ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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# τ – DECAY AND ππ – INTERACTION

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# $\tau$ -DECAY AND $\pi\pi$ -INTERACTION

*Integral equations are derived for the determination of the $\tau$- decay. An effective range analysis gives the values of the $s$ - wave scattering-lengths for the $\pi\pi$ - interaction, which are suitable to reproduce the experimental $\tau$ - spectrum. A good fit is obtained for $a_2 \approx 0{,}2$ ; $a_0 \approx -0{,}3$ . These values of $a_2$ and $a_0$ in combination with the integral equations for $\pi\pi$ - scattering, hint at the existence of a $(T=2)$ - resonance of the $\pi\pi$ - interaction.*

## 1. Introduction

The $\tau$- decay is one of the few processes which enables the investigation of the $\pi\pi$- interaction without perturbing influence of other particles. The low kinetic energy of the outgoing pions facilitates the theoretical analysis, and the fairly extensive experimental dates allow a comparison of the theory with experiment.

For the theoretical treatment of the $\tau$ - decay the Mandelstam-representation suggests itself. But for decay processes the usual form of this representation does not exactly hold/1/, because in general the spectral functions become complex in this case. In reference/2/it was shown that the imaginary parts of the spectral functions correspond to 3-particle intermediate reactions*. Here we consider the usual 2-particle approximation to which those imaginary parts give no contribution. We look into the consequences of this approach which was first used by Khuri and Treiman/3/.

In the following, integral equations are written down for the $\tau$- decay which also take into account $p$-wave interaction of the pions, and thus in principle offer a possibility for testing solutions of $\pi\pi$-scattering integral equations.

In order to get some first information about the $\pi\pi$-interaction, for example scattering lengths, the $p$-wave terms are not expected to play an important role for $\tau$- decay. Comparison of the calculated matrix element obtained with the experimental $\tau$ - spectrum gives an estimate of the $s$- wave scattering-lengths of the $\pi\pi$-interaction.

The knowledge of these scattering-lengths together with the integral equations of paper/4/ offer further information about the $\pi\pi$-interaction.

## 2. $p$- wave integral equations

In order to obtain equations for $\tau^{+}$- and $\tau'^{+}$- decay we consider the following reactions

$$\tau^{+} + \pi_1^{-} \rightarrow \pi_2^{+} + \pi_3^{-} , \qquad (1)$$

*We wish to thank G.Bonnevay for sending us a preprint in which these questions are also discussed.

$$K^+ + \pi_2^- \to \pi_3^- + \pi_1^+ \,, \qquad \text{(II)}$$

$$K^+ + \pi_3^+ \to \pi_1^+ + \pi_2^+ \,; \qquad \text{(III)}$$

$$K^+ + \pi_1^0 \to \pi_2^0 + \pi_3^+ \,, \qquad \text{(I')}$$

$$K^+ + \pi_2^0 \to \pi_3^+ + \pi_1^0 \,, \qquad \text{(II')}$$

$$K^+ + \pi_3^- \to \pi_1^0 + \pi_2^0 \,. \qquad \text{(III')}$$

The invariant variables of these processes take the following form in the c. m. system of reaction III:

$$\begin{aligned} s_1 &= \gamma - 2q_3^2 + 2p_3 q_3 z_3 \,, \\ s_2 &= \gamma - 2q_3^2 - 2p_3 q_3 z_3 \,, \\ s_3 &= 4(q_3^2 + \mu^2) \,. \end{aligned} \qquad (1)$$

Here $m$, $\mu$ denote the mass of the K--meson and pion respectively; $\gamma = \frac{1}{2}(m^2 - \mu^2)$. $p_3$ and $q_3$ are the momenta of the particles before and after the collisions, $z_3$ denotes the cosine of the scattering angle. Analogous relations hold in the c.m. systems of the other reactions. The $s_i$ obey the condition

$$s_1 + s_2 + s_3 = m^2 + 3\mu^2 . \qquad (2)$$

In the following we also use the invariant combinations $\eta_i$ defined by

$$\begin{aligned} 2\eta_1 &= s_2 - s_3 = 4 p_1 q_1 z_1 \,, \\ 2\eta_2 &= s_3 - s_1 = 4 p_2 q_2 z_2 \,, \\ 2\eta_3 &= s_1 - s_2 = 4 p_3 q_3 z_3 \,. \end{aligned} \qquad (3)$$

We are particularly interested in the values of the invariant variables for $z_i = \pm 1$ ; one obtains

$$s_1 s_2 s_3 = \mu^2 (m^2 - \mu^2)^2 \qquad \text{for } z_i = \pm 1 . \qquad (4)$$

Fig. 1 shows the relation (4). Here I,II,III are the physical regions of the corresponding scattering processes IV corresponds to the decay process, which in the case of a non-real K-meson mass $m = 3\mu$ shrinks into a point. For real $s_i = 4(v_i + 1)$ * with $m = 3.6$ the product $(p_i q_i)$ becomes complex for

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* We set $v_i = q_i^2$ ; $\mu^2 = 1$.

