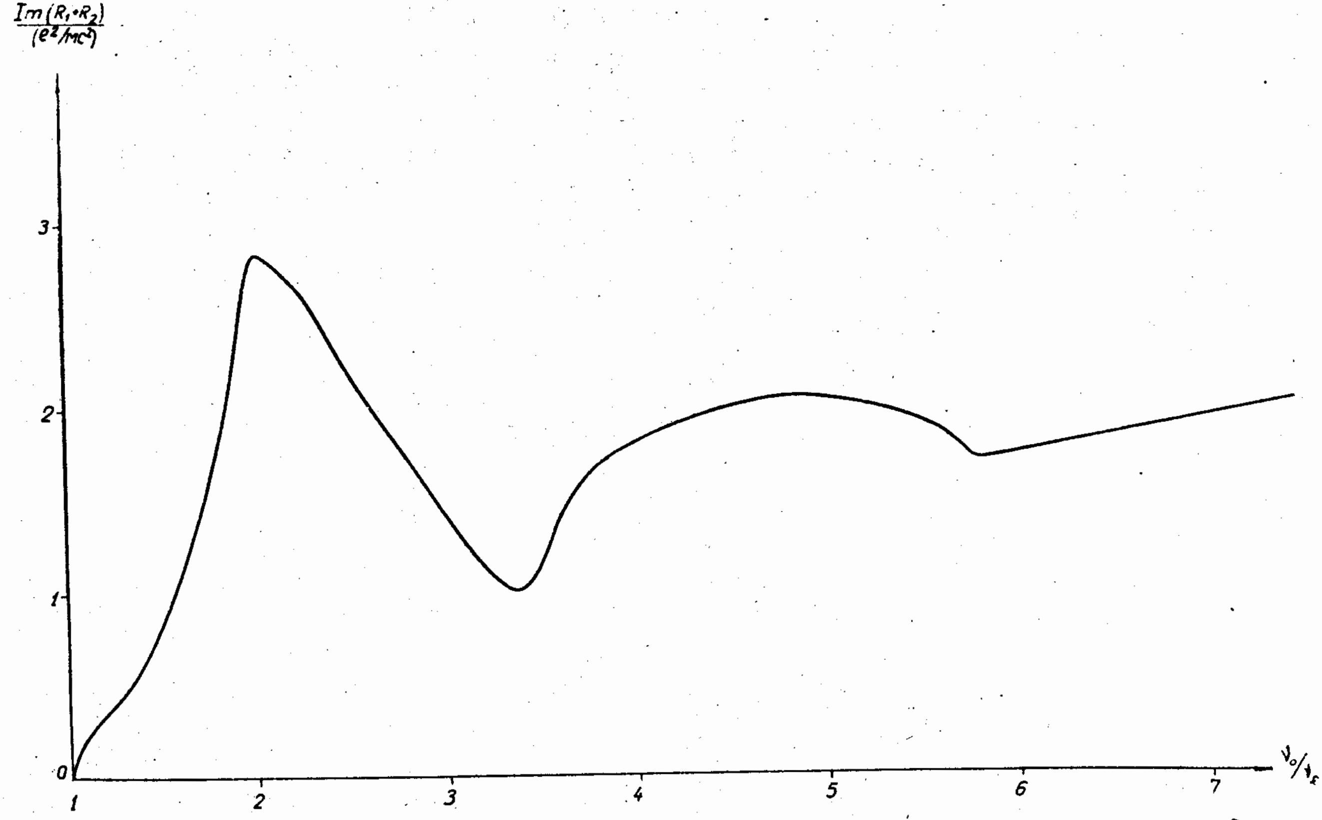


Fig. 1

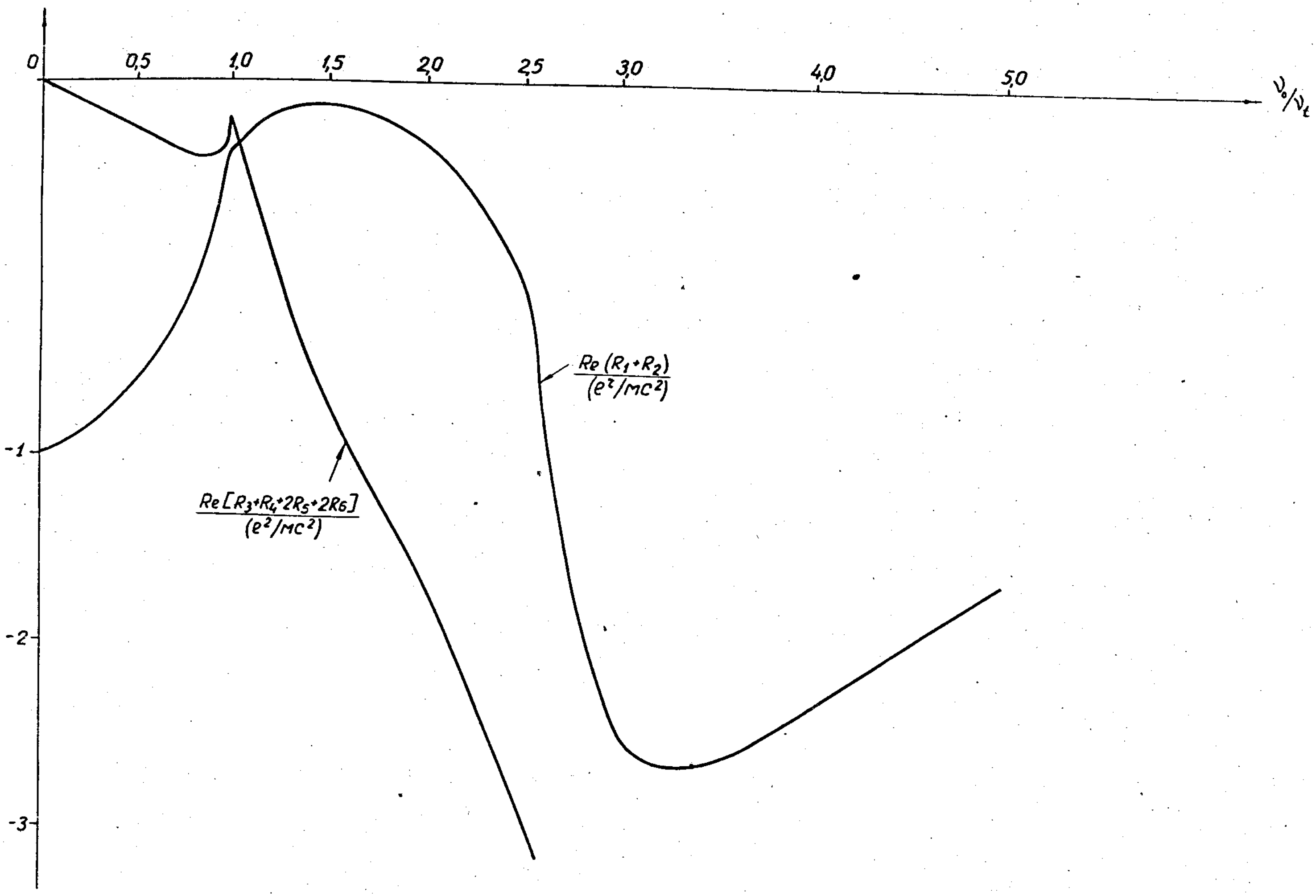

0
0,5
1,0
1,5
2,0
2,5
3,0
4,0
5,0
ν_0/ν_t
-1
-2
-3