We assume the following representation

$$A(s_1,s_2,s_3)=\frac{1}{\pi}\int_4^\infty \frac{\rho(s')+\eta_1\sigma(s')}{s'-s_1}ds'+\frac{1}{\pi}\int_4^\infty \frac{\rho(s')-\eta_2\sigma(s')}{s'-s_2}ds'+\frac{1}{\pi}\int_4^\infty \frac{\lambda(s')}{s'-s_3}ds' \quad , \tag{5}$$

which for $m \leq 3$ can be obtained as the Cini–Fubini $p$-wave approximation/5/from the usual Mandelstam representation. With $\sigma = 0$ , as $s$-wave approximation, relation (5) has been used in references/3,6/.

From (5) and from the unitarity conditions for the processes I and III we have to obtain equations, which determine the functions $\rho$ , $\lambda$ , $\sigma$ .

We write the partial-wave development of the scattering amplitudes in the following form

$$A(\nu,z) = A_0(\nu) + 3\,pq\,z\,A_1(\nu) \tag{6}$$

and define $s$- and $p$-waves respectively by

$$A_0(\nu) = \frac{1}{2}\left\{A(+)+A(-)\right\} , \tag{7a}$$

$$A_1(\nu) = \frac{1}{6pq}\left\{A(+)-A(-)\right\} . \tag{7b}$$

Here, $A(\pm) = A(\nu, z=\pm 1)$ ; $d$- and higher waves are neglected.
In the following we denote the $s$-wave amplitudes of process III and I by $F_0$ and $G_0$ , and those of III′ and I′ by $f_0$ and $g_0$ , respectively; the $p$-waves are denoted accordingly by $G_1$ , $g_1$ . From (5), (7) and (1) we get the following equations

$$\begin{aligned} F_0(s) = \lambda^F &+ \frac{s-4}{\pi}\int_4^\infty \frac{ds'\,\lambda(s')}{(s'-4)(s'-s)} + \frac{2}{\pi}\int_4^\infty \frac{ds'\,\rho(s')}{s'-\gamma}\cdot\frac{(\alpha-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} \\ &- \int_4^\infty ds'\,\sigma(s')\,\frac{(6\nu+4-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} - \frac{\gamma-4}{\pi}\int_4^\infty \frac{ds'\,\sigma(s')}{s'-\gamma} \quad , \end{aligned} \tag{8}$$

$$\begin{aligned} G_0(s) = \lambda^G &+ \frac{s-4}{\pi}\int_4^\infty \frac{ds'\,\rho(s')}{(s'-4)(s'-s)} + \frac{1}{\pi}\int_4^\infty \frac{ds'}{s'-\gamma}\left\{\rho(s')+\lambda(s')\right\}\frac{(\alpha-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} \\ &+ \frac{1}{2\pi}\int_4^\infty ds'\,\sigma(s')\,\frac{(6\nu+4-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} + \frac{\gamma-4}{2\pi}\int_4^\infty \frac{ds'\,\sigma(s')}{s'-\gamma} \quad , \end{aligned} \tag{9}$$

where

$$\alpha = \frac{m^2-1}{2} - 2\nu = \gamma - 2\nu$$

For $F_0$ a subtraction was made at the point $s_1 = s_2 = \gamma$, $s_3 = 4$, and for $G_0$ in the point $s_1 = 4$, $s_2 = s_3 = \gamma$, where $\Lambda^F = A(\gamma,\gamma,4)$ and $\Lambda^G = A(4,\gamma,\gamma)$. These complex constants are connected by relation (5). Note, that actually only the real parts of $\Lambda^F$, $\Lambda^G$ enter the relations (8) and (9); therefore altogether only one constant appears.

For the $p$-wave we get the equation

$$G_1(s) = \frac{1}{\pi}\int_4^\infty \frac{ds' \frac{2}{3}\sigma(s')}{s'-s} + \frac{1}{\pi}\int_4^\infty ds' \frac{2}{3}\,\frac{\rho(s') - \lambda(s')}{(s'-\alpha)^2 - 4p'^2q'^2} + \frac{1}{\pi}\int_4^\infty ds' \frac{\sigma(s')}{3}\,\frac{s' - 2\alpha + s}{(s'-\alpha)^2 - 4p'^2q'^2}. \qquad (10)$$

We remark that in (8), (9), (10) the quantity ($pq$) occurs only in even powers, therefore the imaginary part of the amplitude vanishes in the regions where $(pq)^2$ becomes negative.

For $\rho$, $\lambda$ and $\sigma$ the unitarity conditions give

$$\rho = \sqrt{\frac{\nu}{\nu+1}}\left\{G_0 \dot{\pi}_0^0 - \frac{1}{6}F_0\left(\dot{\pi}_0^0 - \dot{\pi}_0^2\right)\right\}, \qquad (11a)$$

$$\lambda = \sqrt{\frac{\nu}{\nu+1}}\, F_0 \dot{\pi}_0^2, \qquad (11b)$$

$$\frac{2}{3}\sigma = \sqrt{\frac{\nu}{\nu+1}}\, G_1 \dot{\pi}_1^1. \qquad (11c)$$

The $\pi_\ell^T$ are the $\pi\pi$- scattering partial-wave amplitudes for isospin T.

The assumed isospin selection rule $|\Delta T| = \frac{1}{2}$ gives the following relations between the amplitudes determing $\tau$- and $\tau'$- decay:

$$F_0 = 2g_0, \qquad (12a)$$

$$G_0 = f_0 + g_0, \qquad (12b)$$

$$G_1 = -g_1. \qquad (12c)$$

The equations (8), (9), (10) can be written in the form of dispersion relations in the $\nu$-plane. For $F_0(\nu)$ we get for example

$$F_0(\nu) = \Lambda^F + \frac{\nu}{\pi}\int_0^\infty \frac{d\nu'\,\lambda(\nu')}{\nu'(\nu'-\nu)} + \frac{\nu}{\pi}\int_0^{0,7} d\nu'\,\frac{\rho(\alpha'+2p'q') + \left\{\frac{3}{2}\alpha' - 2 - 2\nu' - p'q'\right\}\sigma(\alpha'+2p'q')}{\nu'(\nu'-\nu)} +$$

$$+\left\{\int_{-\infty}^{-1}+\int_{0}^{0,1}\right\}d\nu'\,\frac{\rho(\alpha'-2p'q')+\left\{\tfrac{1}{2}\alpha'-2-2\nu'+p'q'\right\}\sigma(\alpha'-2p'q')}{\nu'(\nu'-\nu)}\,, \tag{13}$$

where $\alpha=\alpha(\nu')$ , $p'=p(\nu')$ , etc.

Table I shows the variation of $\nu^{\pm}=\alpha'\pm 2p'q'$ within the limits of integration of $\nu'$ .

| $\nu'$ | $-\infty$ | $-1,65$ | $1$ | $0$ | $0,5$ | $0,7$ |
|---|---|---|---|---|---|---|
| $\nu^{+}$ | $+\infty$ | $+4,3$ | $+\infty$ | 0,5 | 1 | 0,15 |
| $\nu^{-}$ | – | – | – | 0,5 | 0 | 0,15 |

Table I

We see, that the region $1<\nu^{+}<4,3$ gives no contribution to the dispersion relations. Therefore a possible $p$-wave resonance of the $\pi\pi$ - interaction, which is expected approximately in this region, will practically be of little importance for $F_0$ and $G_0$ . It should be further remembered, that the region of physical $\tau$- decay is limited by $\nu\simeq 0,1$ ; therefore amplitudes with higher values of $\nu^{+}$ will have only a limited influence on the decay. But despite an expected weak influence of the $p$-waves it may be important to take them into account for a test of solutions of the $\pi\pi$- integral equations.

### 3. $s$-wave equations and effective range approximation

A system of integral equations for $s$-waves only can obtained by neglecting the $p$-wave terms in (8), (9). However, in order to get a first numerical estimate it is more convenient, to use a system of integral equations obtained from (5) by integrating along the lines $z_i=0$ . Therefore, instead of (6), (7) we choose the following definition for the $s$- waves:

$$A(\nu,z=0)=A_0(\nu). \tag{14}$$

This leads to the relations

$$\mathrm{Re}\,F_0(\nu)=\lambda+P\frac{1}{\pi}\int_0^{\infty}d\nu'\,\lambda(\nu')\left\{\frac{1}{\nu'-\nu}-\frac{1}{\nu'-\nu_0}\right\}+P\frac{2}{\pi}\int_0^{\infty}d\nu'\,\rho(\nu')\left\{\frac{1}{\nu'-\frac{1-\nu}{2}}-\frac{1}{\nu'-\nu_0}\right\}, \tag{15a}$$

$$\mathrm{Re}\,G_0(\nu)=\lambda+P\frac{1}{\pi}\int_0^{\infty}d\nu'\,\rho(\nu')\left\{\frac{1}{\nu'-\nu}-\frac{1}{\nu'-\nu_0}\right\}+\frac{P}{\pi}\int_0^{\infty}d\nu'\left\{\rho(\nu')+\lambda(\nu')\right\}\left\{\frac{1}{\nu'-\frac{1-\nu}{2}}-\frac{1}{\nu'-\nu_0}\right\}. \tag{15b}$$