$\frac{Re\,(R_1+R_2)}{(e^2/mc^2)}$
$\frac{Re\,[R_3+R_4+2R_5+2R_6]}{(e^2/mc^2)}$

Fig. 2

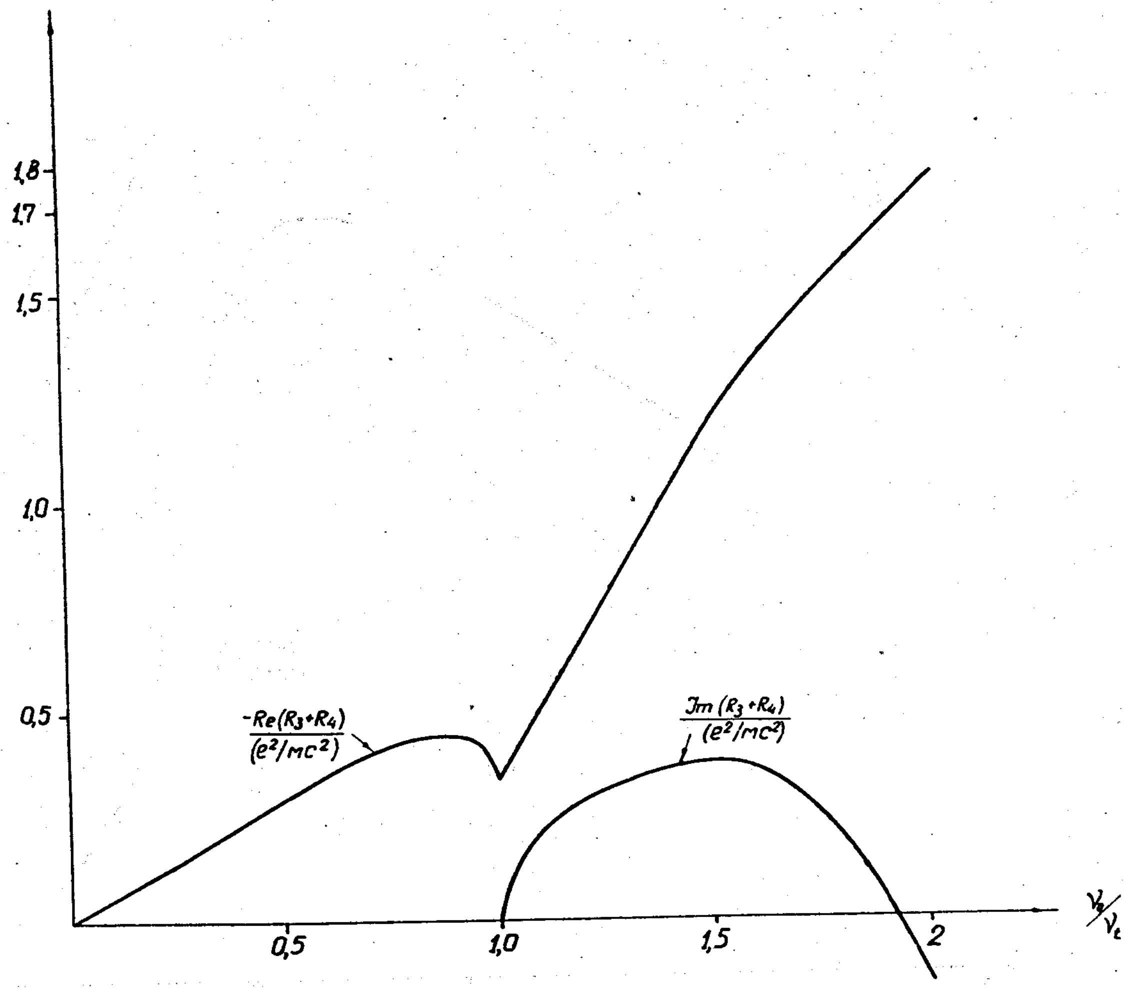
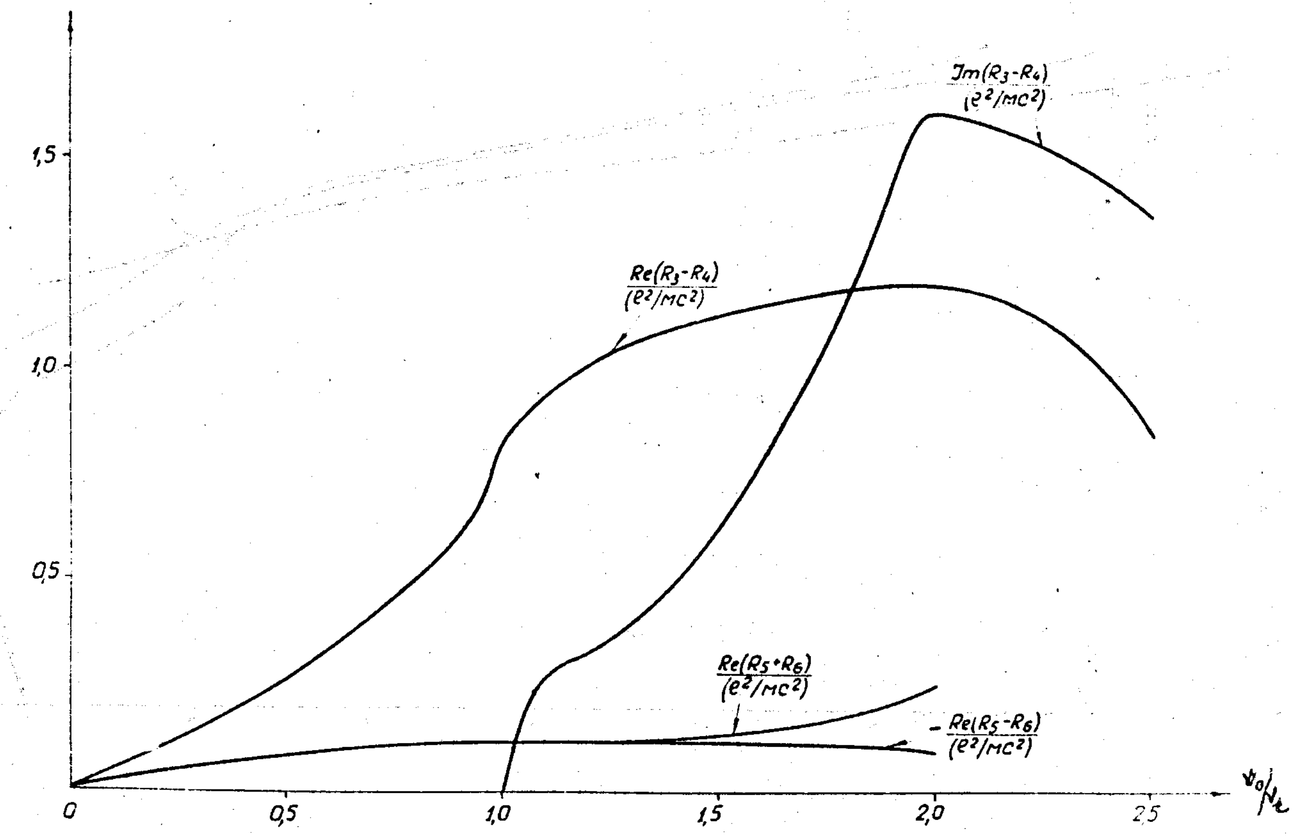


Fig. 3



$\frac{Jm(R_3-R_4)}{(e^2/mc^2)}$
$\frac{Re(R_3-R_4)}{(e^2/mc^2)}$
$\frac{Re(R_5+R_6)}{(e^2/mc^2)}$
$-\frac{Re(R_5-R_6)}{(e^2/mc^2)}$
1,5
1,0
0,5
0
0,5
1,0
1,5
2,0
2,5
ν_0/ν_k

Fig. 4

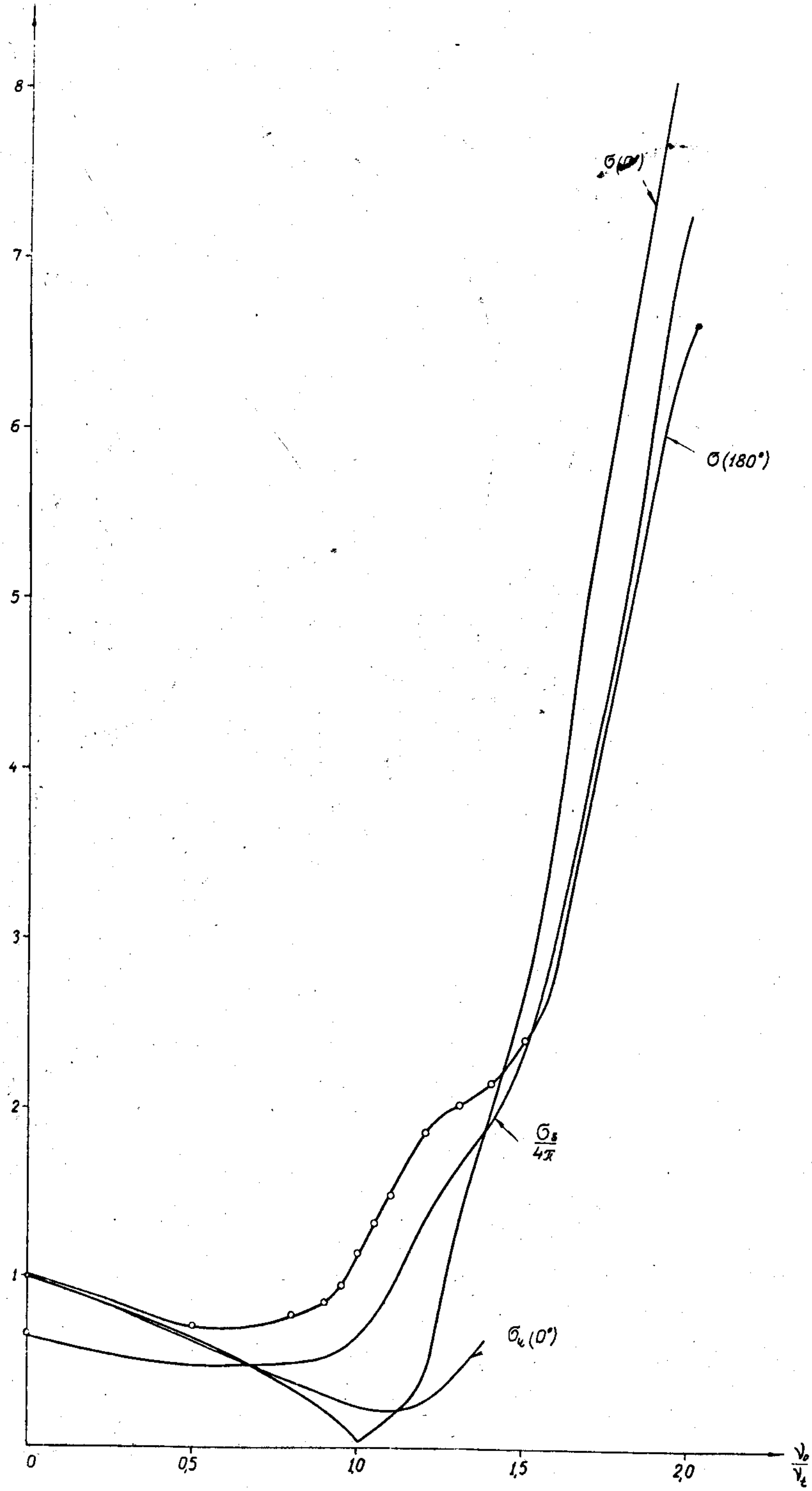

8
7
6
5
4
3
2
1
0
0,5
1,0
1,5
2,0
σ(0°)
σ(180°)
$\frac{\sigma_s}{4\pi}$
$\sigma_u(0°)$
$\frac{\nu_0}{\nu_t}$