A subtraction was made in the point $\nu_1 = \nu_2 = \nu_3 = \frac{1}{3} = \nu_0$ , where $\lambda = \mathrm{Re}\, A(\nu_0, \nu_0, \nu_0)$ . In the region of physical $\tau$-decay we have

$$\left.\begin{aligned} \mathrm{Im}\, F_0(\nu) &= \lambda(\nu) + 2\rho\left(\tfrac{1-\nu}{2}\right) , && (16a) \\ \mathrm{Im}\, G_0(\nu) &= \rho(\nu) + \rho\left(\tfrac{1-\nu}{2}\right) + \lambda\left(\tfrac{1-\nu}{2}\right) . && (16b) \end{aligned}\right\} \; 0 \le \nu \le 1 .$$

The relations (15), (16) are used for an effective range approximation, which should be a good one in view of the weak energy dependence of the $\tau$-decay. In such a way, Khuri and Treiman/3/ got by comparison with experiment a condition for the s-wave $\pi\pi$-scattering lengths. Their investigations, however, should be refined for example by taking into account the imaginary part of the amplitude.

We start with the following approximation:

$$\sqrt{\frac{\nu}{\nu+1}}\, \Pi_0^T(\nu) = \frac{a_T\sqrt{\nu}}{1 - i a_T\sqrt{\nu}} \approx a_T\sqrt{\nu}\, ; \quad F_0 \approx G_0 \approx \lambda . \tag{17}$$

Putting (17) into (15), (16) we get

$$\frac{1}{\lambda}\mathrm{Re}\, F_0(\nu) = 1 + \frac{a_2}{\pi} Z_1(\nu) + \frac{5a_0 + a_2}{3\pi} Z_2(\nu) , \tag{18a}$$

$$\frac{1}{\lambda}\mathrm{Re}\, G_0(\nu) = 1 + \frac{5a_0 + a_2}{6\pi} Z_1(\nu) + \frac{5a_0 + 7a_2}{6\pi} Z_2(\nu) ; \tag{18b}$$

$$\frac{1}{\lambda}\mathrm{Im}\, F_0(\nu) = a_2\sqrt{\nu} + \frac{5a_0 + a_2}{3}\sqrt{\frac{1-\nu}{2}} , \tag{19a}$$

$$\frac{1}{\lambda}\mathrm{Im}\, G_0(\nu) = \frac{5a_0 + a_2}{6}\sqrt{\nu} + \frac{5a_0 + 7a_2}{6}\sqrt{\frac{1-\nu}{2}} , \tag{19b}$$

where

$$Z_1(\nu) = \sqrt{\nu}\, \ln\frac{1-\sqrt{\nu}}{1+\sqrt{\nu}} - \sqrt{\nu_0}\, \ln\frac{1-\sqrt{\nu_0}}{1+\sqrt{\nu_0}} , \tag{20a}$$

$$Z_2(\nu) = \sqrt{\frac{1-\nu}{2}}\, \ln\frac{1-\sqrt{\frac{1-\nu}{2}}}{1+\sqrt{\frac{1-\nu}{2}}} - \sqrt{\nu_0}\, \ln\frac{1-\sqrt{\nu_0}}{1+\sqrt{\nu_0}} . \tag{20b}$$

With (18), (19) and the approximation

$$\sqrt{\frac{\nu}{\nu+1}}\, \Pi_0^T(\nu) \approx \sqrt{\nu}\, a_T + i\nu a_T^2 \tag{14b}$$

for the $\pi\pi$-amplitudes we obtain an expression for $\rho$ , $\lambda$ which is put into (5). After integration and development in powers of $\nu_i$ , taking into account $\nu_1+\nu_2+\nu_3=1$ , the real part of the decay amplitude takes the form:

$$\frac{\pi}{\lambda}\,\mathrm{Re}\,A = \text{const.} - \frac{\pi}{\alpha} R(a_2,a_0)\cdot \nu_3 . \tag{21}$$

Here

$$\frac{\pi}{\alpha} R(a_2,a_0) = \frac{5}{3}(a_2-a_0) + \frac{5}{3}(a_2^2-a_0^2) - \frac{5}{9\pi}\left(a_2^2+a_2a_0-2a_0^2\right) - \frac{5}{3}\left(a_2^3-a_0^3\right) \tag{22}$$

and

$$\alpha = \frac{1}{4}\left(m^2-1\right)^2 - 1 = 0{,}7 \quad \text{for } m^2 = 13.$$

For $\nu_i$ we have

$$\nu_i = \alpha(1-t_i) \; ; \tag{23}$$

here $t_i$ is the kinetic energy of the $i^{th}$ $\pi$-meson in the rest system of the $\tau$-meson.