Fig. 5

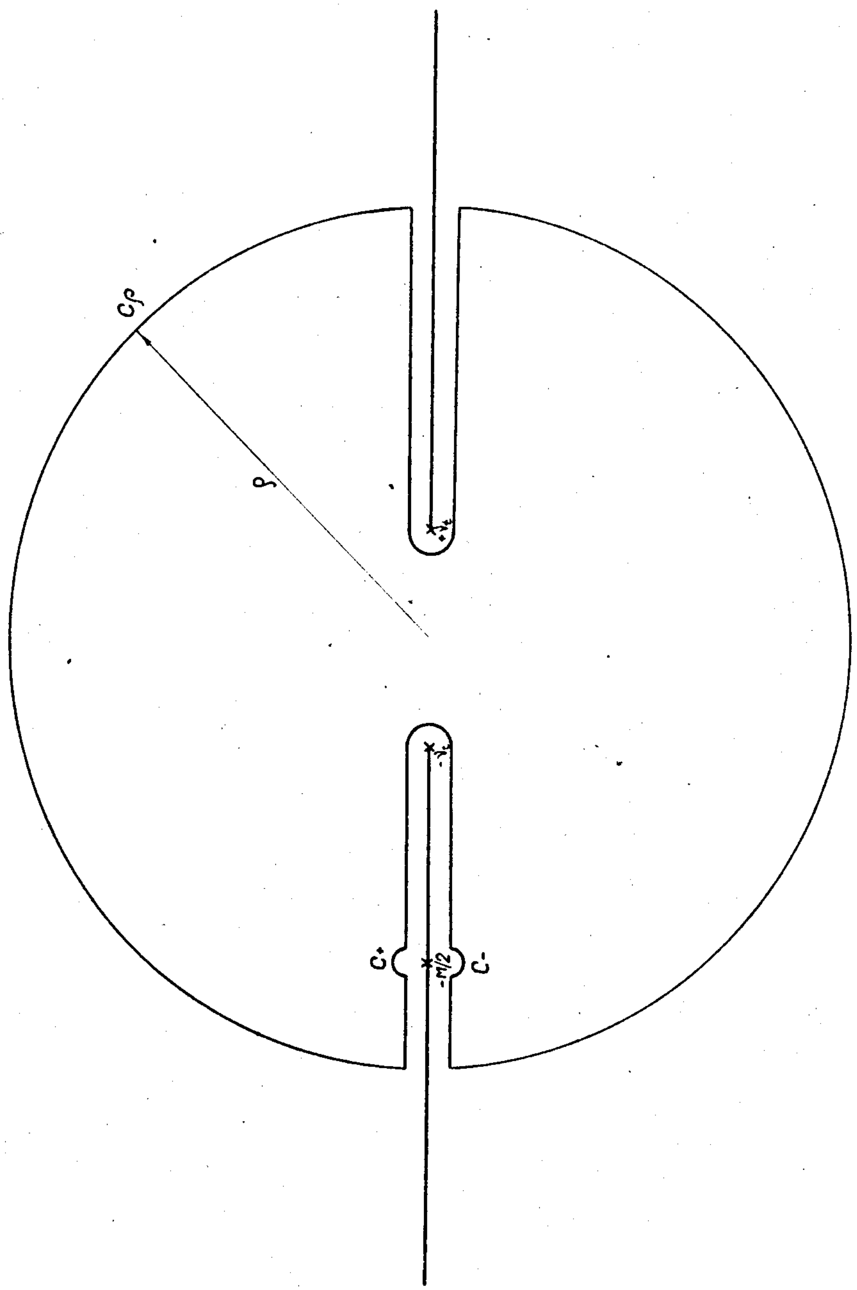


Fig. 6

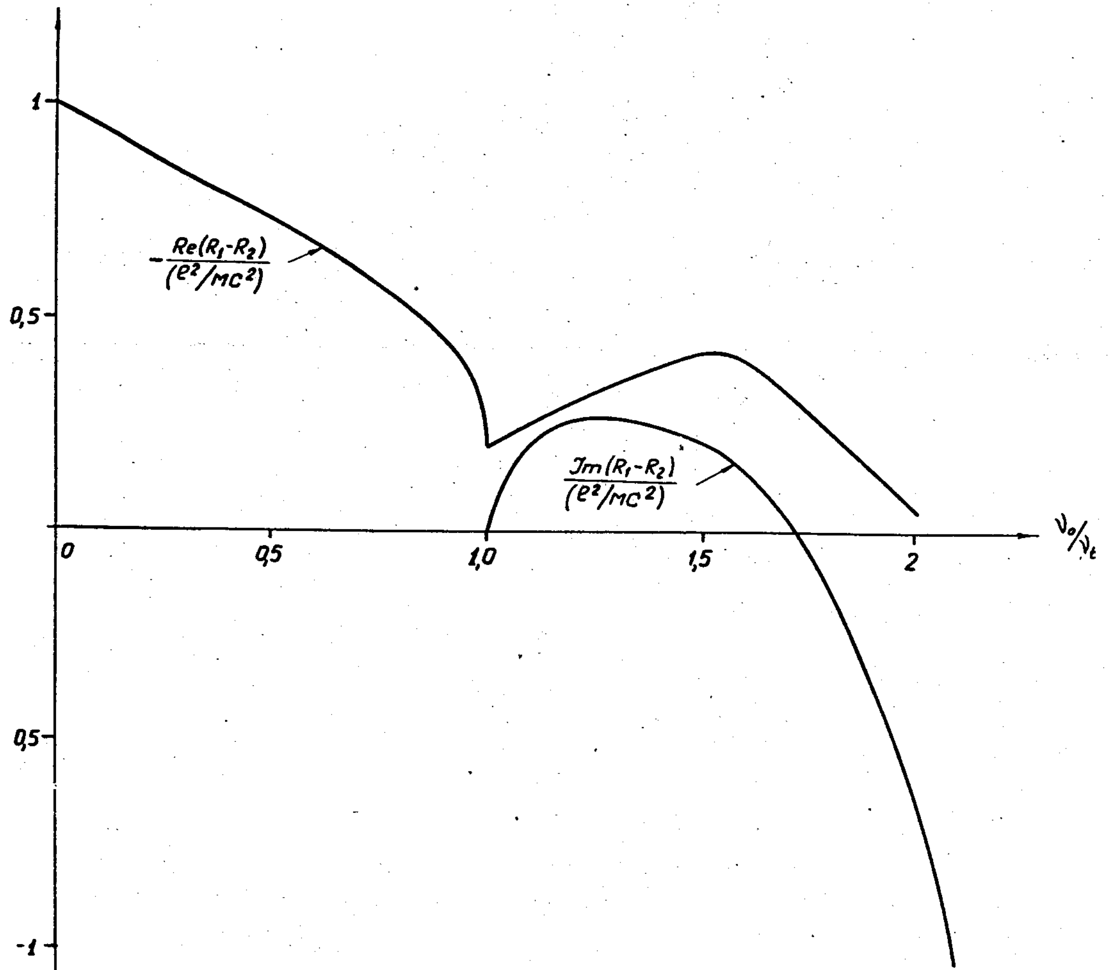

1
0,5
0,5
-1
0
0,5
1,0
1,5
2
ν_0/ν_t
$-\frac{Re(R_1-R_2)}{(e^2/mc^2)}$
$\frac{Jm(R_1-R_2)}{(e^2/mc^2)}$