With a subtraction at the symmetrical point $t_i = t_0 = \frac{1}{2}$ , where $\lambda = \mathrm{Re}A(t_0)$ , we get for the square of the real part of $A$:

$$\frac{1}{\lambda^2}\left\{\mathrm{Re}\,A(t_3)\right\}^2 = 1 + (2t_3-1)\cdot R(a_2,a_0) + \Gamma^2. \tag{24}$$

Here

$$\Gamma^2 = \frac{1}{4}(2t-1)^2\left\{R(a_2,a_0)\right\}^2. \tag{25}$$

For the imaginary part of the decay amplitude we get with

$$\mathrm{Im}\,A(\nu_1,\nu_2,\nu_3) = \rho(\nu_1) + \rho(\nu_2) + \lambda(\nu_3) \tag{26}$$

and the equations (11), (14b), (18) and (19) the following expression

$$\frac{1}{\lambda}\mathrm{Im}\,A(\nu_1,\nu_2,\nu_3) \equiv I =$$

$$= \left(\sqrt{\nu_1}+\sqrt{\nu_2}\right)\left\{\frac{1}{6}(5a_0+a_2) - \frac{5}{54\pi}(a_2-a_0)(4a_0-a_2)\right\} + \sqrt{\nu_3}\,a_2\left\{1+\frac{5}{9\pi}(a_2-a_0)\right\} +$$

$$+\left(v_1^{1/2}+v_2^{1/2}\right)\left\{\frac{1}{6}\left(5a_0^2+a_2^2\right)+\frac{5}{18\pi}(a_2-a_0)(4a_0-a_2)\right\}+v_3^{1/2}a_2\left\{a_2^2-\frac{5}{3\pi}(a_2-a_0)\right\}+$$

$$+\left(v_1\sqrt{\frac{v_3+v_2}{2}}+v_2\sqrt{\frac{v_3+v_1}{2}}\right)\left\{\frac{5}{9}a_0^2(a_0+2a_2)+\frac{1}{18}a_2^2(5a_0+a_2)\right\}+v_3\sqrt{\frac{v_1+v_2}{2}}\left\{\frac{1}{3}a_2^2(5a_0+a_2)\right\}. \tag{27}$$

With the help of (23), (24)we have for the square of the Feynman amplitude

$$\sigma' = \frac{1}{\lambda^2}|A|^2 = 1+(2t_3-1)R(a_2,a_0)+K(a_2,a_0;v_i), \tag{28}$$

where

$$K(a_2,a_0;v_i) = \Gamma^2 + I^2. \tag{29}$$

The quantity (28) is compared with the following one, which fairly well represents the experimental data/7/:

$$\sigma = |M|^2 = 1+0{,}2(2t_3-1). \tag{30}$$

Between $\sigma$ and $\sigma'$ we have the relation

$$\sigma = \frac{\sigma'}{1+I^2(v_0)} . \tag{31}$$

In Fig. 2 the quantities $R(a_2,a_0)=0{,}2$ and $\mathrm{Im}\,A(v_i=\frac{1}{3})=0$ are shown as functions of $a_2$, $a_0$. If we put the values $a_2=-0.3$ and $a_0=-1$ which were proposed in/3/ into our relations, we obtain $R(-0{,}3;-1)=-0{,}35$ and

$$\frac{1}{\lambda^2}(\mathrm{Im}A)^2 = \begin{cases}2{,}6\\ 1{,}6\\ 0{,}6\end{cases};\quad \sigma = \begin{cases}0{,}5\\ 1\\ 0{,}6\end{cases}\quad \text{for}\quad \begin{cases}v_3=0;\ v_1=v_2=0{,}5\ ,\\ v_3=v_1=v_2=\frac{1}{3}\ ,\\ v_3=0{,}7;\ v_1=v_2=0{,}15.\end{cases} \tag{32}$$

Note, that according to (30) the experimental energy–dependence of $\sigma$ for the energies in (32) has the form 1.2; 1; 0.8; therefore the above–mentioned values of $a_2$, $a_0$ are not suitable to reproduce the experimental dates of the $\tau$–decay.