Fig. 7

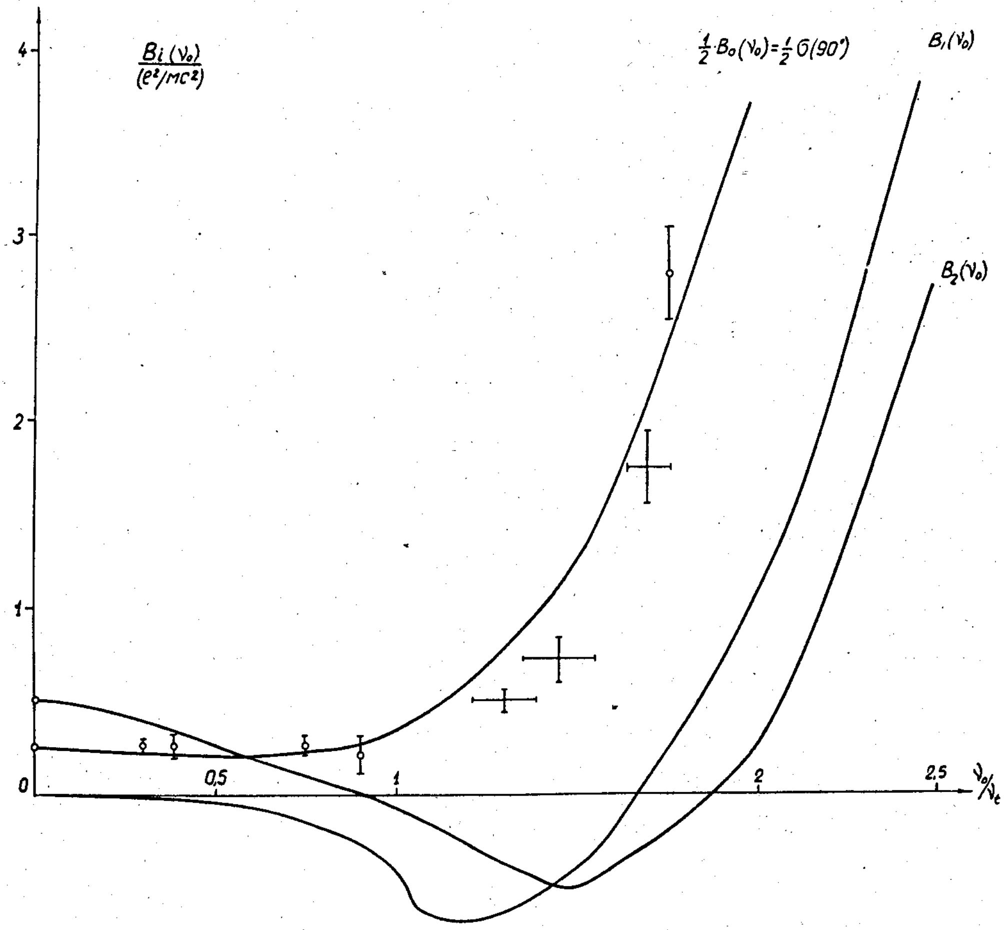

$\frac{B_i(\nu_0)}{(e^2/mc^2)}$
$\frac{1}{2} B_0(\nu_0) = \frac{1}{2} \sigma(90°)$
$B_1(\nu_0)$
$B_2(\nu_0)$
ν_0/ν_t
0
1
2
3
4
0,5
1
2
2,5