Agreement with the experimental energy–dependence can be achieved for example with the values $a_2=0.2$, $a_0=-0.3$. We obtain $R(0.2;\ -0.3)=0.16$ and

$$\frac{1}{\lambda^2}(\mathrm{Im}A)^2 = \begin{cases}0{,}07\\ 0{,}02\\ 0{,}00\end{cases};\qquad \sigma = \begin{cases}1{,}2\ ,\\ 1\ ,\\ 0{,}8\ .\end{cases} \tag{33}$$

The following table gives examples which characterize the dependence of the theoretical value of $\sigma$ on the scattering lengths $a_2$ , $a_0$ .

| $a_2$ | –1 | –0,3 | –0,333 | –0,2 | 0 | 0 | 0 | 0,1 | 0,1 | 0,15 |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_0$ | –0,465 | –1 | –0,135 | –0,7 | –0,5 | –0,3 | 0 | –0,65 | –0,45 | –0,4 |
| $\sigma$ $\nu_3=0;\ \nu_1=\nu_2=0,5$ | 0,64 | 0,45 | 0,89 | 1,21 | 1,23 | 1,08 | 1 | 1,41 | 1,28 | 1,28 |
| $\sigma$ $\nu_3=0,7;\ \nu_1=\nu_2=0,15$ | 1,41 | 0,61 | 1,05 | 0,68 | 0,79 | 0,85 | 1 | 0,71 | 0,79 | 0,81 |

| $a_2$ | 0,2 | 0,2 | 0,25 | 0,3 | 0,333 | 0,333 | 0,4 | 0,5 | 0,6 | 0,7 |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_0$ | –0,3 | 0,2 | –0,35 | –0,5 | –0,2 | 0,135 | –0,3 | 0 | –0,8 | 0,11 |
| $\sigma$ $\nu_3=0;\ \nu_1=\nu_2=0,5$ | 1,22 | 1,20 | 1,32 | 1,13 | 1,23 | 1,0 | 1,36 | 1,11 | 1,45 | 0,95 |
| $\sigma$ $\nu_3=0,7;\ \nu_1=\nu_2=0,15$ | 0,82 | 0,87 | 0,78 | 0,76 | 0,83 | 0,78 | 0,79 | 0,95 | 0,49 | 1,03 |

Table 2

$\sigma$ as a function of $a_2$ , $a_0$ ( by definition $\sigma(\nu_1=\nu_2=\nu_3=\frac{1}{3})=1$ ).

Allowing for an uncertainty of 40% in the determination of the energy–dependent part of $\sigma$ , one finds that the values of $a_2$ , $a_0$ within the hatched region of Fig.2 reproduce the experimental data with sufficient accuracy.

The values given for $\sigma$ refer to equal energies $\nu_1$ , $\nu_2$ . We remark that for $\nu_1 \neq \nu_2$ in the hatched region values are obtained, which differ little from the former ones, e.g. for $a_2$ = 0.2; $a_0$ = –0.3:

$$\sigma(\nu_3=0;\ \nu_1=0,3;\ \nu_2=0,7) = 1,24 ,$$

$$\sigma(\nu_3=\tfrac{1}{3};\ \nu_1=\tfrac{2}{3};\ \nu_2=0) = 1,00 , \qquad (34)$$

$$\sigma(\nu_3=0,7;\ \nu_1=0,3;\ \nu_2=0) = 0,84 .$$

Our values for the scattering–lengths reproduce also the data of the $\pi$– meson–spectra in reference/8/. With $a_2$ = 0,2; $a_0$ = – 0.3 we get for the $\pi^+$– spectrum

$$W(t_1) = \begin{cases} 0.92 \\ 1 \\ 1.11 \end{cases} \quad \text{for} \quad \begin{cases} \nu_1=0;\ \nu_2=\nu_3=0,5 , \\ \nu_i=\frac{1}{3} , \\ \nu_1=0,7;\ \nu_2=\nu_3=0,15 . \end{cases} \qquad (35)$$

The $\pi\pi$-scattering-lengths $a_2$, $a_0$ thus obtained from $\tau$-decay lead to certain consequences for the relative magnitude of the partial-waves of the $\pi\pi$ interaction. From the integral-equations for $\pi\pi$-scattering of reference/4/ the following relation between $a_2$ and $a_0$ can be derived:

$$2a_0 - 5a_2 = \frac{1}{\pi}\int_0^\infty \frac{d\nu}{\nu(\nu+1)} \left\{ 2\,\mathrm{Im}\,\Pi_0^0(\nu) + 9\,\mathrm{Im}\,\Pi_1^1(\nu) - 5\,\Pi_0^2(\nu) \right\}. \tag{36}$$

For all pairs of $a_2$ and $a_0$ reproducing the experimental $\tau$-spectrum, the combination ($2a_0 - 5a_2$) is negative.

This statement is not necessarily in contradiction with the possibility of a $p$-wave-resonance, but it should hint at the existence of a ($T=2$)-resonance in the $\pi\pi$ interaction.

## References


1. L.Okun. Proceedings of the 1960 Annual Conference of High Energy Physics, Rochester.
2. S.Fubini, R.Stroffolini. Nuovo Cim., XYII, 2, 263 (1960).
3. N.Khuri, S.Treiman. Phys. Rev., 119, 1115 (1960).
4. Сянь Дин-чан, Хэ Цзо-сю, В.Целлнер. ЖЭТФ, 39, 1668 (1960).
5. M.Cini, S.Fubini. Ann. Phys., 3, 352 (1960).
6. R.Sawyer, K.Wali. Phys. Rev., 119, 1429 (1960).
7. M.Gell-Mann, A.Rosenfeld. Annual Review of Nuclear Science, vol., 7, p.463.
8. S.McKenna, et al. Nuovo Cim., 10, 763 (1958).


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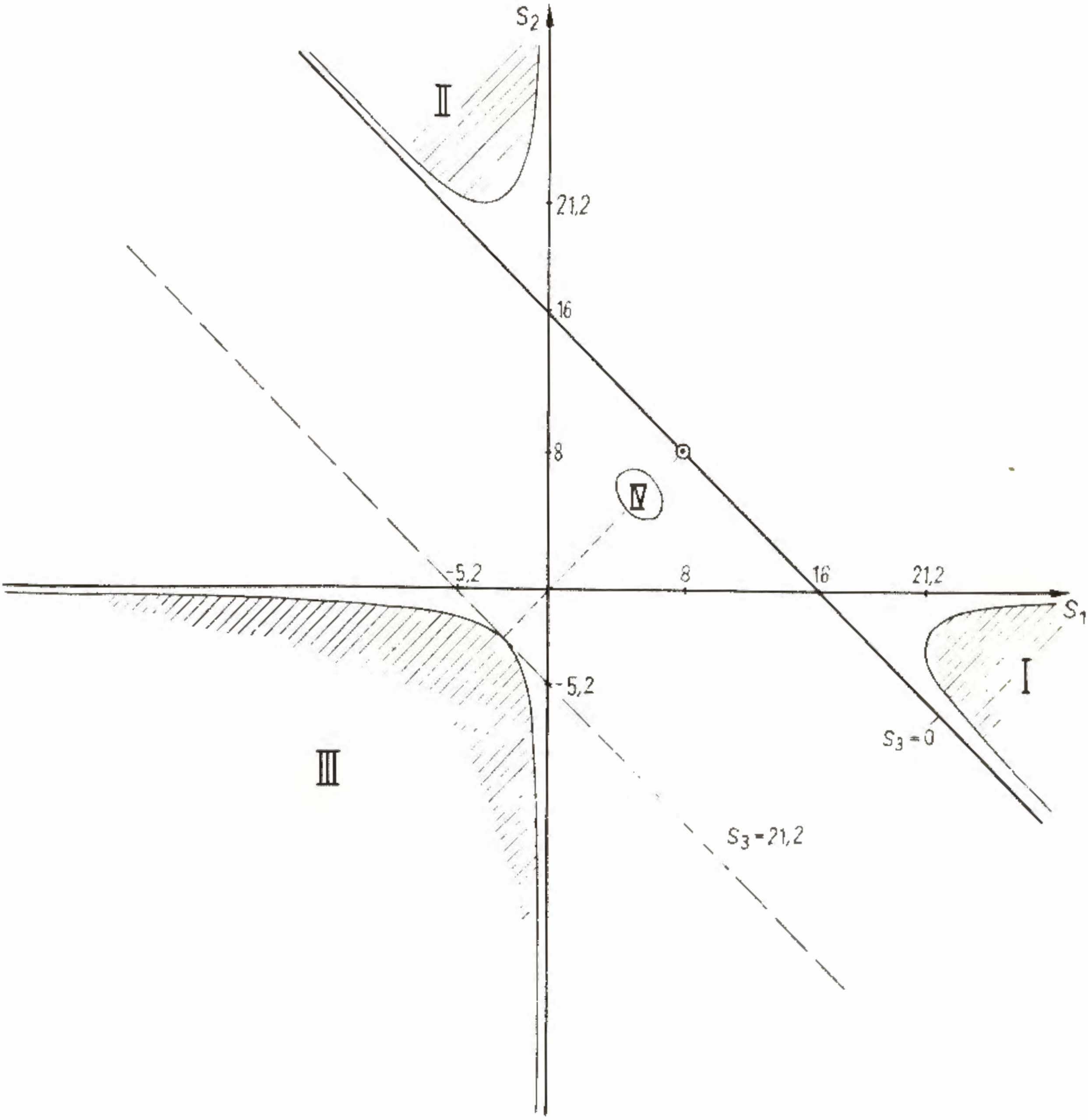
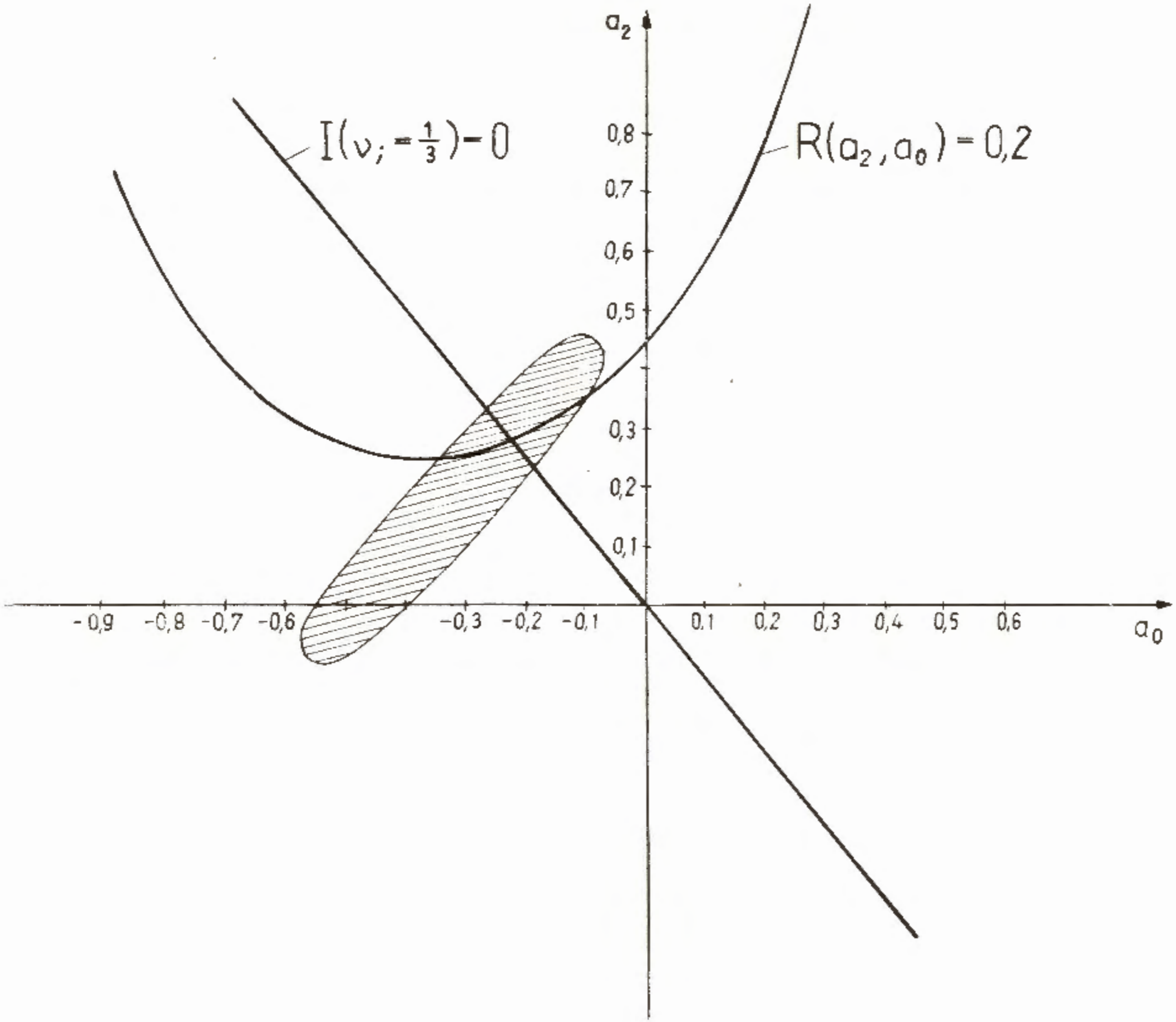



*Fig. 1*



$a_2$
$I(\nu_i = \frac{1}{3}) = 0$
$R(a_2, a_0) = 0{,}2$
0,8
0,7
0,6
0,5
0,3
0,2
0,1
-0,9
-0,8
-0,7
-0,6
-0,3
-0,2
-0,1
0,1
0,2
0,3
0,4
0,5
0,6
$a_0$


*Fig. 2*