Fig. 8

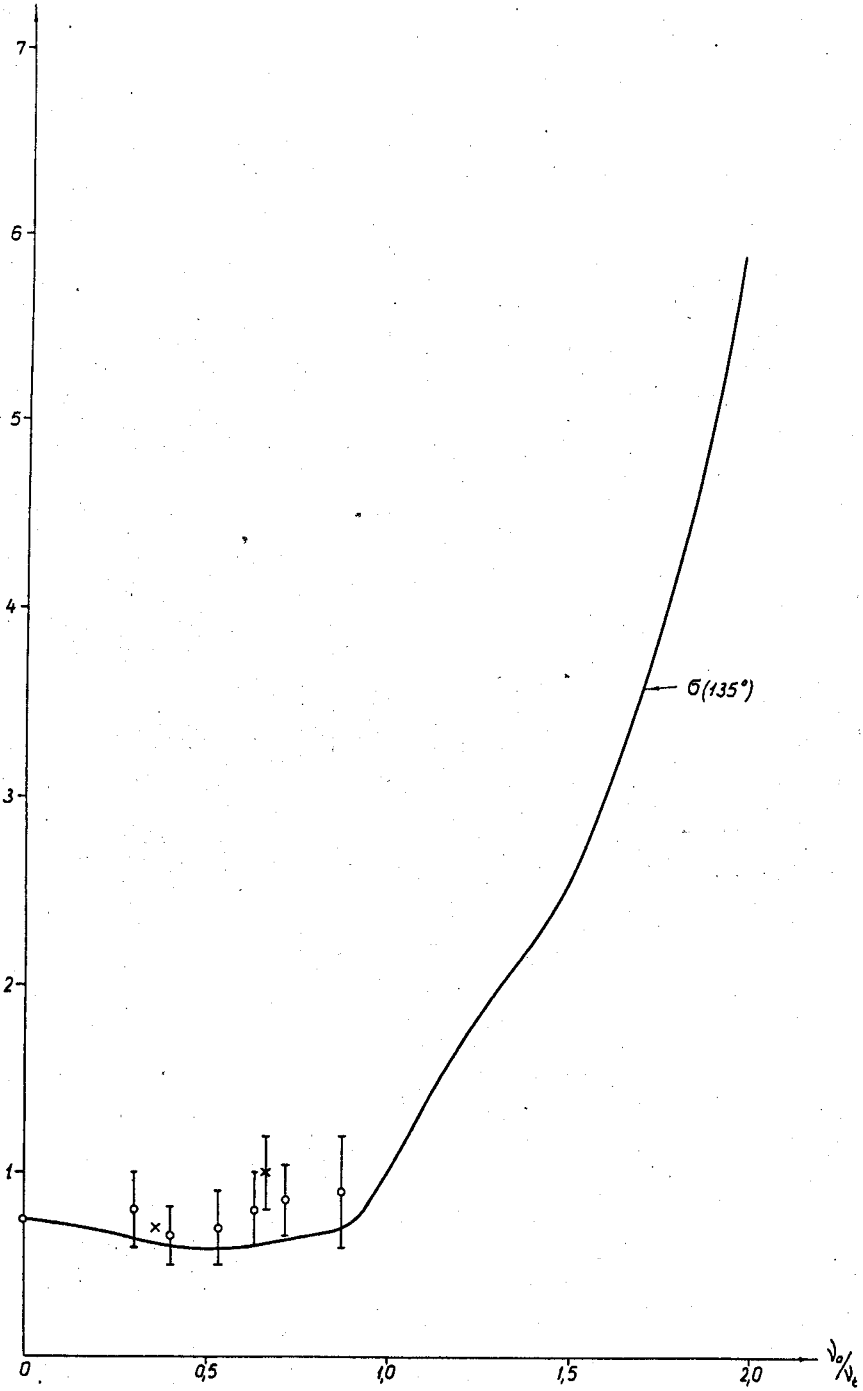


Fig. 9

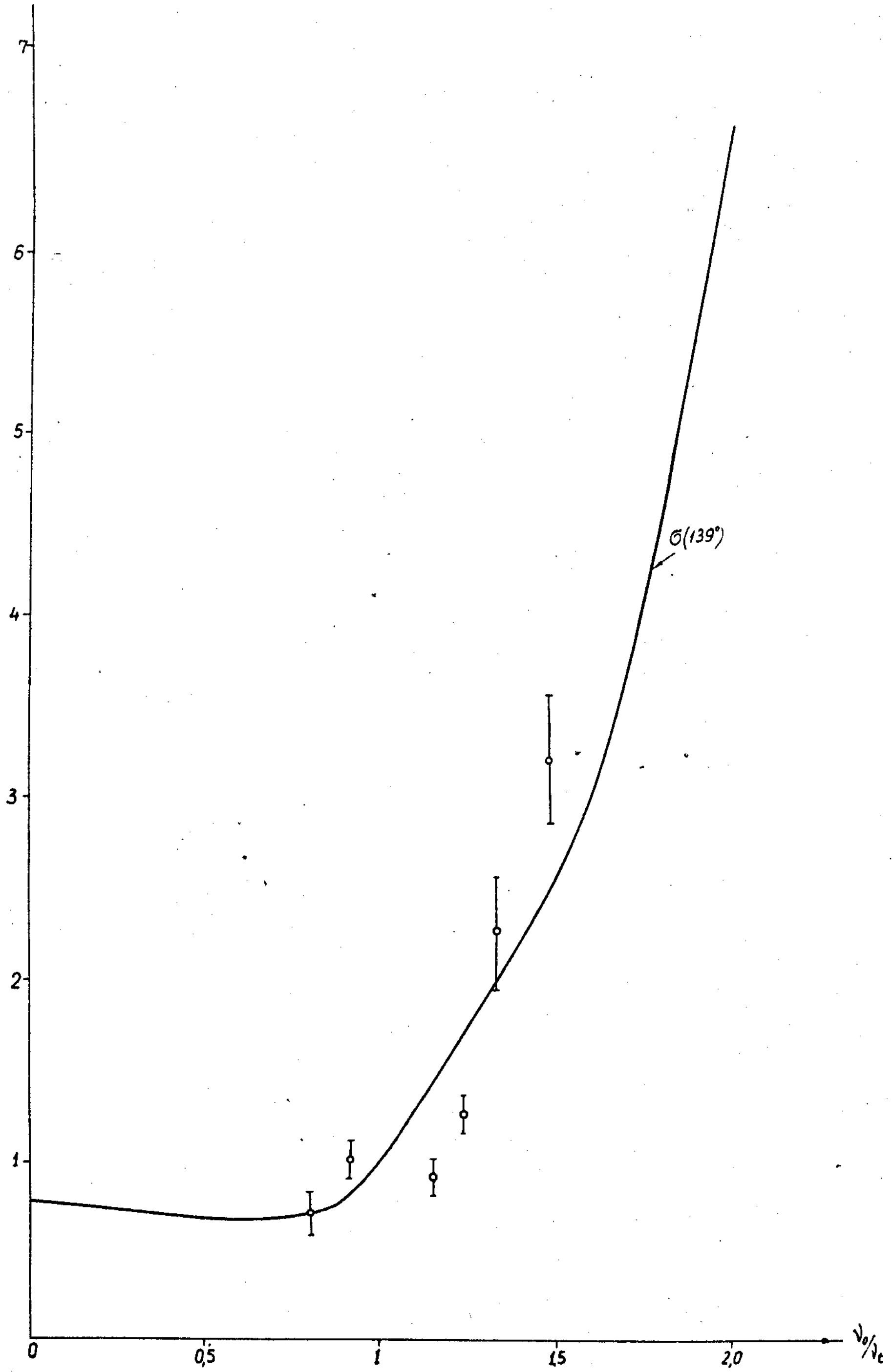
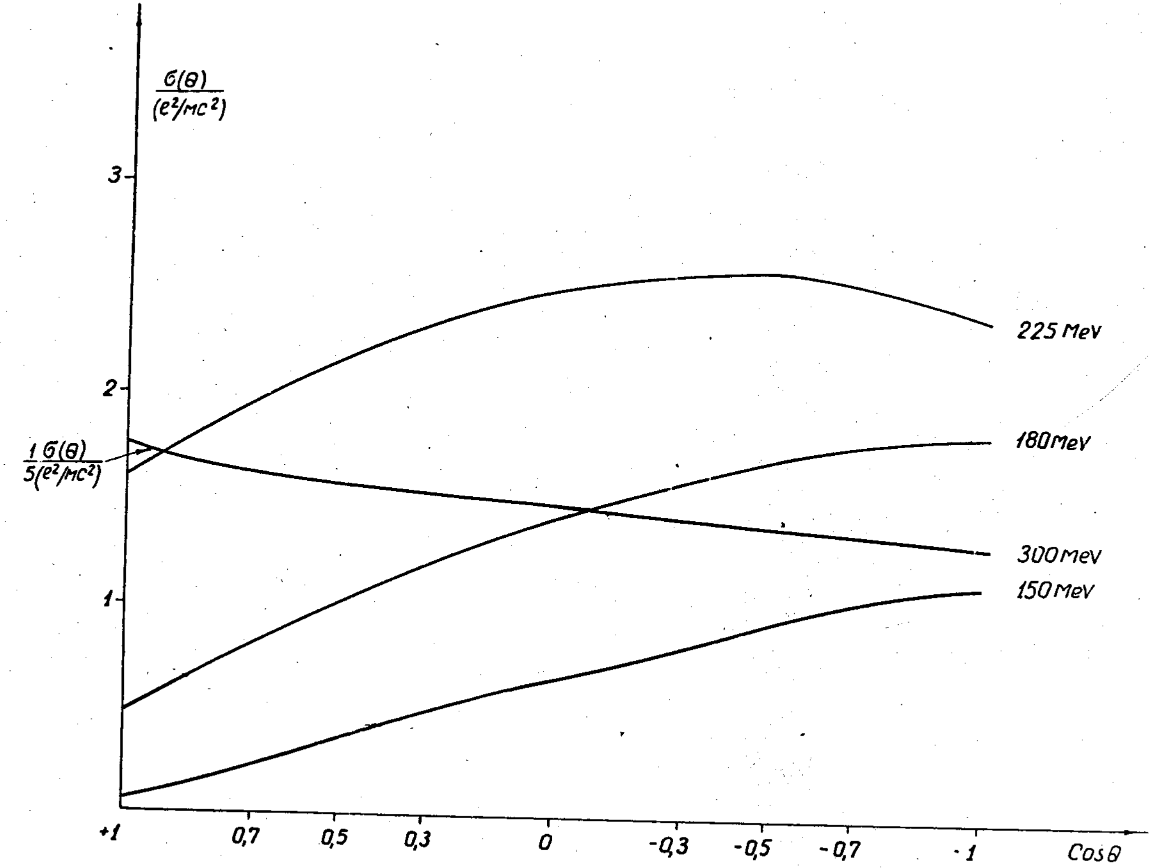


Fig. 10



$\frac{\sigma(\theta)}{(e^2/mc^2)}$
3
2
1
$\frac{1}{5}\frac{\sigma(\theta)}{(e^2/mc^2)}$
225 MeV
180 MeV
300 MeV
150 MeV
+1
0,7
0,5
0,3
0
-0,3
-0,5
-0,7
-1
Cos θ

Fig. 11

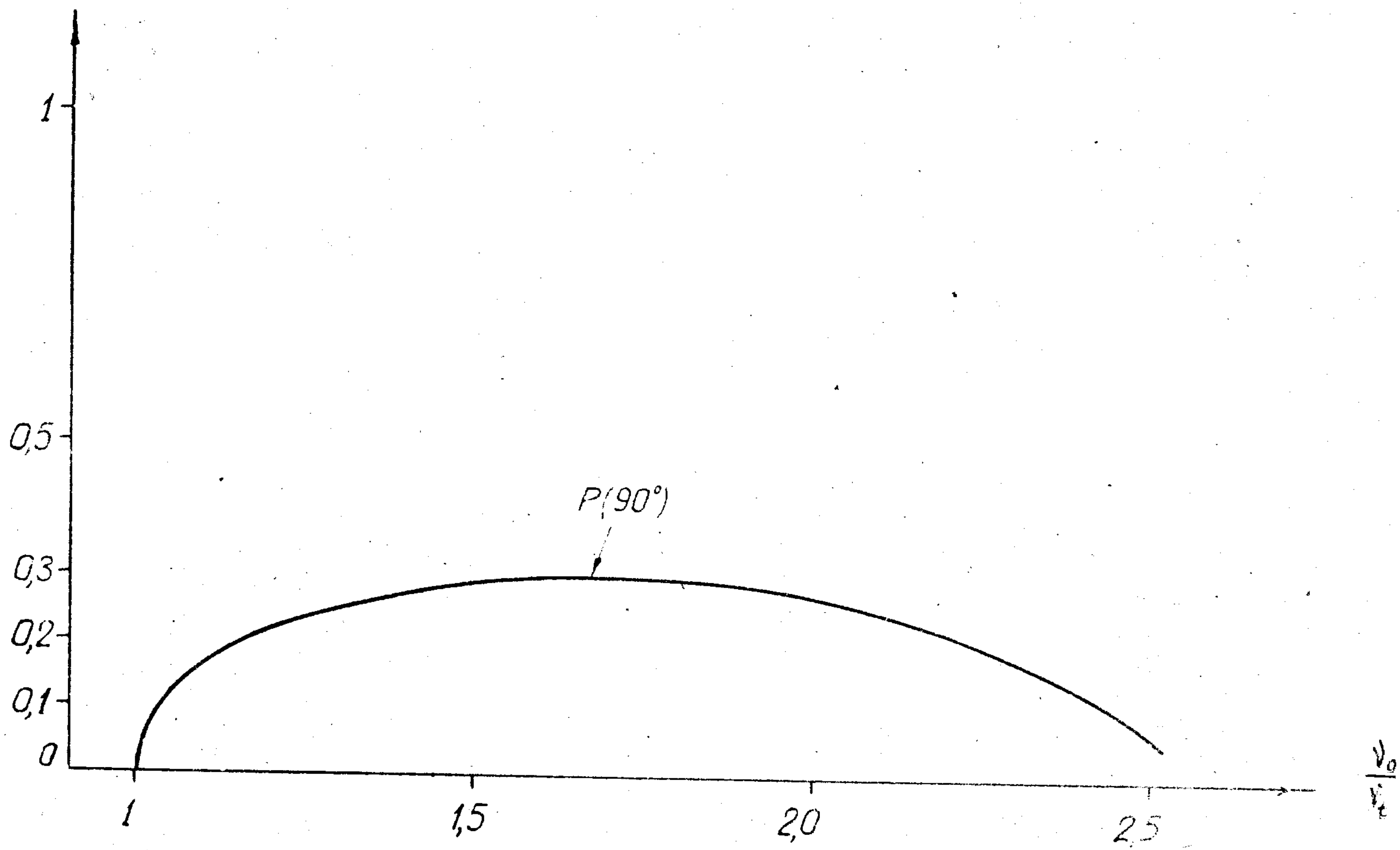


Fig. 12

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Работа поступила в издательский отдел
6 мая 1961 г.